Computer algebra independent integration tests

1-Algebraic-functions/1.1-Binomial-products/1.1.1-Linear/1.1.1.4-a+b-x-^m-c+d-x-^n-e+f-x-^p-g+h-x-^q

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [159]. This is test number [15].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 99.37 (158)	% 0.63 (1)
Mathematica	% 97.48 (155)	% 2.52 (4)
Maple	% 80.50 (128)	% 19.50 (31)
Maxima	% 24.53 (39)	% 75.47 (120)
Fricas	% 30.19 (48)	% 69.81 (111)
Sympy	% 25.16 (40)	% 74.84 (119)
Giac	% 26.42 (42)	% 73.58 (117)
Mupad	% 30.82 (49)	% 69.18 (110)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

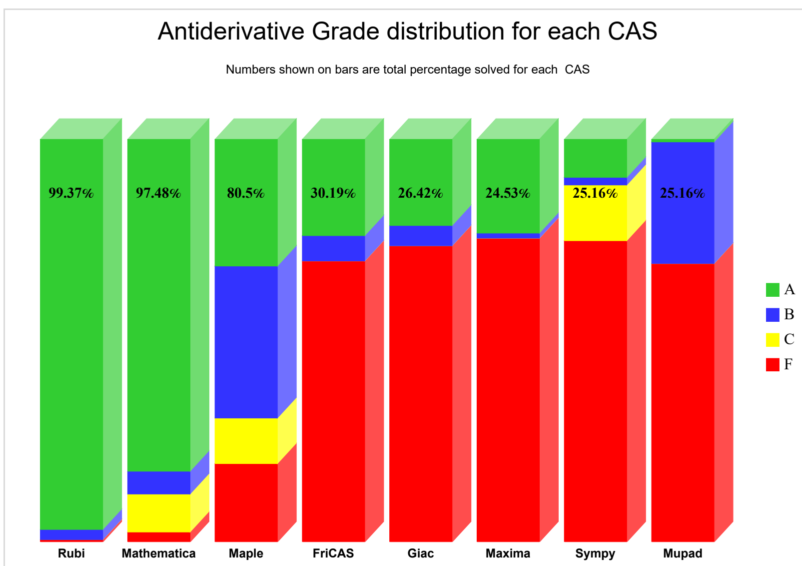
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

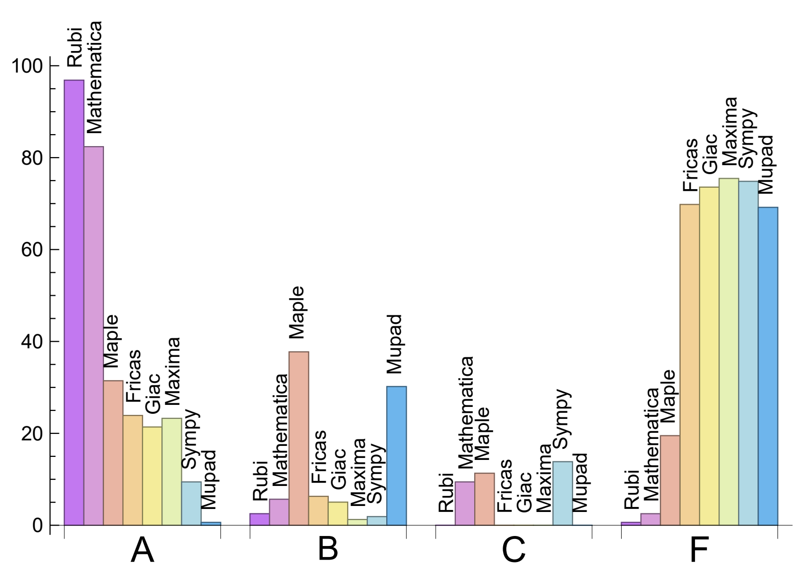
System	% A grade	% B grade	% C grade	% F grade
Rubi	96.86	2.52	0.00	0.63
Mathematica	82.39	5.66	9.43	2.52
Maple	31.45	37.74	11.32	19.50
Maxima	23.27	1.26	0.00	75.47
Fricas	23.90	6.29	0.00	69.81
Sympy	9.43	1.89	13.84	74.84
Giac	21.38	5.03	0.00	73.58
Mupad	0.63	30.19	0.00	69.18

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	1	100.00 %	0.00 %	0.00 %
Mathematica	4	100.00 %	0.00 %	0.00 %
Maple	31	100.00 %	0.00 %	0.00 %
Maxima	120	95.00 %	0.00 %	5.00 %
Fricas	111	87.39 %	12.61 %	0.00 %
Sympy	119	38.66 %	54.62 %	6.72 %
Giac	117	88.89 %	6.84 %	4.27 %
Mupad	110	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

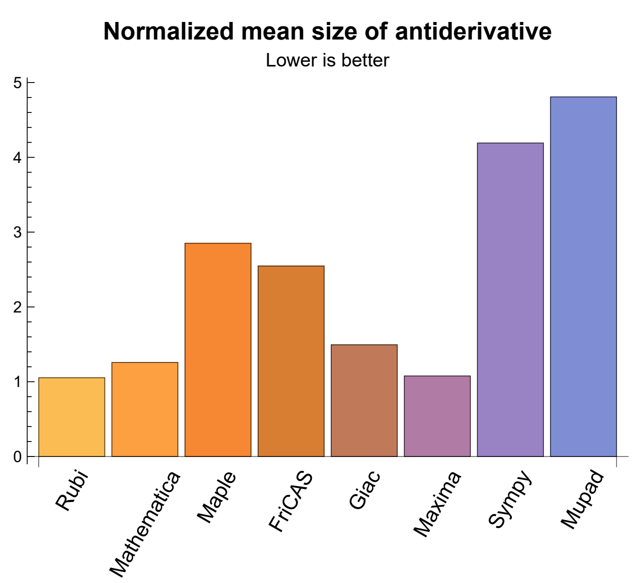
1.3 Performance

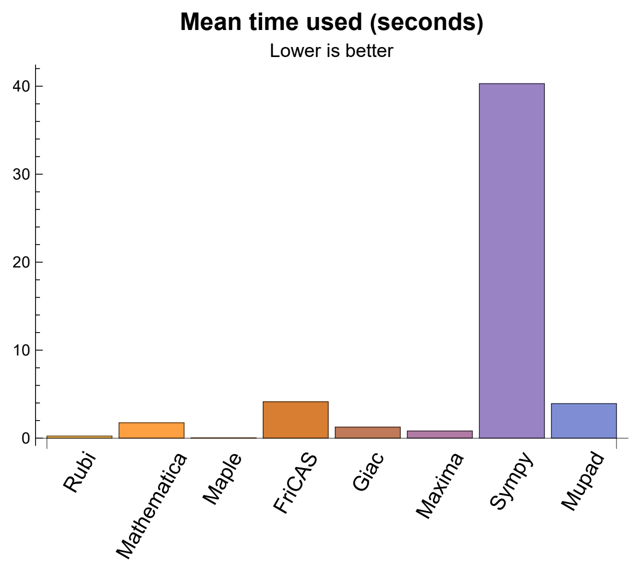
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.24	223.98	1.05	191.50	1.00
Mathematica	1.75	366.78	1.26	153.00	0.90
Maple	0.03	1062.70	2.85	196.00	1.81
Maxima	0.81	99.97	1.08	84.00	1.05
Fricas	4.14	428.58	2.55	87.00	1.44
Sympy	40.29	451.55	4.19	217.00	2.90
Giac	1.25	166.81	1.49	108.50	1.35
Mupad	3.92	693.47	4.81	244.00	2.38

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{143}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {136}

Mathematica {77, 78, 79, 80, 81, 82, 85, 86, 87, 88, 89, 93, 94, 95, 96, 99, 101, 102, 107, 111, 132, 136, 146, 154, 155, 156, 157}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
```

```
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))
```

```
except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

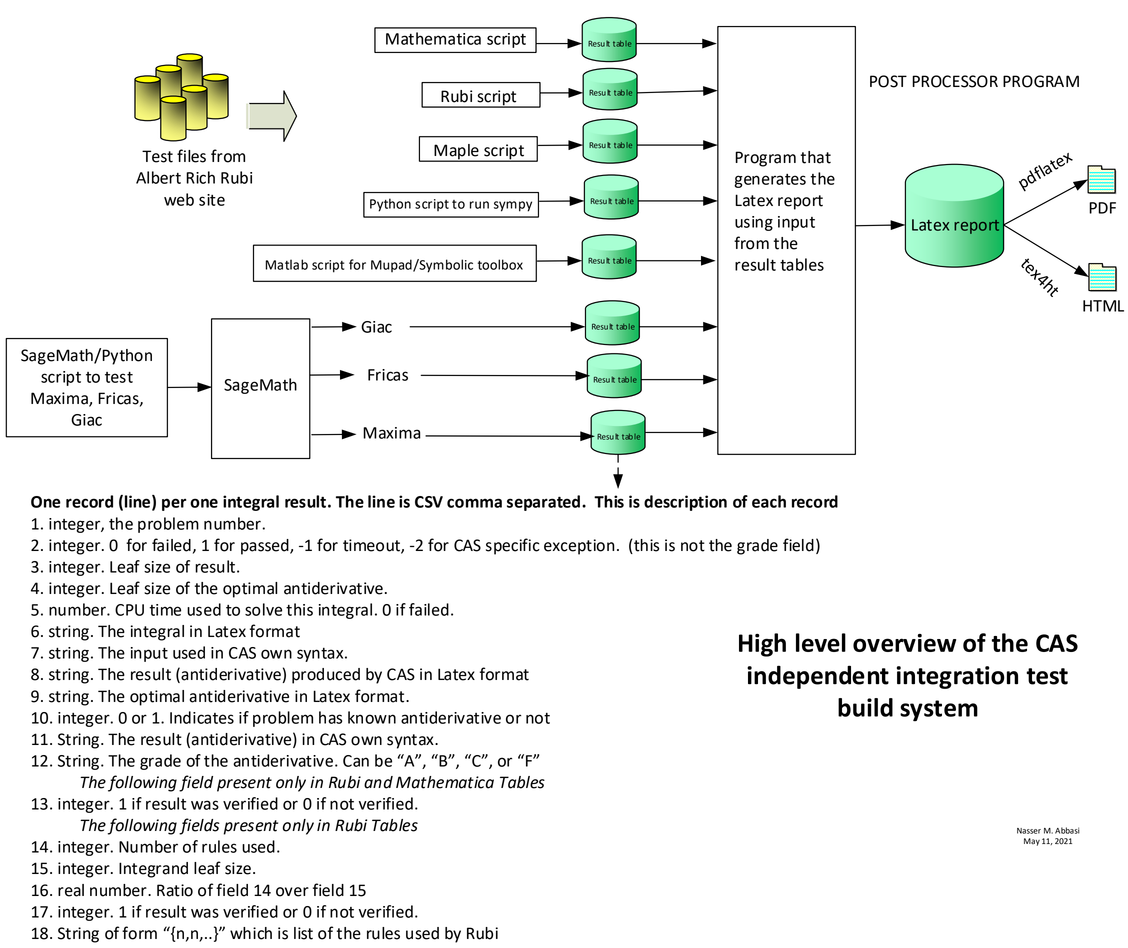
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 158, 159 }

B grade: { 97, 155, 156, 157 }

C grade: { }

F grade: { 111 }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 35, 36, 37, 38, 39, 40, 41, 42, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 60, 61, 62, 63, 64, 65, 66, 67, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 98, 100, 101, 102, 103, 104, 105, 106, 109, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 133, 134, 135, 136, 137, 138, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 157, 158, 159 }

B grade: { 54, 97, 99, 107, 108, 110, 111, 155, 156 }

C grade: { 33, 34, 43, 58, 59, 68, 69, 70, 71, 72, 73, 74, 75, 76, 132 }

F grade: { 139, 140, 141, 144 }

2.1.3 Maple

A grade: { 1, 4, 5, 6, 7, 8, 9, 10, 11, 13, 15, 16, 17, 18, 19, 20, 21, 27, 28, 29, 30, 31, 32, 35, 36, 37, 38, 39, 44, 45, 46, 47, 48, 51, 52, 53, 55, 58, 60, 61, 62, 63, 65, 70, 88, 95, 102, 104, 109, 143 }

B grade: { 2, 3, 12, 14, 33, 34, 40, 41, 42, 43, 49, 50, 56, 57, 59, 66, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 96, 97, 98, 99, 100, 101, 103, 105, 106, 107, 108, 110, 111, 119, 130, 131, 134, 135 }

C grade: { 22, 23, 24, 25, 26, 54, 64, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159 }

F grade: { 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 132, 133, 136, 137, 138, 139, 140, 141, 142, 144, 145, 146, 147, 148 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 15, 16, 17, 18, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 143, 149, 150, 151, 152, 153, 154, 156, 157, 158, 159 }

B grade: { 119, 155 }

C grade: { }

F grade: { 12, 13, 14, 19, 20, 21, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 144, 145, 146, 147, 148 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 15, 16, 17, 18, 19, 22, 23, 24, 25, 27, 28, 29, 30, 31, 32, 143, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159 }

B grade: { 13, 14, 20, 21, 26, 119, 130, 131, 134, 135 }

C grade: { }

F grade: { 5, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 132, 133, 136, 137, 138, 139, 140, 141, 142, 144, 145, 146, 147, 148 }

2.1.6 Sympy

A grade: { 1, 2, 6, 7, 8, 9, 10, 11, 12, 15, 16, 17, 18, 19, 119 }

B grade: { 3, 13, 20 }

C grade: { 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159 }

F grade: { 4, 5, 14, 21, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 31, 32, 143, 149, 150, 154, 155, 156, 157 }

B grade: { 5, 27, 28, 29, 30, 119, 158, 159 }

C grade: { }

F grade: { 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 144, 145, 146, 147, 148, 151, 152, 153 }

2.1.8 Mupad

A grade: { 143 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 119, 130, 131, 134, 135, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159 }

C grade: { }

F grade: { 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 132, 133, 136, 137, 138, 139, 140, 141, 142, 144, 145, 146, 147, 148 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	112	109	108	142	148	150	115
normalized size	1	1.00	1.00	0.97	0.96	1.27	1.32	1.34	1.03
time (sec)	N/A	0.158	0.052	0.001	0.445	0.605	0.094	1.121	2.383

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	123	246	162	163	146	208	174
normalized size	1	1.00	0.98	1.95	1.29	1.29	1.16	1.65	1.38
time (sec)	N/A	0.211	0.088	0.005	0.443	0.944	0.580	1.189	2.537

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	85	196	104	117	507	112	105
normalized size	1	1.00	1.01	2.33	1.24	1.39	6.04	1.33	1.25
time (sec)	N/A	0.086	0.062	0.009	0.439	0.790	20.494	1.251	2.970

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	102	179	134	160	0	162	127
normalized size	1	1.00	0.94	1.66	1.24	1.48	0.00	1.50	1.18
time (sec)	N/A	0.110	0.082	0.010	0.443	152.519	0.000	1.137	4.168

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	164	164	310	0	0	363	317
normalized size	1	1.00	1.01	1.01	1.90	0.00	0.00	2.23	1.94
time (sec)	N/A	0.212	0.201	0.010	0.489	0.000	0.000	1.189	6.622

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	19	19	20	22	19
normalized size	1	1.00	1.00	0.87	0.83	0.83	0.87	0.96	0.83
time (sec)	N/A	0.011	0.006	0.007	0.430	1.013	0.153	1.251	0.076

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	33	34	34	53	32	31	29
normalized size	1	1.00	0.77	0.79	0.79	1.23	0.74	0.72	0.67
time (sec)	N/A	0.037	0.024	0.009	0.433	1.079	0.161	1.176	0.123

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	205	301	239	649	274	338	413
normalized size	1	1.00	0.90	1.33	1.05	2.86	1.21	1.49	1.82
time (sec)	N/A	0.257	0.275	0.010	0.991	1.309	37.787	1.405	0.161

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	145	176	152	405	167	201	263
normalized size	1	1.00	0.99	1.21	1.04	2.77	1.14	1.38	1.80
time (sec)	N/A	0.098	0.183	0.010	0.963	1.150	27.596	1.362	2.621

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	81	89	91	219	92	105	136
normalized size	1	1.00	1.05	1.16	1.18	2.84	1.19	1.36	1.77
time (sec)	N/A	0.024	0.159	0.008	0.979	1.287	25.991	1.328	0.091

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	55	46	60	111	54	57	45
normalized size	1	1.00	1.02	0.85	1.11	2.06	1.00	1.06	0.83
time (sec)	N/A	0.017	0.048	0.007	0.979	1.268	5.984	1.244	0.072

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	101	196	0	449	97	112	2368
normalized size	1	1.00	1.00	1.94	0.00	4.45	0.96	1.11	23.45
time (sec)	N/A	0.114	0.122	0.016	0.000	1.650	27.334	1.352	2.871

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	124	192	0	1018	1204	142	1827
normalized size	1	1.00	0.98	1.51	0.00	8.02	9.48	1.12	14.39
time (sec)	N/A	0.109	0.439	0.019	0.000	1.105	133.870	1.423	0.599

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	260	424	0	2216	0	300	4852
normalized size	1	1.00	1.25	2.04	0.00	10.65	0.00	1.44	23.33
time (sec)	N/A	0.274	0.602	0.020	0.000	2.642	0.000	1.394	4.543

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	204	301	238	641	274	338	413
normalized size	1	1.00	0.90	1.33	1.05	2.84	1.21	1.50	1.83
time (sec)	N/A	0.252	0.281	0.011	0.974	0.792	37.946	1.394	2.528

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	146	176	152	403	167	201	263
normalized size	1	1.00	1.01	1.21	1.05	2.78	1.15	1.39	1.81
time (sec)	N/A	0.094	0.179	0.010	0.980	0.958	26.173	1.368	0.089

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	87	89	90	217	92	105	136
normalized size	1	1.00	1.13	1.16	1.17	2.82	1.19	1.36	1.77
time (sec)	N/A	0.024	0.132	0.009	0.979	1.076	25.966	1.262	2.487

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	55	46	60	111	54	57	45
normalized size	1	1.00	1.02	0.85	1.11	2.06	1.00	1.06	0.83
time (sec)	N/A	0.016	0.050	0.007	0.974	0.934	6.083	1.201	0.069

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	100	103	0	450	100	112	2355
normalized size	1	1.00	0.99	1.02	0.00	4.46	0.99	1.11	23.32
time (sec)	N/A	0.119	0.227	0.016	0.000	1.120	24.251	1.219	2.819

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	122	137	0	1008	1149	142	1814
normalized size	1	1.00	0.95	1.07	0.00	7.88	8.98	1.11	14.17
time (sec)	N/A	0.116	0.196	0.017	0.000	1.263	142.242	1.323	2.952

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	259	221	0	2211	0	301	4839
normalized size	1	1.00	1.26	1.08	0.00	10.79	0.00	1.47	23.60
time (sec)	N/A	0.279	0.532	0.019	0.000	2.479	0.000	1.401	4.632

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	89	132	105	65	484	63	345
normalized size	1	1.00	0.80	1.19	0.95	0.59	4.36	0.57	3.11
time (sec)	N/A	0.045	0.089	0.036	0.974	0.879	35.797	1.245	7.778

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	81	111	83	57	393	53	269
normalized size	1	1.00	0.93	1.28	0.95	0.66	4.52	0.61	3.09
time (sec)	N/A	0.034	0.031	0.015	0.993	1.224	25.601	1.341	5.922

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	73	90	61	49	269	40	191
normalized size	1	1.00	1.16	1.43	0.97	0.78	4.27	0.63	3.03
time (sec)	N/A	0.023	0.030	0.016	0.962	0.636	20.771	1.275	4.527

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	61	70	41	43	133	28	118
normalized size	1	1.00	1.65	1.89	1.11	1.16	3.59	0.76	3.19
time (sec)	N/A	0.013	0.026	0.019	0.955	0.954	11.713	1.239	3.447

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	53	69	41	47	71	44	47
normalized size	1	1.00	1.83	2.38	1.41	1.62	2.45	1.52	1.62
time (sec)	N/A	0.016	0.025	0.019	0.953	0.977	25.620	1.233	2.985

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	29	25	42	27	107	88	24
normalized size	1	1.00	0.64	0.56	0.93	0.60	2.38	1.96	0.53
time (sec)	N/A	0.010	0.013	0.004	0.958	0.924	15.238	1.271	2.747

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	37	33	62	35	189	130	32
normalized size	1	1.00	0.51	0.45	0.85	0.48	2.59	1.78	0.44
time (sec)	N/A	0.019	0.016	0.005	0.962	0.617	17.839	1.242	2.731

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	45	41	84	43	274	175	40
normalized size	1	1.00	0.46	0.42	0.87	0.44	2.82	1.80	0.41
time (sec)	N/A	0.027	0.018	0.007	0.973	0.706	23.154	1.406	2.766

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	53	49	106	51	359	217	48
normalized size	1	1.00	0.44	0.40	0.88	0.42	2.97	1.79	0.40
time (sec)	N/A	0.038	0.019	0.006	0.966	0.933	33.865	1.315	2.829

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	48	44	21	40	117	43	65
normalized size	1	1.00	1.23	1.13	0.54	1.03	3.00	1.10	1.67
time (sec)	N/A	0.007	0.016	0.018	0.962	0.818	35.801	1.269	4.080

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	48	44	21	40	117	43	444
normalized size	1	1.00	1.23	1.13	0.54	1.03	3.00	1.10	11.38
time (sec)	N/A	0.011	0.006	0.005	0.976	0.821	72.778	1.291	5.273

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	309	624	0	0	0	0	-1
normalized size	1	1.00	2.13	4.30	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.108	1.511	0.049	0.000	0.943	0.000	0.000	0.000

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	312	940	0	0	0	0	-1
normalized size	1	1.00	1.41	4.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.199	2.249	0.045	0.000	1.085	0.000	0.000	0.000

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	135	160	0	0	0	0	-1
normalized size	1	1.00	0.48	0.57	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.389	0.436	0.053	0.000	1.045	0.000	0.000	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	130	155	0	0	0	0	-1
normalized size	1	1.00	0.53	0.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.300	0.283	0.013	0.000	1.161	0.000	0.000	0.000

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	125	150	0	0	0	0	-1
normalized size	1	1.00	0.65	0.78	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.077	0.246	0.013	0.000	0.923	0.000	0.000	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	120	145	0	0	0	0	-1
normalized size	1	1.00	0.74	0.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.180	0.012	0.000	0.920	0.000	0.000	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	139	183	0	0	0	0	-1
normalized size	1	1.00	0.76	1.01	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.213	0.818	0.024	0.000	1.092	0.000	0.000	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	130	320	0	0	0	0	-1
normalized size	1	1.00	0.69	1.69	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.211	0.815	0.029	0.000	0.987	0.000	0.000	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	134	461	0	0	0	0	-1
normalized size	1	1.00	0.59	2.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.309	0.750	0.031	0.000	0.721	0.000	0.000	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	139	602	0	0	0	0	-1
normalized size	1	1.00	0.53	2.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.397	0.787	0.033	0.000	1.102	0.000	0.000	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	570	570	1820	3678	0	0	0	0	-1
normalized size	1	1.00	3.19	6.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.286	14.191	0.072	0.000	0.000	0.000	0.000	0.000

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	130	155	0	0	0	0	-1
normalized size	1	1.00	0.53	0.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.297	0.389	0.026	0.000	1.043	0.000	0.000	0.000

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	125	150	0	0	0	0	-1
normalized size	1	1.00	0.61	0.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.213	0.318	0.015	0.000	0.822	0.000	0.000	0.000

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	120	145	0	0	0	0	-1
normalized size	1	1.00	0.74	0.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.206	0.013	0.000	0.936	0.000	0.000	0.000

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	115	140	0	0	0	0	-1
normalized size	1	1.00	0.88	1.07	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.195	0.012	0.000	0.930	0.000	0.000	0.000

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	95	76	0	0	0	0	-1
normalized size	1	1.00	0.63	0.50	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.121	0.418	0.015	0.000	0.940	0.000	0.000	0.000

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	130	320	0	0	0	0	-1
normalized size	1	1.00	0.69	1.69	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.214	0.659	0.018	0.000	0.858	0.000	0.000	0.000

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	135	461	0	0	0	0	-1
normalized size	1	1.00	0.60	2.05	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.301	0.580	0.019	0.000	0.915	0.000	0.000	0.000

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	125	150	0	0	0	0	-1
normalized size	1	1.00	0.61	0.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.212	0.373	0.028	0.000	1.055	0.000	0.000	0.000

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	120	145	0	0	0	0	-1
normalized size	1	1.00	0.72	0.87	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.145	0.300	0.017	0.000	0.913	0.000	0.000	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	115	140	0	0	0	0	-1
normalized size	1	1.00	0.88	1.07	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.193	0.015	0.000	0.944	0.000	0.000	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	111	55	0	0	0	0	-1
normalized size	1	1.00	2.36	1.17	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.015	0.364	0.014	0.000	0.893	0.000	0.000	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	70	56	0	0	0	0	-1
normalized size	1	1.00	0.68	0.54	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.096	0.424	0.017	0.000	1.259	0.000	0.000	0.000

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	130	320	0	0	0	0	-1
normalized size	1	1.00	0.69	1.69	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.210	0.691	0.022	0.000	0.953	0.000	0.000	0.000

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	142	461	0	0	0	0	-1
normalized size	1	1.00	0.63	2.05	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.310	0.461	0.023	0.000	1.042	0.000	0.000	0.000

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	202	382	0	0	0	0	-1
normalized size	1	1.00	0.69	1.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.504	1.770	0.051	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	1198	968	0	0	0	0	-1
normalized size	1	1.00	2.67	2.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.672	7.744	0.033	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	125	150	0	0	0	0	-1
normalized size	1	1.00	0.62	0.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.216	0.530	0.030	0.000	1.404	0.000	0.000	0.000

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	120	145	0	0	0	0	-1
normalized size	1	1.00	0.73	0.88	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.153	0.372	0.019	0.000	1.406	0.000	0.000	0.000

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	115	140	0	0	0	0	-1
normalized size	1	1.00	0.89	1.09	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.257	0.020	0.000	1.255	0.000	0.000	0.000

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	187	57	0	0	0	0	-1
normalized size	1	1.00	1.91	0.58	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.037	0.478	0.019	0.000	1.218	0.000	0.000	0.000

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	79	36	0	0	0	0	-1
normalized size	1	1.00	1.65	0.75	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.014	0.112	0.016	0.000	1.072	0.000	0.000	0.000

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	99	37	0	0	0	0	-1
normalized size	1	1.00	1.94	0.73	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.070	0.487	0.018	0.000	1.015	0.000	0.000	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	130	320	0	0	0	0	-1
normalized size	1	1.00	0.69	1.69	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.220	0.703	0.023	0.000	0.970	0.000	0.000	0.000

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	142	461	0	0	0	0	-1
normalized size	1	1.00	0.63	2.05	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.310	0.460	0.021	0.000	0.816	0.000	0.000	0.000

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	180	552	0	0	0	0	-1
normalized size	1	1.00	1.31	4.03	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.599	0.027	0.000	0.892	0.000	0.000	0.000

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	319	559	0	0	0	0	-1
normalized size	1	1.00	1.12	1.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.169	1.974	0.039	0.000	1.116	0.000	0.000	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	226	223	0	0	0	0	-1
normalized size	1	1.00	1.37	1.35	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.374	1.405	0.030	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	393	393	322	2842	0	0	0	0	-1
normalized size	1	1.00	0.82	7.23	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.624	4.051	0.085	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	875	875	721	17330	0	0	0	0	-1
normalized size	1	1.00	0.82	19.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.339	15.768	0.283	0.000	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	203	184	0	0	0	0	-1
normalized size	1	1.00	2.74	2.49	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.160	0.905	0.105	0.000	0.000	0.000	0.000	0.000

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	203	181	0	0	0	0	-1
normalized size	1	1.00	2.74	2.45	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.176	0.168	0.028	0.000	0.000	0.000	0.000	0.000

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	218	212	0	0	0	0	-1
normalized size	1	1.00	2.53	2.47	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.160	0.858	0.147	0.000	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	218	205	0	0	0	0	-1
normalized size	1	1.00	2.53	2.38	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.175	0.162	0.030	0.000	0.000	0.000	0.000	0.000

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	471	471	350	895	0	0	0	0	-1
normalized size	1	1.00	0.74	1.90	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.665	4.599	0.081	0.000	1.272	0.000	0.000	0.000

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	429	429	345	890	0	0	0	0	-1
normalized size	1	1.00	0.80	2.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.542	4.331	0.023	0.000	0.876	0.000	0.000	0.000

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	340	885	0	0	0	0	-1
normalized size	1	1.00	0.87	2.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.429	4.267	0.020	0.000	1.181	0.000	0.000	0.000

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	347	880	0	0	0	0	-1
normalized size	1	1.00	0.99	2.51	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.324	3.069	0.035	0.000	1.261	0.000	0.000	0.000

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	330	875	0	0	0	0	-1
normalized size	1	1.00	0.95	2.51	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.319	2.977	0.046	0.000	0.953	0.000	0.000	0.000

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	366	1149	0	0	0	0	-1
normalized size	1	1.00	0.94	2.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.429	3.226	0.047	0.000	0.934	0.000	0.000	0.000

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	251	973	0	0	0	0	-1
normalized size	1	1.00	0.76	2.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.395	2.458	0.048	0.000	0.641	0.000	0.000	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	370	370	259	1160	0	0	0	0	-1
normalized size	1	1.00	0.70	3.14	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.515	2.806	0.051	0.000	0.742	0.000	0.000	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	429	429	345	890	0	0	0	0	-1
normalized size	1	1.00	0.80	2.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.526	3.852	0.042	0.000	0.831	0.000	0.000	0.000

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	340	885	0	0	0	0	-1
normalized size	1	1.00	0.87	2.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.412	3.771	0.024	0.000	0.753	0.000	0.000	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	347	880	0	0	0	0	-1
normalized size	1	1.00	0.99	2.51	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.316	4.214	0.022	0.000	0.807	0.000	0.000	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	365	365	318	875	0	0	0	0	-1
normalized size	1	1.00	0.87	2.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.216	1.289	0.026	0.000	0.881	0.000	0.000	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	326	870	0	0	0	0	-1
normalized size	1	1.00	1.17	3.12	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.195	2.467	0.029	0.000	0.756	0.000	0.000	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	246	786	0	0	0	0	-1
normalized size	1	1.00	0.85	2.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.320	1.950	0.030	0.000	0.947	0.000	0.000	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	251	973	0	0	0	0	-1
normalized size	1	1.00	0.76	2.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.398	1.868	0.031	0.000	0.752	0.000	0.000	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	370	370	258	1160	0	0	0	0	-1
normalized size	1	1.00	0.70	3.14	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.521	2.451	0.035	0.000	0.705	0.000	0.000	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	340	885	0	0	0	0	-1
normalized size	1	1.00	0.87	2.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.421	2.617	0.041	0.000	0.956	0.000	0.000	0.000

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	349	880	0	0	0	0	-1
normalized size	1	1.00	0.99	2.51	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.322	3.341	0.027	0.000	0.811	0.000	0.000	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	365	365	347	875	0	0	0	0	-1
normalized size	1	1.00	0.95	2.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.202	1.472	0.026	0.000	0.894	0.000	0.000	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	170	172	0	0	0	0	-1
normalized size	1	1.00	1.68	1.70	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.037	0.588	0.028	0.000	0.817	0.000	0.000	0.000

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	B	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	195	237	330	0	0	0	0	-1
normalized size	1	3.25	3.95	5.50	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.131	1.860	0.030	0.000	0.759	0.000	0.000	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	248	786	0	0	0	0	-1
normalized size	1	1.00	0.86	2.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.296	1.916	0.032	0.000	0.878	0.000	0.000	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	721	721	6667	18077	0	0	0	0	-1
normalized size	1	1.00	9.25	25.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.675	15.046	0.157	0.000	0.000	0.000	0.000	0.000

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	208	206	4590	0	0	0	0	-1
normalized size	1	1.29	1.28	28.51	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.093	5.206	0.128	0.000	3.356	0.000	0.000	0.000

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	347	880	0	0	0	0	-1
normalized size	1	1.00	0.99	2.51	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.319	2.606	0.041	0.000	0.714	0.000	0.000	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	469	469	347	875	0	0	0	0	-1
normalized size	1	1.00	0.74	1.87	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.280	1.323	0.029	0.000	0.654	0.000	0.000	0.000

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	95	170	0	0	0	0	-1
normalized size	1	1.00	0.95	1.70	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.038	0.171	0.026	0.000	0.781	0.000	0.000	0.000

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	90	134	0	0	0	0	-1
normalized size	1	1.00	1.27	1.89	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.163	0.026	0.000	0.760	0.000	0.000	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	270	237	599	0	0	0	0	-1
normalized size	1	1.38	1.22	3.07	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.183	1.691	0.032	0.000	0.589	0.000	0.000	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	246	786	0	0	0	0	-1
normalized size	1	1.00	0.85	2.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.303	1.826	0.035	0.000	0.699	0.000	0.000	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	968	968	6638	16526	0	0	0	0	-1
normalized size	1	1.00	6.86	17.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.882	14.158	0.102	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	584	2465	0	0	0	0	-1
normalized size	1	1.00	2.56	10.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.150	5.576	0.060	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	198	227	270	0	0	0	0	-1
normalized size	1	1.23	1.41	1.68	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	1.337	0.066	0.000	5.871	0.000	0.000	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	429	429	3247	4660	0	0	0	0	-1
normalized size	1	1.00	7.57	10.86	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.256	14.142	0.113	0.000	0.947	0.000	0.000	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	B	B	F	F	F(-1)	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	786	0	7075	21102	0	0	0	0	-1
normalized size	1	0.00	9.00	26.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.010	17.287	0.253	0.000	6.358	0.000	0.000	0.000

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	285	0	0	0	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.279	0.986	0.261	0.000	0.802	0.000	0.000	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	174	0	0	0	0	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.154	0.427	0.275	0.000	0.718	0.000	0.000	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	153	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.107	0.074	0.252	0.000	0.965	0.000	0.000	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	116	0	0	0	0	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.036	0.247	0.000	0.948	0.000	0.000	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	116	0	0	0	0	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.033	0.248	0.000	0.870	0.000	0.000	0.000

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	170	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.192	0.248	0.000	0.875	0.000	0.000	0.000

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	177	0	0	0	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.151	0.278	0.249	0.000	0.720	0.000	0.000	0.000

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	149	726	474	877	8221	1665	819
normalized size	1	1.00	0.89	4.35	2.84	5.25	49.23	9.97	4.90
time (sec)	N/A	0.131	0.231	0.010	0.542	0.923	8.124	1.028	2.945

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	120	0	0	0	0	0	-1
normalized size	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.198	0.277	0.000	0.939	0.000	0.000	0.000

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	115	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.079	0.253	0.000	0.799	0.000	0.000	0.000

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	193	0	0	0	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.195	0.369	0.260	0.000	0.854	0.000	0.000	0.000

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	104	0	0	0	0	0	-1
normalized size	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.231	0.250	0.000	0.722	0.000	0.000	0.000

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	195	0	0	0	0	0	-1
normalized size	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.165	0.240	0.244	0.000	0.958	0.000	0.000	0.000

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	195	0	0	0	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.155	0.265	0.238	0.000	0.918	0.000	0.000	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	189	0	0	0	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.138	0.218	0.240	0.000	1.104	0.000	0.000	0.000

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	221	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.186	0.208	0.242	0.000	0.910	0.000	0.000	0.000

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	205	198	0	0	0	0	0	-1
normalized size	1	1.01	0.98	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.107	0.231	0.289	0.000	0.997	0.000	0.000	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	237	0	0	0	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.160	0.322	0.260	0.000	0.943	0.000	0.000	0.000

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	362	220	894	0	1659	0	0	1895
normalized size	1	1.00	0.61	2.47	0.00	4.58	0.00	0.00	5.23
time (sec)	N/A	0.362	0.478	0.010	0.000	1.079	0.000	0.000	4.487

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	507	507	279	2343	0	3441	0	0	3720
normalized size	1	1.00	0.55	4.62	0.00	6.79	0.00	0.00	7.34
time (sec)	N/A	0.586	0.702	0.017	0.000	1.177	0.000	0.000	6.752

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	815	803	3579	0	0	0	0	0	-1
normalized size	1	0.99	4.39	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.435	44.242	0.240	0.000	0.990	0.000	0.000	0.000

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	572	566	422	0	0	0	0	0	-1
normalized size	1	0.99	0.74	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.662	1.907	0.237	0.000	1.052	0.000	0.000	0.000

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	360	227	906	0	1608	0	0	1890
normalized size	1	0.99	0.63	2.50	0.00	4.43	0.00	0.00	5.21
time (sec)	N/A	0.397	0.548	0.010	0.000	1.051	0.000	0.000	4.277

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	186	181	509	0	905	0	0	869
normalized size	1	0.99	0.96	2.71	0.00	4.81	0.00	0.00	4.62
time (sec)	N/A	0.098	0.128	0.010	0.000	0.870	0.000	0.000	3.407

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	177	190	199	0	0	0	0	0	-1
normalized size	1	1.07	1.12	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.399	0.247	0.000	0.600	0.000	0.000	0.000

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	220	174	0	0	0	0	0	-1
normalized size	1	0.94	0.75	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.127	0.140	0.254	0.000	0.664	0.000	0.000	0.000

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	184	0	0	0	0	0	-1
normalized size	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.217	0.226	0.257	0.000	1.030	0.000	0.000	0.000

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	530	530	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.179	4.532	0.361	0.000	3.375	0.000	0.000	0.000

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	393	393	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.493	1.529	0.251	0.000	3.521	0.000	0.000	0.000

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.210	0.834	0.256	0.000	2.275	0.000	0.000	0.000

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	121	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.083	0.133	0.000	0.000	1.056	0.000	0.000	0.000

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.008	0.252	0.262	0.000	0.811	0.000	0.000	0.000

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.212	0.646	0.252	0.000	1.193	0.000	0.000	0.000

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	208	0	0	0	0	0	-1
normalized size	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.208	0.326	0.287	0.000	1.238	0.000	0.000	0.000

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	215	0	0	0	0	0	-1
normalized size	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.147	0.435	0.253	0.000	1.486	0.000	0.000	0.000

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	261	223	0	0	0	0	0	-1
normalized size	1	0.99	0.85	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.227	0.271	0.250	0.000	2.173	0.000	0.000	0.000

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	558	558	508	0	0	0	0	0	-1
normalized size	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.981	1.942	0.253	0.000	2.547	0.000	0.000	0.000

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	57	139	87	78	313	101	244
normalized size	1	1.00	0.72	1.76	1.10	0.99	3.96	1.28	3.09
time (sec)	N/A	0.142	0.066	0.043	0.967	0.787	82.403	1.339	7.441

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	45	117	57	67	282	76	232
normalized size	1	1.00	0.71	1.86	0.90	1.06	4.48	1.21	3.68
time (sec)	N/A	0.061	0.035	0.020	0.970	0.537	49.786	1.315	6.988

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	96	57	81	245	0	122
normalized size	1	1.00	1.00	2.00	1.19	1.69	5.10	0.00	2.54
time (sec)	N/A	0.184	0.060	0.028	0.969	0.614	55.199	0.000	3.924

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	97	57	84	221	0	114
normalized size	1	1.00	1.00	2.02	1.19	1.75	4.60	0.00	2.38
time (sec)	N/A	0.183	0.060	0.023	0.971	0.844	49.853	0.000	3.741

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	56	108	98	65	218	0	312
normalized size	1	1.00	0.79	1.52	1.38	0.92	3.07	0.00	4.39
time (sec)	N/A	0.188	0.050	0.022	0.967	0.702	80.293	0.000	5.854

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	87	151	149	137	100	73	308	105	318
normalized size	1	1.74	1.71	1.57	1.15	0.84	3.54	1.21	3.66
time (sec)	N/A	0.146	0.349	0.026	0.434	0.578	78.825	1.303	12.354

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	B	C	B	A	C	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	52	135	126	120	90	61	277	80	312
normalized size	1	2.60	2.42	2.31	1.73	1.17	5.33	1.54	6.00
time (sec)	N/A	0.071	0.225	0.018	0.426	0.762	48.292	1.253	12.400

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	B	C	A	A	C	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	55	135	128	95	56	73	240	71	118
normalized size	1	2.45	2.33	1.73	1.02	1.33	4.36	1.29	2.15
time (sec)	N/A	0.184	0.418	0.023	0.962	0.599	47.403	1.295	3.975

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	C	A	A	C	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	55	135	89	96	56	82	216	83	118
normalized size	1	2.45	1.62	1.75	1.02	1.49	3.93	1.51	2.15
time (sec)	N/A	0.182	0.185	0.023	0.965	0.642	45.925	1.347	3.859

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	129	82	103	61	69	212	145	316
normalized size	1	1.55	0.99	1.24	0.73	0.83	2.55	1.75	3.81
time (sec)	N/A	0.189	0.144	0.023	0.972	0.589	74.795	1.350	9.891

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	171	94	123	86	90	219	197	304
normalized size	1	1.47	0.81	1.06	0.74	0.78	1.89	1.70	2.62
time (sec)	N/A	0.221	0.133	0.023	0.983	0.626	129.782	1.348	9.436

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [72] had the largest ratio of [.3429]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.00	21	0.048
2	A	2	1	1.00	23	0.043
3	A	2	1	1.00	25	0.040
4	A	2	1	1.00	27	0.037
5	A	2	1	1.00	29	0.034
6	A	2	1	1.00	17	0.059
7	A	3	2	1.00	22	0.091
8	A	6	5	1.00	25	0.200
9	A	5	5	1.00	25	0.200
10	A	4	4	1.00	23	0.174
11	A	4	4	1.00	18	0.222
12	A	6	4	1.00	25	0.160
13	A	6	4	1.00	25	0.160
14	A	7	5	1.00	25	0.200
15	A	6	5	1.00	25	0.200
16	A	5	5	1.00	25	0.200
17	A	4	4	1.00	23	0.174
18	A	4	4	1.00	18	0.222
19	A	6	5	1.00	25	0.200
20	A	6	5	1.00	25	0.200
21	A	7	6	1.00	25	0.240
22	A	8	6	1.00	26	0.231
23	A	7	6	1.00	26	0.231
24	A	6	6	1.00	24	0.250
25	A	4	4	1.00	23	0.174
26	A	5	5	1.00	26	0.192
27	A	3	3	1.00	26	0.115
28	A	4	4	1.00	26	0.154
29	A	5	4	1.00	26	0.154
30	A	6	4	1.00	26	0.154
31	A	4	4	1.00	24	0.167
32	A	5	5	1.00	36	0.139
33	A	3	3	1.00	45	0.067
34	A	5	5	1.00	39	0.128
35	A	10	8	1.00	35	0.229

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
36	A	9	8	1.00	35	0.229
37	A	8	6	1.00	33	0.182
38	A	7	7	1.00	28	0.250
39	A	10	10	1.00	35	0.286
40	A	10	10	1.00	35	0.286
41	A	11	11	1.00	35	0.314
42	A	12	11	1.00	35	0.314
43	A	12	10	1.00	35	0.286
44	A	9	8	1.00	35	0.229
45	A	8	8	1.00	35	0.229
46	A	7	6	1.00	33	0.182
47	A	6	6	1.00	28	0.214
48	A	9	9	1.00	35	0.257
49	A	10	10	1.00	35	0.286
50	A	11	11	1.00	35	0.314
51	A	8	8	1.00	35	0.229
52	A	7	7	1.00	35	0.200
53	A	6	6	1.00	33	0.182
54	A	2	2	1.00	28	0.071
55	A	6	6	1.00	35	0.171
56	A	10	10	1.00	35	0.286
57	A	11	11	1.00	35	0.314
58	A	8	6	1.00	35	0.171
59	A	11	8	1.00	35	0.229
60	A	8	8	1.00	35	0.229
61	A	7	7	1.00	35	0.200
62	A	7	7	1.00	35	0.200
63	A	5	5	1.00	33	0.152
64	A	2	2	1.00	28	0.071
65	A	3	3	1.00	35	0.086
66	A	10	10	1.00	35	0.286
67	A	11	11	1.00	35	0.314
68	A	3	3	1.00	36	0.083
69	A	6	5	1.00	33	0.152
70	A	4	3	1.00	35	0.086
71	A	10	8	1.00	35	0.229

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
72	A	18	12	1.00	35	0.343
73	A	3	3	1.00	36	0.083
74	A	4	4	1.00	31	0.129
75	A	3	3	1.00	40	0.075
76	A	4	4	1.00	31	0.129
77	A	12	10	1.00	37	0.270
78	A	11	10	1.00	37	0.270
79	A	10	10	1.00	37	0.270
80	A	9	9	1.00	37	0.243
81	A	9	9	1.00	37	0.243
82	A	10	10	1.00	37	0.270
83	A	9	9	1.00	37	0.243
84	A	10	9	1.00	37	0.243
85	A	11	10	1.00	37	0.270
86	A	10	10	1.00	37	0.270
87	A	9	9	1.00	37	0.243
88	A	9	8	1.00	37	0.216
89	A	7	7	1.00	37	0.189
90	A	8	8	1.00	37	0.216
91	A	9	9	1.00	37	0.243
92	A	10	9	1.00	37	0.243
93	A	10	10	1.00	37	0.270
94	A	9	9	1.00	37	0.243
95	A	9	8	1.00	37	0.216
96	A	2	2	1.00	37	0.054
97	B	5	5	3.25	37	0.135
98	A	8	8	1.00	37	0.216
99	A	7	7	1.00	37	0.189
100	A	2	2	1.29	37	0.054
101	A	9	9	1.00	37	0.243
102	A	12	10	1.00	37	0.270
103	A	2	2	1.00	37	0.054
104	A	2	2	1.00	37	0.054
105	A	8	7	1.38	37	0.189
106	A	8	8	1.00	37	0.216
107	A	10	8	1.00	37	0.216

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
108	A	2	2	1.00	37	0.054
109	A	2	2	1.23	37	0.054
110	A	5	5	1.00	37	0.135
111	F	0	0	N/A	0	N/A
112	A	8	3	1.00	25	0.120
113	A	6	3	1.00	25	0.120
114	A	4	2	1.00	25	0.080
115	A	3	2	1.00	23	0.087
116	A	3	2	1.00	22	0.091
117	A	5	3	1.00	25	0.120
118	A	6	3	1.00	25	0.120
119	A	2	1	1.00	23	0.043
120	A	2	2	1.00	25	0.080
121	A	3	2	1.00	27	0.074
122	A	5	2	1.00	29	0.069
123	A	6	3	1.00	25	0.120
124	A	3	3	1.00	25	0.120
125	A	3	3	1.00	29	0.103
126	A	3	3	1.00	27	0.111
127	A	3	3	1.00	29	0.103
128	A	3	3	1.01	29	0.103
129	A	3	3	1.00	29	0.103
130	A	3	3	1.00	29	0.103
131	A	4	3	1.00	29	0.103
132	A	10	9	0.99	31	0.290
133	A	9	8	0.99	31	0.258
134	A	3	3	0.99	29	0.103
135	A	3	3	0.99	24	0.125
136	A	5	5	1.07	27	0.185
137	A	7	5	0.94	29	0.172
138	A	7	4	1.00	29	0.138
139	A	31	5	1.00	29	0.172
140	A	15	5	1.00	29	0.172
141	A	7	4	1.00	27	0.148
142	A	3	3	1.00	22	0.136
143	A	0	0	0.00	0	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
144	A	7	4	1.00	33	0.121
145	A	7	4	1.00	34	0.118
146	A	5	5	1.00	34	0.147
147	A	3	3	0.99	34	0.088
148	A	4	3	1.00	34	0.088
149	A	4	4	1.00	31	0.129
150	A	4	4	1.00	30	0.133
151	A	7	7	1.00	33	0.212
152	A	7	7	1.00	33	0.212
153	A	6	6	1.00	33	0.182
154	A	5	5	1.74	30	0.167
155	B	5	5	2.60	29	0.172
156	B	8	8	2.45	32	0.250
157	B	8	8	2.45	32	0.250
158	A	6	6	1.55	32	0.188
159	A	7	7	1.47	32	0.219

Chapter 3

Listing of integrals

3.1 $\int (a+bx)(c+dx)(e+fx)(g+hx)\,dx$

Optimal. Leaf size=112

$$\frac{1}{4}x^4(adfh+b(cfh+deh+dfg))+\frac{1}{3}x^3(a(cfh+deh+dfg)+b(ceh+cfg+deg))+\frac{1}{2}x^2(a(ceh+cfg+deg)+bceg)+acegx$$

```
[Out] a*c*e*g*x+1/2*(b*c*e*g+a*(c*e*h+c*f*g+d*e*g))*x^2+1/3*(b*(c*e*h+c*f*g+d*e*g
)+a*(c*f*h+d*e*h+d*f*g))*x^3+1/4*(a*d*f*h+b*(c*f*h+d*e*h+d*f*g))*x^4+1/5*b*
d*f*h*x^5
```

Rubi [A] time = 0.16, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {142}

$$\frac{1}{4}x^4(adfh+b(cfh+deh+dfg))+\frac{1}{3}x^3(a(cfh+deh+dfg)+b(ceh+cfg+deg))+\frac{1}{2}x^2(a(ceh+cfg+deg)+bceg)+acegx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(c + d*x)*(e + f*x)*(g + h*x),x]
```

```
[Out] a*c*e*g*x + ((b*c*e*g + a*(d*e*g + c*f*g + c*e*h))*x^2)/2 + ((b*(d*e*g + c*
f*g + c*e*h) + a*(d*f*g + d*e*h + c*f*h))*x^3)/3 + ((a*d*f*h + b*(d*f*g + d
*e*h + c*f*h))*x^4)/4 + (b*d*f*h*x^5)/5
```

Rule 142

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c +
d*x)^n*(e + f*x)*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 (IGtQ[m, 0] || IntegersQ[m, n])
```

Rubi steps

$$\int (a+bx)(c+dx)(e+fx)(g+hx)\,dx=\int \Big(aceg+(bceg+a(deg+cfg+ceh))x+(b(deg+cfg+ceh)+a(d$$
$$=acegx+\frac{1}{2}(bceg+a(deg+cfg+ceh))x^2+\frac{1}{3}(b(deg+cfg+ceh)+a($$

Mathematica [A] time = 0.05, size = 112, normalized size = 1.00

$$\frac{1}{4}x^4(adfh+bcfh+bdeh+bdfg)+\frac{1}{3}x^3(acfh+adeh+adfg+bceh+bcfg+bdeg)+\frac{1}{2}x^2(aceh+acfg+adeg+bceg)+ace$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(c + d*x)*(e + f*x)*(g + h*x),x]
```

```
[Out] a*c*e*g*x + ((b*c*e*g + a*d*e*g + a*c*f*g + a*c*e*h)*x^2)/2 + ((b*d*e*g + b
*c*f*g + a*d*f*g + b*c*e*h + a*d*e*h + a*c*f*h)*x^3)/3 + ((b*d*f*g + b*d*e*
h + b*c*f*h + a*d*f*h)*x^4)/4 + (b*d*f*h*x^5)/5
```

fricas [A] time = 0.60, size = 142, normalized size = 1.27

$$\frac{1}{5}x^5hfdb+\frac{1}{4}x^4gfdb+\frac{1}{4}x^4hedb+\frac{1}{4}x^4hfcb+\frac{1}{4}x^4hfda+\frac{1}{3}x^3gedb+\frac{1}{3}x^3gfcb+\frac{1}{3}x^3hecb+\frac{1}{3}x^3gfda+\frac{1}{3}x^3heda+\frac{1}{3}x^3hfc$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g),x, algorithm="fricas")
```

```
[Out] 1/5*x^5*h*f*d*b + 1/4*x^4*g*f*d*b + 1/4*x^4*h*e*d*b + 1/4*x^4*h*f*c*b + 1/4
*x^4*h*f*d*a + 1/3*x^3*g*e*d*b + 1/3*x^3*g*f*c*b + 1/3*x^3*h*e*c*b + 1/3*x^
3*g*f*d*a + 1/3*x^3*h*e*d*a + 1/3*x^3*h*f*c*a + 1/2*x^2*g*e*c*b + 1/2*x^2*g
*e*d*a + 1/2*x^2*g*f*c*a + 1/2*x^2*h*e*c*a + x*g*e*c*a
```

giac [A] time = 1.12, size = 150, normalized size = 1.34

$$\frac{1}{5}bdfhx^5+\frac{1}{4}bdfgx^4+\frac{1}{4}bcfhx^4+\frac{1}{4}adfhx^4+\frac{1}{4}bdhx^4e+\frac{1}{3}bcfgx^3+\frac{1}{3}adfgx^3+\frac{1}{3}acfhx^3+\frac{1}{3}bdgx^3e+\frac{1}{3}bchx^3e+\frac{1}{3}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g),x, algorithm="giac")
```

```
[Out] 1/5*b*d*f*h*x^5 + 1/4*b*d*f*g*x^4 + 1/4*b*c*f*h*x^4 + 1/4*a*d*f*h*x^4 + 1/4
*b*d*h*x^4*e + 1/3*b*c*f*g*x^3 + 1/3*a*d*f*g*x^3 + 1/3*a*c*f*h*x^3 + 1/3*b*
d*g*x^3*e + 1/3*b*c*h*x^3*e + 1/3*a*d*h*x^3*e + 1/2*a*c*f*g*x^2 + 1/2*b*c*g
*x^2*e + 1/2*a*d*g*x^2*e + 1/2*a*c*h*x^2*e + a*c*g*x*e
```

maple [A] time = 0.00, size = 109, normalized size = 0.97

$$\frac{bdfh\,x^5}{5}+acegx+\frac{\left(bdfg+\left(bde+(ad+bc)\,f\right)h\right)x^4}{4}+\frac{\left(\left(bde+(ad+bc)\,f\right)g+\left(acf+(ad+bc)\,e\right)h\right)x^3}{3}+\frac{\left(aceh+\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g),x)
```

```
[Out] 1/5*b*d*f*h*x^5+1/4*(((a*d+b*c)*f+b*d*e)*h+b*d*f*g)*x^4+1/3*((a*c*f+(a*d+b*
c)*e)*h+((a*d+b*c)*f+b*d*e)*g)*x^3+1/2*(a*c*e*h+(a*c*f+(a*d+b*c)*e)*g)*x^2+
a*c*e*g*x
```

maxima [A] time = 0.45, size = 108, normalized size = 0.96

$$\frac{1}{5}bdfhx^5+acegx+\frac{1}{4}\left(bdfg+\left(bde+(bc+ad)f\right)h\right)x^4+\frac{1}{3}\left(\left(bde+(bc+ad)f\right)g+\left(acf+(bc+ad)e\right)h\right)x^3+\frac{1}{2}\left(aceh\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g),x, algorithm="maxima")
```

```
[Out] 1/5*b*d*f*h*x^5 + a*c*e*g*x + 1/4*(b*d*f*g + (b*d*e + (b*c + a*d)*f)*h)*x^4
 + 1/3*((b*d*e + (b*c + a*d)*f)*g + (a*c*f + (b*c + a*d)*e)*h)*x^3 + 1/2*(a
*c*e*h + (a*c*f + (b*c + a*d)*e)*g)*x^2
```

mupad [B] time = 2.38, size = 115, normalized size = 1.03

$$\frac{b\,d\,f\,h\,x^5}{5}+\left(\frac{a\,d\,f\,h}{4}+\frac{b\,c\,f\,h}{4}+\frac{b\,d\,e\,h}{4}+\frac{b\,d\,f\,g}{4}\right)x^4+\left(\frac{a\,c\,f\,h}{3}+\frac{a\,d\,e\,h}{3}+\frac{a\,d\,f\,g}{3}+\frac{b\,c\,e\,h}{3}+\frac{b\,c\,f\,g}{3}+\frac{b\,d\,e\,g}{3}\right)x^3+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)*(g + h*x)*(a + b*x)*(c + d*x),x)
```

```
[Out] x^3*((a*c*f*h)/3 + (a*d*e*h)/3 + (a*d*f*g)/3 + (b*c*e*h)/3 + (b*c*f*g)/3 +
(b*d*e*g)/3) + x^2*((a*c*e*h)/2 + (a*c*f*g)/2 + (a*d*e*g)/2 + (b*c*e*g)/2)
+ x^4*((a*d*f*h)/4 + (b*c*f*h)/4 + (b*d*e*h)/4 + (b*d*f*g)/4) + a*c*e*g*x +
 (b*d*f*h*x^5)/5
```

sympy [A] time = 0.09, size = 148, normalized size = 1.32

$$acegx+\frac{bdfhx^5}{5}+x^4\left(\frac{adfh}{4}+\frac{bcfh}{4}+\frac{bdeh}{4}+\frac{bdfg}{4}\right)+x^3\left(\frac{acfh}{3}+\frac{adeh}{3}+\frac{adfg}{3}+\frac{bceh}{3}+\frac{bcfg}{3}+\frac{bdeg}{3}\right)+x^2\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g),x)
```

```
[Out] a*c*e*g*x + b*d*f*h*x**5/5 + x**4*(a*d*f*h/4 + b*c*f*h/4 + b*d*e*h/4 + b*d*
f*g/4) + x**3*(a*c*f*h/3 + a*d*e*h/3 + a*d*f*g/3 + b*c*e*h/3 + b*c*f*g/3 +
b*d*e*g/3) + x**2*(a*c*e*h/2 + a*c*f*g/2 + a*d*e*g/2 + b*c*e*g/2)
```

3.2 $\int \frac{(a+bx)(c+dx)(e+fx)}{g+hx}\,dx$

Optimal. Leaf size=126

$$-\frac{(bg-ah)(dg-ch)(fg-eh)\log(g+hx)}{h^4}+\frac{x(b(dg-ch)(fg-eh)-ah(-cfh-deh+dfg))}{h^3}+\frac{x^2(adfh-b(-cfh-$$

```
[Out] (b*(-c*h+d*g)*(-e*h+f*g)-a*h*(-c*f*h-d*e*h+d*f*g))*x/h^3+1/2*(a*d*f*h-b*(-c
*f*h-d*e*h+d*f*g))*x^2/h^2+1/3*b*d*f*x^3/h-(-a*h+b*g)*(-c*h+d*g)*(-e*h+f*g)
*ln(h*x+g)/h^4
```

Rubi [A] time = 0.21, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {142}

$$\frac{x^2(adfh-b(-cfh-deh+dfg))}{2h^2}+\frac{x(b(dg-ch)(fg-eh)-ah(-cfh-deh+dfg))}{h^3}-\frac{(bg-ah)(dg-ch)(fg-eh)\,\mathrm{lo}}{h^4}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)*(c + d*x)*(e + f*x))/(g + h*x),x]
```

```
[Out] ((b*(d*g - c*h)*(f*g - e*h) - a*h*(d*f*g - d*e*h - c*f*h))*x)/h^3 + ((a*d*f
*h - b*(d*f*g - d*e*h - c*f*h))*x^2)/(2*h^2) + (b*d*f*x^3)/(3*h) - ((b*g -
a*h)*(d*g - c*h)*(f*g - e*h)*Log[g + h*x])/h^4
```

Rule 142

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c +
d*x)^n*(e + f*x)*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 (IGtQ[m, 0] || IntegersQ[m, n])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)(c+dx)(e+fx)}{g+hx}\,dx &= \int \left(\frac{b(dg-ch)(fg-eh)-ah(dfg-deh-cfh)}{h^3}+\frac{(adfh-b(dfg-deh-cfh))x}{h^2}\right.\\ &= \frac{(b(dg-ch)(fg-eh)-ah(dfg-deh-cfh))x}{h^3}+\frac{(adfh-b(dfg-deh-cfh))x^2}{2h^2}\end{aligned}$$

Mathematica [A] time = 0.09, size = 123, normalized size = 0.98

$$\frac{hx\left(3ah(2cfh+d(2eh-2fg+fhx))+b\left(3ch(2eh-2fg+fhx)+3deh(hx-2g)+df\left(6g^2-3ghx+2h^2x^2\right)\right)\right)-}{6h^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*(c + d*x)*(e + f*x))/(g + h*x),x]
```

```
[Out] (h*x*(3*a*h*(2*c*f*h + d*(-2*f*g + 2*e*h + f*h*x)) + b*(3*d*e*h*(-2*g + h*x
) + 3*c*h*(-2*f*g + 2*e*h + f*h*x) + d*f*(6*g^2 - 3*g*h*x + 2*h^2*x^2))) -
6*(b*g - a*h)*(d*g - c*h)*(f*g - e*h)*Log[g + h*x])/(6*h^4)
```

fricas [A] time = 0.94, size = 163, normalized size = 1.29

$$\frac{2\,bdfh^3x^3-3\left(bdfgh^2-\left(bde+(bc+ad)f\right)h^3\right)x^2+6\left(bdfg^2h-\left(bde+(bc+ad)f\right)gh^2+\left(acf+(bc+ad)e\right)h^3\right)x}{6\,h^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)*(f*x+e)/(h*x+g),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*d*f*h^3*x^3 - 3*(b*d*f*g*h^2 - (b*d*e + (b*c + a*d)*f)*h^3)*x^2 +
6*(b*d*f*g^2*h - (b*d*e + (b*c + a*d)*f)*g*h^2 + (a*c*f + (b*c + a*d)*e)*h^
3)*x - 6*(b*d*f*g^3 - a*c*e*h^3 - (b*d*e + (b*c + a*d)*f)*g^2*h + (a*c*f +
(b*c + a*d)*e)*g*h^2)*log(h*x + g))/h^4
```

giac [A] time = 1.19, size = 208, normalized size = 1.65

$$\frac{2\,bdfh^2x^3 - 3\,bdfghx^2 + 3\,bcfh^2x^2 + 3\,adfh^2x^2 + 3\,bdh^2x^2e + 6\,bdfg^2x - 6\,bcfghx - 6\,adfghx + 6\,acfh^2x -}{6\,h^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)*(f*x+e)/(h*x+g),x, algorithm="giac")
```

```
[Out] 1/6*(2*b*d*f*h^2*x^3 - 3*b*d*f*g*h*x^2 + 3*b*c*f*h^2*x^2 + 3*a*d*f*h^2*x^2
+ 3*b*d*h^2*x^2*e + 6*b*d*f*g^2*x - 6*b*c*f*g*h*x - 6*a*d*f*g*h*x + 6*a*c*f
*h^2*x - 6*b*d*g*h*x*e + 6*b*c*h^2*x*e + 6*a*d*h^2*x*e)/h^3 - (b*d*f*g^3 -
b*c*f*g^2*h - a*d*f*g^2*h + a*c*f*g*h^2 - b*d*g^2*h*e + b*c*g*h^2*e + a*d*g
*h^2*e - a*c*h^3*e)*log(abs(h*x + g))/h^4
```

maple [B] time = 0.00, size = 246, normalized size = 1.95

$$\frac{bdf\,x^3}{3h}+\frac{adf\,x^2}{2h}+\frac{bcf\,x^2}{2h}+\frac{bde\,x^2}{2h}-\frac{bdfg\,x^2}{2h^2}+\frac{ace\ln\left(hx+g\right)}{h}-\frac{acfg\ln\left(hx+g\right)}{h^2}+\frac{acfx}{h}-\frac{adeg\ln\left(hx+g\right)}{h^2}+\frac{adex}{h}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)*(f*x+e)/(h*x+g),x)
```

```
[Out] 1/3*b*d*f*x^3/h+1/2/h*x^2*a*d*f+1/2/h*x^2*b*c*f+1/2/h*x^2*b*d*e-1/2/h^2*x^2
*b*d*f*g+1/h*x*a*c*f+1/h*x*a*d*e-1/h^2*x*a*d*f*g+1/h*x*b*c*e-1/h^2*x*b*c*f*
g-1/h^2*x*b*d*e*g+1/h^3*x*b*d*f*g^2+1/h*ln(h*x+g)*a*c*e-1/h^2*ln(h*x+g)*a*c
*f*g-1/h^2*ln(h*x+g)*a*d*e*g+1/h^3*ln(h*x+g)*a*d*f*g^2-1/h^2*ln(h*x+g)*b*c*
e*g+1/h^3*ln(h*x+g)*b*c*f*g^2+1/h^3*ln(h*x+g)*b*d*e*g^2-1/h^4*ln(h*x+g)*b*d
*f*g^3
```

maxima [A] time = 0.44, size = 162, normalized size = 1.29

$$\frac{2\,bdfh^2x^3 - 3\left(bdfgh - \left(bde + (bc+ad)f\right)h^2\right)x^2 + 6\left(bdfg^2 - \left(bde + (bc+ad)f\right)gh + \left(acf + (bc+ad)e\right)h^2\right)x}{6\,h^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)*(f*x+e)/(h*x+g),x, algorithm="maxima")
```

```
[Out] 1/6*(2*b*d*f*h^2*x^3 - 3*(b*d*f*g*h - (b*d*e + (b*c + a*d)*f)*h^2)*x^2 + 6*
(b*d*f*g^2 - (b*d*e + (b*c + a*d)*f)*g*h + (a*c*f + (b*c + a*d)*e)*h^2)*x)/
h^3 - (b*d*f*g^3 - a*c*e*h^3 - (b*d*e + (b*c + a*d)*f)*g^2*h + (a*c*f + (b*
c + a*d)*e)*g*h^2)*log(h*x + g)/h^4
```

mupad [B] time = 2.54, size = 174, normalized size = 1.38

$$x\left(\frac{a\,c\,f+a\,d\,e+b\,c\,e}{h}-\frac{g\left(\frac{a\,d\,f+b\,c\,f+b\,d\,e}{h}-\frac{b\,d\,f\,g}{h^2}\right)}{h}\right)+x^2\left(\frac{a\,d\,f+b\,c\,f+b\,d\,e}{2\,h}-\frac{b\,d\,f\,g}{2\,h^2}\right)+\frac{\ln\left(g+h\,x\right)\,\left(a\,c\,e\,h^3-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(a + b*x)*(c + d*x))/(g + h*x),x)
```

```
[Out] x*((a*c*f + a*d*e + b*c*e)/h - (g*((a*d*f + b*c*f + b*d*e)/h - (b*d*f*g)/h^
2))/h) + x^2*((a*d*f + b*c*f + b*d*e)/(2*h) - (b*d*f*g)/(2*h^2)) + (log(g +
 h*x)*(a*c*e*h^3 - b*d*f*g^3 - a*c*f*g*h^2 - a*d*e*g*h^2 - b*c*e*g*h^2 + a*
d*f*g^2*h + b*c*f*g^2*h + b*d*e*g^2*h))/h^4 + (b*d*f*x^3)/(3*h)
```

sympy [A] time = 0.58, size = 146, normalized size = 1.16

$$\frac{bdfx^3}{3h}+x^2\left(\frac{adf}{2h}+\frac{bcf}{2h}+\frac{bde}{2h}-\frac{bdfg}{2h^2}\right)+x\left(\frac{acf}{h}+\frac{ade}{h}-\frac{adfg}{h^2}+\frac{bce}{h}-\frac{bcfg}{h^2}-\frac{bdeg}{h^2}+\frac{bdfg^2}{h^3}\right)+\frac{(ah-bg)(ch-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)*(f*x+e)/(h*x+g),x)
```

```
[Out] b*d*f*x**3/(3*h) + x**2*(a*d*f/(2*h) + b*c*f/(2*h) + b*d*e/(2*h) - b*d*f*g/
(2*h**2)) + x*(a*c*f/h + a*d*e/h - a*d*f*g/h**2 + b*c*e/h - b*c*f*g/h**2 -
b*d*e*g/h**2 + b*d*f*g**2/h**3) + (a*h - b*g)*(c*h - d*g)*(e*h - f*g)*log(g
 + h*x)/h**4
```

3.3 $\int \frac{(a+bx)(c+dx)}{(e+fx)(g+hx)} \, dx$

Optimal. Leaf size=84

$$\frac{(be-af)(de-cf)\log(e+fx)}{f^2(fg-eh)} - \frac{(bg-ah)(dg-ch)\log(g+hx)}{h^2(fg-eh)} + \frac{bdx}{fh}$$

```
[Out] b*d*x/f/h+(-a*f+b*e)*(-c*f+d*e)*ln(f*x+e)/f^2/(-e*h+f*g)-(-a*h+b*g)*(-c*h+d
*g)*ln(h*x+g)/h^2/(-e*h+f*g)
```

Rubi [A] time = 0.09, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {142}

$$\frac{(be-af)(de-cf)\log(e+fx)}{f^2(fg-eh)} - \frac{(bg-ah)(dg-ch)\log(g+hx)}{h^2(fg-eh)} + \frac{bdx}{fh}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)*(c + d*x))/((e + f*x)*(g + h*x)),x]
```

```
[Out] (b*d*x)/(f*h) + ((b*e - a*f)*(d*e - c*f)*Log[e + f*x])/(f^2*(f*g - e*h)) -
((b*g - a*h)*(d*g - c*h)*Log[g + h*x])/(h^2*(f*g - e*h))
```

Rule 142

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c +
d*x)^n*(e + f*x)*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 (IGtQ[m, 0] || IntegersQ[m, n])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)(c+dx)}{(e+fx)(g+hx)} \, dx &= \int \left(\frac{bd}{fh} + \frac{(-be+af)(-de+cf)}{f(fg-eh)(e+fx)} + \frac{(-bg+ah)(-dg+ch)}{h(-fg+eh)(g+hx)}\right) dx \\
&= \frac{bdx}{fh} + \frac{(be-af)(de-cf)\log(e+fx)}{f^2(fg-eh)} - \frac{(bg-ah)(dg-ch)\log(g+hx)}{h^2(fg-eh)}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 85, normalized size = 1.01

$$\frac{f(bdhx(fg-eh)-f(bg-ah)(dg-ch)\log(g+hx))+h^2(be-af)(de-cf)\log(e+fx)}{f^2h^2(fg-eh)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*(c + d*x))/((e + f*x)*(g + h*x)),x]
```

```
[Out] ((b*e - a*f)*(d*e - c*f)*h^2*Log[e + f*x] + f*(b*d*h*(f*g - e*h)*x - f*(b*g
 - a*h)*(d*g - c*h)*Log[g + h*x]))/(f^2*h^2*(f*g - e*h))
```

fricas [A] time = 0.79, size = 117, normalized size = 1.39

$$\frac{\Big(bde^2 + acf^2 - (bc+ad)ef\Big)h^2 \log\Big(fx+e\Big) + \Big(bdf^2gh - bdefh^2\Big)x - \Big(bdf^2g^2 + acf^2h^2 - (bc+ad)f^2gh\Big)\log\Big(}{f^3gh^2 - ef^2h^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="fricas")

[Out] ((b*d*e^2 + a*c*f^2 - (b*c + a*d)*e*f)*h^2*log(f*x + e) + (b*d*f^2*g*h - b*d*e*f*h^2)*x - (b*d*f^2*g^2 + a*c*f^2*h^2 - (b*c + a*d)*f^2*g*h)*log(h*x + g))/(f^3*g*h^2 - e*f^2*h^3)

giac [A] time = 1.25, size = 112, normalized size = 1.33

$$\frac{bdx}{fh} + \frac{\left(acf^2 - bcfe - adfe + bde^2\right)\log\left(\left|fx + e\right|\right)}{f^3g - f^2he} - \frac{\left(bdg^2 - bcgh - adgh + ach^2\right)\log\left(\left|hx + g\right|\right)}{fgh^2 - h^3e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="giac")

[Out] b*d*x/(f*h) + (a*c*f^2 - b*c*f*e - a*d*f*e + b*d*e^2)*log(abs(f*x + e))/(f^3*g - f^2*h*e) - (b*d*g^2 - b*c*g*h - a*d*g*h + a*c*h^2)*log(abs(h*x + g))/(f*g*h^2 - h^3*e)

maple [B] time = 0.01, size = 196, normalized size = 2.33

$$-\frac{ac\ln\left(fx+e\right)}{eh-fg} + \frac{ac\ln\left(hx+g\right)}{eh-fg} + \frac{ade\ln\left(fx+e\right)}{\left(eh-fg\right)f} - \frac{adg\ln\left(hx+g\right)}{\left(eh-fg\right)h} + \frac{bce\ln\left(fx+e\right)}{\left(eh-fg\right)f} - \frac{bcg\ln\left(hx+g\right)}{\left(eh-fg\right)h} - \frac{bd\,e^2\ln\left(fx\right.}{\left(eh-fg\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)*(d*x+c)/(f*x+e)/(h*x+g),x)

[Out] b*d*x/f/h+1/(e*h-f*g)*ln(h*x+g)*a*c-1/h/(e*h-f*g)*ln(h*x+g)*a*d*g-1/h/(e*h-f*g)*ln(h*x+g)*b*c*g+1/h^2/(e*h-f*g)*ln(h*x+g)*b*d*g^2-1/(e*h-f*g)*ln(f*x+e)*a*c+1/f/(e*h-f*g)*ln(f*x+e)*a*d*e+1/f/(e*h-f*g)*ln(f*x+e)*b*c*e-1/f^2/(e*h-f*g)*ln(f*x+e)*b*d*e^2

maxima [A] time = 0.44, size = 104, normalized size = 1.24

$$\frac{bdx}{fh} + \frac{\left(bde^2 + acf^2 - (bc + ad)ef\right)\log\left(fx + e\right)}{f^3g - ef^2h} - \frac{\left(bdg^2 + ach^2 - (bc + ad)gh\right)\log\left(hx + g\right)}{fgh^2 - eh^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)*(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="maxima")

[Out] b*d*x/(f*h) + (b*d*e^2 + a*c*f^2 - (b*c + a*d)*e*f)*log(f*x + e)/(f^3*g - e*f^2*h) - (b*d*g^2 + a*c*h^2 - (b*c + a*d)*g*h)*log(h*x + g)/(f*g*h^2 - e*h^3)

mupad [B] time = 2.97, size = 105, normalized size = 1.25

$$\frac{\ln\left(e+f\,x\right)\,\left(a\,c\,f^2-f\,\left(a\,d\,e+b\,c\,e\right)+b\,d\,e^2\right)}{f^3\,g-e\,f^2\,h}+\frac{\ln\left(g+h\,x\right)\,\left(a\,c\,h^2-h\,\left(a\,d\,g+b\,c\,g\right)+b\,d\,g^2\right)}{e\,h^3-f\,g\,h^2}+\frac{b\,d\,x}{f\,h}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b*x)*(c + d*x))/((e + f*x)*(g + h*x)),x)

[Out] (log(e + f*x)*(a*c*f^2 - f*(a*d*e + b*c*e) + b*d*e^2))/(f^3*g - e*f^2*h) + (log(g + h*x)*(a*c*h^2 - h*(a*d*g + b*c*g) + b*d*g^2))/(e*h^3 - f*g*h^2) + (b*d*x)/(f*h)

sympy [B] time = 20.49, size = 507, normalized size = 6.04

$$\frac{bdx}{fh}+\frac{\left(ah-bg\right)\left(ch-dg\right)\log\left(x+\frac{acefh^2+acf^2gh-2adefgh-2bcefgh+bde^2gh+bdefg^2-\frac{e^2fh\left(ah-bg\right)\left(ch-dg\right)}{eh-fg}+\frac{2ef^2g\left(ah-bg\right)\left(ch-dg\right)}{eh-fg}-\frac{f^3g^2(a}{h}}{2acf^2h^2-adefh^2-adf^2gh-bcefh^2-bcf^2gh+bde^2h^2+bdf^2g^2}\right.}{h^2\left(eh-fg\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)/(f*x+e)/(h*x+g),x)
```

```
[Out] b*d*x/(f*h) + (a*h - b*g)*(c*h - d*g)*log(x + (a*c*e*f*h**2 + a*c*f**2*g*h
- 2*a*d*e*f*g*h - 2*b*c*e*f*g*h + b*d*e**2*g*h + b*d*e*f*g**2 - e**2*f*h*(a
*h - b*g)*(c*h - d*g)/(e*h - f*g) + 2*e*f**2*g*(a*h - b*g)*(c*h - d*g)/(e*h
 - f*g) - f**3*g**2*(a*h - b*g)*(c*h - d*g)/(h*(e*h - f*g)))/(2*a*c*f**2*h*
*2 - a*d*e*f*h**2 - a*d*f**2*g*h - b*c*e*f*h**2 - b*c*f**2*g*h + b*d*e**2*h
**2 + b*d*f**2*g**2))/(h**2*(e*h - f*g)) - (a*f - b*e)*(c*f - d*e)*log(x +
(a*c*e*f*h**2 + a*c*f**2*g*h - 2*a*d*e*f*g*h - 2*b*c*e*f*g*h + b*d*e**2*g*h
 + b*d*e*f*g**2 + e**2*h**3*(a*f - b*e)*(c*f - d*e)/(f*(e*h - f*g)) - 2*e*g
*h**2*(a*f - b*e)*(c*f - d*e)/(e*h - f*g) + f*g**2*h*(a*f - b*e)*(c*f - d*e
)/(e*h - f*g))/(2*a*c*f**2*h**2 - a*d*e*f*h**2 - a*d*f**2*g*h - b*c*e*f*h**
2 - b*c*f**2*g*h + b*d*e**2*h**2 + b*d*f**2*g**2))/(f**2*(e*h - f*g))
```

3.4 $\int \frac{a+bx}{(c+dx)(e+fx)(g+hx)}\,dx$

Optimal. Leaf size=108

$$-\frac{(bc-ad)\log(c+dx)}{(de-cf)(dg-ch)}+\frac{(be-af)\log(e+fx)}{(de-cf)(fg-eh)}-\frac{(bg-ah)\log(g+hx)}{(dg-ch)(fg-eh)}$$

```
[Out] -(-a*d+b*c)*ln(d*x+c)/(-c*f+d*e)/(-c*h+d*g)+(-a*f+b*e)*ln(f*x+e)/(-c*f+d*e)
/(-e*h+f*g)-(-a*h+b*g)*ln(h*x+g)/(-c*h+d*g)/(-e*h+f*g)
```

Rubi [A] time = 0.11, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {148}

$$-\frac{(bc-ad)\log(c+dx)}{(de-cf)(dg-ch)}+\frac{(be-af)\log(e+fx)}{(de-cf)(fg-eh)}-\frac{(bg-ah)\log(g+hx)}{(dg-ch)(fg-eh)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/((c + d*x)*(e + f*x)*(g + h*x)),x]
```

```
[Out] -(((b*c - a*d)*Log[c + d*x])/((d*e - c*f)*(d*g - c*h))) + ((b*e - a*f)*Log[
e + f*x])/((d*e - c*f)*(f*g - e*h)) - ((b*g - a*h)*Log[g + h*x])/((d*g - c*
h)*(f*g - e*h))
```

Rule 148

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(
c + d*x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{(c+dx)(e+fx)(g+hx)}\,dx &= \int \left(\frac{d(-bc+ad)}{(de-cf)(dg-ch)(c+dx)}+\frac{f(-be+af)}{(de-cf)(-fg+eh)(e+fx)}+\frac{h(-bg+\ldots}{(dg-ch)(fg-\ldots}\right.\\ &= -\frac{(bc-ad)\log(c+dx)}{(de-cf)(dg-ch)}+\frac{(be-af)\log(e+fx)}{(de-cf)(fg-eh)}-\frac{(bg-ah)\log(g+hx)}{(dg-ch)(fg-eh)}\end{aligned}$$

Mathematica [A] time = 0.08, size = 102, normalized size = 0.94

$$\frac{(bc-ad)\log(c+dx)(fg-eh)-(be-af)(dg-ch)\log(e+fx)+(bg-ah)(de-cf)\log(g+hx)}{(de-cf)(dg-ch)(eh-fg)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/((c + d*x)*(e + f*x)*(g + h*x)),x]
```

```
[Out] ((b*c - a*d)*(f*g - e*h)*Log[c + d*x] - (b*e - a*f)*(d*g - c*h)*Log[e + f*x
] + (d*e - c*f)*(b*g - a*h)*Log[g + h*x])/((d*e - c*f)*(d*g - c*h)*(-(f*g)
+ e*h))
```

fricas [A] time = 152.52, size = 160, normalized size = 1.48

$$-\frac{\left((bc-ad)fg-(bc-ad)eh\right)\log\left(dx+c\right)-\left(\left(bde-adf\right)g-\left(bce-acf\right)h\right)\log\left(fx+e\right)+\left(\left(bde-bcf\right)g-\left(ade-\ldots\right.\right.}{\left(d^2ef-cdf^2\right)g^2-\left(d^2e^2-c^2f^2\right)gh+\left(cde^2-c^2ef\right)h^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="fricas")
```

```
[Out] -(((b*c - a*d)*f*g - (b*c - a*d)*e*h)*log(d*x + c) - ((b*d*e - a*d*f)*g - (
b*c*e - a*c*f)*h)*log(f*x + e) + ((b*d*e - b*c*f)*g - (a*d*e - a*c*f)*h)*lo
g(h*x + g))/((d^2*e*f - c*d*f^2)*g^2 - (d^2*e^2 - c^2*f^2)*g*h + (c*d*e^2 -
 c^2*e*f)*h^2)
```

giac [A] time = 1.14, size = 162, normalized size = 1.50

$$\frac{\left(bcd-ad^2\right)\log\left(|dx+c|\right)}{cd^2fg-c^2dfh-d^3ge+cd^2he}+\frac{\left(af^2-bfe\right)\log\left(\left|fx+e\right|\right)}{cf^3g-df^2ge-cf^2he+dfhe^2}-\frac{\left(bgh-ah^2\right)\log\left(\left|hx+g\right|\right)}{dfg^2h-cfgh^2-dgh^2e+ch^3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="giac")
```

```
[Out] (b*c*d - a*d^2)*log(abs(d*x + c))/(c*d^2*f*g - c^2*d*f*h - d^3*g*e + c*d^2*
h*e) + (a*f^2 - b*f*e)*log(abs(f*x + e))/(c*f^3*g - d*f^2*g*e - c*f^2*h*e +
 d*f*h*e^2) - (b*g*h - a*h^2)*log(abs(h*x + g))/(d*f*g^2*h - c*f*g*h^2 - d*
g*h^2*e + c*h^3*e)
```

maple [A] time = 0.01, size = 179, normalized size = 1.66

$$\frac{ad\ln\left(dx+c\right)}{\left(cf-de\right)\left(ch-dg\right)}-\frac{af\ln\left(fx+e\right)}{\left(cf-de\right)\left(eh-fg\right)}+\frac{ah\ln\left(hx+g\right)}{\left(ch-dg\right)\left(eh-fg\right)}-\frac{bc\ln\left(dx+c\right)}{\left(cf-de\right)\left(ch-dg\right)}+\frac{be\ln\left(fx+e\right)}{\left(cf-de\right)\left(eh-fg\right)}-\frac{b}{\left(ch\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x)
```

```
[Out] 1/(c*f-d*e)/(c*h-d*g)*ln(d*x+c)*a*d-1/(c*f-d*e)/(c*h-d*g)*ln(d*x+c)*b*c+1/(
c*h-d*g)/(e*h-f*g)*ln(h*x+g)*a*h-1/(c*h-d*g)/(e*h-f*g)*ln(h*x+g)*b*g-1/(c*f
-d*e)/(e*h-f*g)*ln(f*x+e)*a*f+1/(c*f-d*e)/(e*h-f*g)*ln(f*x+e)*b*e
```

maxima [A] time = 0.44, size = 134, normalized size = 1.24

$$-\frac{(bc-ad)\log\left(dx+c\right)}{\left(d^2e-cdf\right)g-\left(cde-c^2f\right)h}+\frac{\left(be-af\right)\log\left(fx+e\right)}{\left(def-cf^2\right)g-\left(de^2-cef\right)h}-\frac{\left(bg-ah\right)\log\left(hx+g\right)}{dfg^2+ceh^2-\left(de+cf\right)gh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="maxima")
```

```
[Out] -(b*c - a*d)*log(d*x + c)/((d^2*e - c*d*f)*g - (c*d*e - c^2*f)*h) + (b*e -
a*f)*log(f*x + e)/((d*e*f - c*f^2)*g - (d*e^2 - c*e*f)*h) - (b*g - a*h)*log
(h*x + g)/(d*f*g^2 + c*e*h^2 - (d*e + c*f)*g*h)
```

mupad [B] time = 4.17, size = 127, normalized size = 1.18

$$\frac{\ln\left(e+f\,x\right)\,\left(a\,f-b\,e\right)}{c\,f^2\,g+d\,e^2\,h-c\,e\,f\,h-d\,e\,f\,g}+\frac{\ln\left(g+h\,x\right)\,\left(a\,h-b\,g\right)}{c\,e\,h^2+d\,f\,g^2-c\,f\,g\,h-d\,e\,g\,h}+\frac{\ln\left(c+d\,x\right)\,\left(a\,d-b\,c\right)}{d^2\,e\,g+c^2\,f\,h-c\,d\,e\,h-c\,d\,f\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/((e + f*x)*(g + h*x)*(c + d*x)),x)
```

```
[Out] (log(e + f*x)*(a*f - b*e))/(c*f^2*g + d*e^2*h - c*e*f*h - d*e*f*g) + (log(g
 + h*x)*(a*h - b*g))/(c*e*h^2 + d*f*g^2 - c*f*g*h - d*e*g*h) + (log(c + d*x
)*(a*d - b*c))/(d^2*e*g + c^2*f*h - c*d*e*h - c*d*f*g)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x)
```

```
[Out] Timed out
```

3.5 $\int \frac{1}{(a+bx)(c+dx)(e+fx)(g+hx)} dx$

Optimal. Leaf size=163

$$\frac{b^2 \log(a+bx)}{(bc-ad)(be-af)(bg-ah)} - \frac{d^2 \log(c+dx)}{(bc-ad)(de-cf)(dg-ch)} + \frac{f^2 \log(e+fx)}{(be-af)(de-cf)(fg-eh)} - \frac{h^2 \log(g+hx)}{(bg-ah)(dg-ch)(fg}$$

```
[Out] b^2*ln(b*x+a)/(-a*d+b*c)/(-a*f+b*e)/(-a*h+b*g)-d^2*ln(d*x+c)/(-a*d+b*c)/(-c
*f+d*e)/(-c*h+d*g)+f^2*ln(f*x+e)/(-a*f+b*e)/(-c*f+d*e)/(-e*h+f*g)-h^2*ln(h*
x+g)/(-a*h+b*g)/(-c*h+d*g)/(-e*h+f*g)
```

Rubi [A] time = 0.21, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {180}

$$\frac{b^2 \log(a+bx)}{(bc-ad)(be-af)(bg-ah)} - \frac{d^2 \log(c+dx)}{(bc-ad)(de-cf)(dg-ch)} + \frac{f^2 \log(e+fx)}{(be-af)(de-cf)(fg-eh)} - \frac{h^2 \log(g+hx)}{(bg-ah)(dg-ch)(fg}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)*(e + f*x)*(g + h*x)),x]
```

```
[Out] (b^2*Log[a + b*x])/((b*c - a*d)*(b*e - a*f)*(b*g - a*h)) - (d^2*Log[c + d*x
])/((b*c - a*d)*(d*e - c*f)*(d*g - c*h)) + (f^2*Log[e + f*x])/((b*e - a*f)*
(d*e - c*f)*(f*g - e*h)) - (h^2*Log[g + h*x])/((b*g - a*h)*(d*g - c*h)*(f*g
 - e*h))
```

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rubi steps

$$\int \frac{1}{(a+bx)(c+dx)(e+fx)(g+hx)} dx = \int \left(\frac{b^3}{(bc-ad)(be-af)(bg-ah)(a+bx)} - \frac{d^3}{(bc-ad)(-de+cf)(-dg+}\right.$$
$$= \frac{b^2 \log(a+bx)}{(bc-ad)(be-af)(bg-ah)} - \frac{d^2 \log(c+dx)}{(bc-ad)(de-cf)(dg-ch)} + \frac{f}{(be-af}$$

Mathematica [A] time = 0.20, size = 164, normalized size = 1.01

$$\frac{b^2 \log(a+bx)}{(bc-ad)(be-af)(bg-ah)} - \frac{d^2 \log(c+dx)}{(bc-ad)(cf-de)(ch-dg)} - \frac{f^2 \log(e+fx)}{(be-af)(de-cf)(eh-fg)} - \frac{h^2 \log(g+hx)}{(bg-ah)(dg-ch)(fg}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(c + d*x)*(e + f*x)*(g + h*x)),x]
```

```
[Out] (b^2*Log[a + b*x])/((b*c - a*d)*(b*e - a*f)*(b*g - a*h)) - (d^2*Log[c + d*x
])/((b*c - a*d)*(-(d*e) + c*f)*(-(d*g) + c*h)) - (f^2*Log[e + f*x])/((b*e -
 a*f)*(d*e - c*f)*(-(f*g) + e*h)) - (h^2*Log[g + h*x])/((b*g - a*h)*(d*g -
c*h)*(f*g - e*h))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 1.19, size = 363, normalized size = 2.23

$$-\frac{b^3\log(|bx+a|)}{ab^3cfg-a^2b^2dfg-a^2b^2cfh+a^3bdfh-b^4cge+ab^3dge+ab^3che-a^2b^2dhe}+\frac{d^3}{bc^2d^2fg-acd^3fg-bc^3dfh+ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="giac")
```

```
[Out] -b^3*log(abs(b*x + a))/(a*b^3*c*f*g - a^2*b^2*d*f*g - a^2*b^2*c*f*h + a^3*b
*d*f*h - b^4*c*g*e + a*b^3*d*g*e + a*b^3*c*h*e - a^2*b^2*d*h*e) + d^3*log(a
bs(d*x + c))/(b*c^2*d^2*f*g - a*c*d^3*f*g - b*c^3*d*f*h + a*c^2*d^2*f*h - b
*c*d^3*g*e + a*d^4*g*e + b*c^2*d^2*h*e - a*c*d^3*h*e) + f^3*log(abs(f*x + e
))/(a*c*f^4*g - b*c*f^3*g*e - a*d*f^3*g*e - a*c*f^3*h*e + b*d*f^2*g*e^2 + b
*c*f^2*h*e^2 + a*d*f^2*h*e^2 - b*d*f*h*e^3) - h^3*log(abs(h*x + g))/(b*d*f*
g^3*h - b*c*f*g^2*h^2 - a*d*f*g^2*h^2 + a*c*f*g*h^3 - b*d*g^2*h^2*e + b*c*g
*h^3*e + a*d*g*h^3*e - a*c*h^4*e)
```

maple [A] time = 0.01, size = 164, normalized size = 1.01

$$-\frac{b^2\ln(bx+a)}{(ad-bc)(af-be)(ah-bg)}+\frac{d^2\ln(dx+c)}{(ad-bc)(cf-de)(ch-dg)}-\frac{f^2\ln(fx+e)}{(cf-de)(eh-fg)(af-be)}+\frac{h^2\ln(hx+}{(ch-dg)(ah-bg)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x)
```

```
[Out] d^2/(a*d-b*c)/(c*f-d*e)/(c*h-d*g)*ln(d*x+c)+h^2/(c*h-d*g)/(a*h-b*g)/(e*h-f*
g)*ln(h*x+g)-b^2/(a*d-b*c)/(a*f-b*e)/(a*h-b*g)*ln(b*x+a)-f^2/(c*f-d*e)/(e*h
-f*g)/(a*f-b*e)*ln(f*x+e)
```

maxima [A] time = 0.49, size = 310, normalized size = 1.90

$$\frac{b^2\log(bx+a)}{\left(\left(b^3c-ab^2d\right)e-\left(ab^2c-a^2bd\right)f\right)g-\left(\left(ab^2c-a^2bd\right)e-\left(a^2bc-a^3d\right)f\right)h}-\frac{d^2\log(dx+}{\left(\left(bcd^2-ad^3\right)e-\left(bc^2d-acd^2\right)f\right)g-((}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="maxima")
```

```
[Out] b^2*log(b*x + a)/(((b^3*c - a*b^2*d)*e - (a*b^2*c - a^2*b*d)*f)*g - ((a*b^2
*c - a^2*b*d)*e - (a^2*b*c - a^3*d)*f)*h) - d^2*log(d*x + c)/(((b*c*d^2 - a
*d^3)*e - (b*c^2*d - a*c*d^2)*f)*g - ((b*c^2*d - a*c*d^2)*e - (b*c^3 - a*c^
2*d)*f)*h) + f^2*log(f*x + e)/((b*d*e^2*f + a*c*f^3 - (b*c + a*d)*e*f^2)*g
- (b*d*e^3 + a*c*e*f^2 - (b*c + a*d)*e^2*f)*h) - h^2*log(h*x + g)/(b*d*f*g^
3 - a*c*e*h^3 - (b*d*e + (b*c + a*d)*f)*g^2*h + (a*c*f + (b*c + a*d)*e)*g*h
^2)
```

mupad [B] time = 6.62, size = 317, normalized size = 1.94

$$\frac{b^2\ln(a+bx)}{b^3ceg-a^3dfh-ab^2ceh-ab^2cfg-ab^2deg+a^2bcfh+a^2bdeh+a^2bdfg}+\frac{}{ad^3eg-bc^3fh-acd^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e + f*x)*(g + h*x)*(a + b*x)*(c + d*x)),x)
```

```
[Out] (b^2*log(a + b*x))/(b^3*c*e*g - a^3*d*f*h - a*b^2*c*e*h - a*b^2*c*f*g - a*b
^2*d*e*g + a^2*b*c*f*h + a^2*b*d*e*h + a^2*b*d*f*g) + (d^2*log(c + d*x))/(a
*d^3*e*g - b*c^3*f*h - a*c*d^2*e*h - a*c*d^2*f*g - b*c*d^2*e*g + a*c^2*d*f*
h + b*c^2*d*e*h + b*c^2*d*f*g) + (f^2*log(e + f*x))/(a*c*f^3*g - b*d*e^3*h
- a*c*e*f^2*h - a*d*e*f^2*g - b*c*e*f^2*g + a*d*e^2*f*h + b*c*e^2*f*h + b*d
*e^2*f*g) + (h^2*log(g + h*x))/(a*c*e*h^3 - b*d*f*g^3 - a*c*f*g*h^2 - a*d*e
*g*h^2 - b*c*e*g*h^2 + a*d*f*g^2*h + b*c*f*g^2*h + b*d*e*g^2*h)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x)
```

```
[Out] Timed out
```

3.6 $\int \frac{x}{(1+x)(2+x)(3+x)} \, dx$

Optimal. Leaf size=23

$$-\frac{1}{2}\log(x+1)+2\log(x+2)-\frac{3}{2}\log(x+3)$$

```
[Out] -1/2*ln(1+x)+2*ln(2+x)-3/2*ln(3+x)
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {148}

$$-\frac{1}{2}\log(x+1)+2\log(x+2)-\frac{3}{2}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + x)*(2 + x)*(3 + x)),x]
```

```
[Out] -Log[1 + x]/2 + 2*Log[2 + x] - (3*Log[3 + x])/2
```

Rule 148

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(
c + d*x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(1+x)(2+x)(3+x)} \, dx &= \int \left(-\frac{1}{2(1+x)}+\frac{2}{2+x}-\frac{3}{2(3+x)}\right) dx\\ &= -\frac{1}{2}\log(1+x)+2\log(2+x)-\frac{3}{2}\log(3+x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$-\frac{1}{2}\log(x+1)+2\log(x+2)-\frac{3}{2}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 + x)*(2 + x)*(3 + x)),x]
```

```
[Out] -1/2*Log[1 + x] + 2*Log[2 + x] - (3*Log[3 + x])/2
```

fricas [A] time = 1.01, size = 19, normalized size = 0.83

$$-\frac{3}{2}\log\left(x+3\right)+2\log\left(x+2\right)-\frac{1}{2}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(2+x)/(3+x),x, algorithm="fricas")
```

```
[Out] -3/2*log(x + 3) + 2*log(x + 2) - 1/2*log(x + 1)
```

giac [A] time = 1.25, size = 22, normalized size = 0.96

$$-\frac{3}{2}\log\left(|x+3|\right)+2\log\left(|x+2|\right)-\frac{1}{2}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(2+x)/(3+x),x, algorithm="giac")
```

```
[Out] -3/2*log(abs(x + 3)) + 2*log(abs(x + 2)) - 1/2*log(abs(x + 1))
```

maple [A] time = 0.01, size = 20, normalized size = 0.87

$$-\frac{\ln\left(x+1\right)}{2}+2\ln\left(x+2\right)-\frac{3\ln\left(x+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x+1)/(x+2)/(x+3),x)
```

```
[Out] -1/2*ln(x+1)+2*ln(x+2)-3/2*ln(x+3)
```

maxima [A] time = 0.43, size = 19, normalized size = 0.83

$$-\frac{3}{2}\log\left(x+3\right)+2\log\left(x+2\right)-\frac{1}{2}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(2+x)/(3+x),x, algorithm="maxima")
```

```
[Out] -3/2*log(x + 3) + 2*log(x + 2) - 1/2*log(x + 1)
```

mupad [B] time = 0.08, size = 19, normalized size = 0.83

$$2\ln\left(x+2\right)-\frac{\ln\left(x+1\right)}{2}-\frac{3\ln\left(x+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x + 1)*(x + 2)*(x + 3)),x)
```

```
[Out] 2*log(x + 2) - log(x + 1)/2 - (3*log(x + 3))/2
```

sympy [A] time = 0.15, size = 20, normalized size = 0.87

$$-\frac{\log\left(x+1\right)}{2}+2\log\left(x+2\right)-\frac{3\log\left(x+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(2+x)/(3+x),x)
```

```
[Out] -log(x + 1)/2 + 2*log(x + 2) - 3*log(x + 3)/2
```

3.7 $\int \frac{-x^2+x^3}{(-6+x)(3+5x)^3} \, dx$

Optimal. Leaf size=43

$$\frac{201}{15125(5x+3)}-\frac{12}{1375(5x+3)^2}+\frac{20\log(6-x)}{3993}+\frac{1493\log(5x+3)}{499125}$$

```
[Out] -12/1375/(3+5*x)^2+201/15125/(3+5*x)+20/3993*ln(6-x)+1493/499125*ln(3+5*x)
```

Rubi [A] time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1593, 148}

$$\frac{201}{15125(5x+3)}-\frac{12}{1375(5x+3)^2}+\frac{20\log(6-x)}{3993}+\frac{1493\log(5x+3)}{499125}$$

Antiderivative was successfully verified.

```
[In] Int[(-x^2 + x^3)/((-6 + x)*(3 + 5*x)^3),x]
```

```
[Out] -12/(1375*(3 + 5*x)^2) + 201/(15125*(3 + 5*x)) + (20*Log[6 - x])/3993 + (14
93*Log[3 + 5*x])/499125
```

Rule 148

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(
c + d*x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{-x^2+x^3}{(-6+x)(3+5x)^3} \, dx &= \int \frac{(-1+x)x^2}{(-6+x)(3+5x)^3} \, dx\\
&= \int \left(\frac{20}{3993(-6+x)}+\frac{24}{275(3+5x)^3}-\frac{201}{3025(3+5x)^2}+\frac{1493}{99825(3+5x)}\right) dx\\
&= -\frac{12}{1375(3+5x)^2}+\frac{201}{15125(3+5x)}+\frac{20\log(6-x)}{3993}+\frac{1493\log(3+5x)}{499125}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.77

$$\frac{\frac{99(335x+157)}{(5x+3)^2}+2500\log(x-6)+1493\log(5x+3)}{499125}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^2 + x^3)/((-6 + x)*(3 + 5*x)^3),x]
```

```
[Out] ((99*(157 + 335*x))/(3 + 5*x)^2 + 2500*Log[-6 + x] + 1493*Log[3 + 5*x])/499
125
```

fricas [A] time = 1.08, size = 53, normalized size = 1.23

$$\frac{1493 \left(25 x^2 + 30 x + 9\right) \log(5 x + 3) + 2500 \left(25 x^2 + 30 x + 9\right) \log(x - 6) + 33165 x + 15543}{499125 \left(25 x^2 + 30 x + 9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x^2)/(-6+x)/(3+5*x)^3,x, algorithm="fricas")
```

```
[Out] 1/499125*(1493*(25*x^2 + 30*x + 9)*log(5*x + 3) + 2500*(25*x^2 + 30*x + 9)*
log(x - 6) + 33165*x + 15543)/(25*x^2 + 30*x + 9)
```

giac [A] time = 1.18, size = 31, normalized size = 0.72

$$\frac{3 (335 x + 157)}{15125 (5 x + 3)^2} + \frac{1493}{499125} \log(|5 x + 3|) + \frac{20}{3993} \log(|x - 6|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x^2)/(-6+x)/(3+5*x)^3,x, algorithm="giac")
```

```
[Out] 3/15125*(335*x + 157)/(5*x + 3)^2 + 1493/499125*log(abs(5*x + 3)) + 20/3993
*log(abs(x - 6))
```

maple [A] time = 0.01, size = 34, normalized size = 0.79

$$\frac{1493 \ln(5x + 3)}{499125} + \frac{20 \ln(x - 6)}{3993} - \frac{12}{1375 (5x + 3)^2} + \frac{201}{15125 (5x + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-x^2)/(x-6)/(5*x+3)^3,x)
```

```
[Out] -12/1375/(5*x+3)^2+201/15125/(5*x+3)+1493/499125*ln(5*x+3)+20/3993*ln(x-6)
```

maxima [A] time = 0.43, size = 34, normalized size = 0.79

$$\frac{3 (335 x + 157)}{15125 \left(25 x^2 + 30 x + 9\right)} + \frac{1493}{499125} \log(5 x + 3) + \frac{20}{3993} \log(x - 6)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x^2)/(-6+x)/(3+5*x)^3,x, algorithm="maxima")
```

```
[Out] 3/15125*(335*x + 157)/(25*x^2 + 30*x + 9) + 1493/499125*log(5*x + 3) + 20/3
993*log(x - 6)
```

mupad [B] time = 0.12, size = 29, normalized size = 0.67

$$\frac{20 \ln(x - 6)}{3993} + \frac{1493 \ln\left(x + \frac{3}{5}\right)}{499125} + \frac{\frac{201 x}{75625} + \frac{471}{378125}}{x^2 + \frac{6 x}{5} + \frac{9}{25}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - x^3)/((5*x + 3)^3*(x - 6)),x)
```

```
[Out] (20*log(x - 6))/3993 + (1493*log(x + 3/5))/499125 + ((201*x)/75625 + 471/37
8125)/((6*x)/5 + x^2 + 9/25)
```

sympy [A] time = 0.16, size = 32, normalized size = 0.74

$$\frac{1005x + 471}{378125x^2 + 453750x + 136125} + \frac{20 \log(x - 6)}{3993} + \frac{1493 \log\left(x + \frac{3}{5}\right)}{499125}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-x**2)/(-6+x)/(3+5*x)**3,x)
```

```
[Out] (1005*x + 471)/(378125*x**2 + 453750*x + 136125) + 20*log(x - 6)/3993 + 149
3*log(x + 3/5)/499125
```

3.8 $\int \frac{(a+bx)^3 \sqrt{c+dx}\,(e+fx)}{x}\,dx$

Optimal. Leaf size=227

$$2a^3e\sqrt{c+dx} - 2a^3\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right) + \frac{2(c+dx)^{3/2}\left(2\left(20a^3d^3f+3a^2bd^2(45de-16cf)-9ab^2cd(7de-4c\right.\right.}{}$$

```
[Out] 2/21*(2*a*d*f-2*b*c*f+3*b*d*e)*(b*x+a)^2*(d*x+c)^(3/2)/d^2+2/9*f*(b*x+a)^3*
(d*x+c)^(3/2)/d+2/315*(d*x+c)^(3/2)*(40*a^3*d^3*f+6*a^2*b*d^2*(-16*c*f+45*d
*e)-18*a*b^2*c*d*(-4*c*f+7*d*e)+8*b^3*c^2*(-2*c*f+3*d*e)+3*b*d*(21*a*b*d^2*
e-4*(-a*d+b*c)*(2*a*d*f-2*b*c*f+3*b*d*e))*x)/d^4-2*a^3*e*arctanh((d*x+c)^(1
/2)/c^(1/2))*c^(1/2)+2*a^3*e*(d*x+c)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {153, 147, 50, 63, 208}

$$\frac{2(c+dx)^{3/2}\left(2\left(3a^2bd^2(45de-16cf)+20a^3d^3f-9ab^2cd(7de-4cf)+4b^3c^2(3de-2cf)\right)+3bdx\left(21abd^2e-4\right.\right.}{315d^4}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^3*Sqrt[c + d*x]*(e + f*x))/x,x]
```

```
[Out] 2*a^3*e*Sqrt[c + d*x] + (2*(3*b*d*e - 2*b*c*f + 2*a*d*f)*(a + b*x)^2*(c + d
*x)^(3/2))/(21*d^2) + (2*f*(a + b*x)^3*(c + d*x)^(3/2))/(9*d) + (2*(c + d*x
)^(3/2)*(2*(20*a^3*d^3*f + 3*a^2*b*d^2*(45*d*e - 16*c*f) - 9*a*b^2*c*d*(7*d
*e - 4*c*f) + 4*b^3*c^2*(3*d*e - 2*c*f)) + 3*b*d*(21*a*b*d^2*e - 4*(b*c - a
*d)*(3*b*d*e - 2*b*c*f + 2*a*d*f))*x))/(315*d^4) - 2*a^3*Sqrt[c]*e*ArcTanh[
Sqrt[c + d*x]/Sqrt[c]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
```

```
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx)^3\sqrt{c+dx}\,(e+fx)}{x}\,dx = \frac{2f(a+bx)^3(c+dx)^{3/2}}{9d} + \frac{2\int \frac{(a+bx)^2\sqrt{c+dx}\left(\frac{9ade}{2}+\frac{3}{2}(3bde-2bcf+2adf)x\right)}{x}\,dx}{9d}$$

$$= \frac{2(3bde-2bcf+2adf)(a+bx)^2(c+dx)^{3/2}}{21d^2} + \frac{2f(a+bx)^3(c+dx)^{3/2}}{9d} + 4\int \frac{(a+}{}$$

$$= \frac{2(3bde-2bcf+2adf)(a+bx)^2(c+dx)^{3/2}}{21d^2} + \frac{2f(a+bx)^3(c+dx)^{3/2}}{9d} + 2(c+$$

$$= 2a^3e\sqrt{c+dx} + \frac{2(3bde-2bcf+2adf)(a+bx)^2(c+dx)^{3/2}}{21d^2} + \frac{2f(a+bx)^3(c+}{9d}$$

$$= 2a^3e\sqrt{c+dx} + \frac{2(3bde-2bcf+2adf)(a+bx)^2(c+dx)^{3/2}}{21d^2} + \frac{2f(a+bx)^3(c+}{9d}$$

$$= 2a^3e\sqrt{c+dx} + \frac{2(3bde-2bcf+2adf)(a+bx)^2(c+dx)^{3/2}}{21d^2} + \frac{2f(a+bx)^3(c+}{9d}$$

Mathematica [A] time = 0.28, size = 205, normalized size = 0.90

$$2\left(3de\left(105a^3d^3\sqrt{c+dx} - 105a^3\sqrt{c}\,d^3\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right) + 35b(c+dx)^{3/2}\left(3a^2d^2-3abcd+b^2c^2\right) - 21b^2(c+dx)^{5/2}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^3*Sqrt[c + d*x]*(e + f*x))/x,x]
```

```
[Out] (2*(-(f*(c + d*x)^(3/2)*(105*(b*c - a*d)^3 - 189*b*(b*c - a*d)^2*(c + d*x)
+ 135*b^2*(b*c - a*d)*(c + d*x)^2 - 35*b^3*(c + d*x)^3)) + 3*d*e*(105*a^3*d
^3*Sqrt[c + d*x] + 35*b*(b^2*c^2 - 3*a*b*c*d + 3*a^2*d^2)*(c + d*x)^(3/2) -
 21*b^2*(2*b*c - 3*a*d)*(c + d*x)^(5/2) + 15*b^3*(c + d*x)^(7/2) - 105*a^3*
Sqrt[c]*d^3*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])))/(315*d^4)
```

fricas [A] time = 1.31, size = 649, normalized size = 2.86

$$\left[\frac{315\, a^3\sqrt{c}\, d^4 e \log\left(\frac{dx-2\,\sqrt{dx+c}\,\sqrt{c}+2\,c}{x}\right)+2\left(35\, b^3 d^4 f x^4+5\left(9\, b^3 d^4 e+\left(b^3 c d^3+27\, a b^2 d^4\right) f\right) x^3+3\left(3\left(b^3 c d^3+21\right.\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/315*(315*a^3*sqrt(c)*d^4*e*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x)
+ 2*(35*b^3*d^4*f*x^4 + 5*(9*b^3*d^4*e + (b^3*c*d^3 + 27*a*b^2*d^4)*f)*x^3
+ 3*(3*(b^3*c*d^3 + 21*a*b^2*d^4)*e - (2*b^3*c^2*d^2 - 9*a*b^2*c*d^3 - 63*a
^2*b*d^4)*f)*x^2 + 3*(8*b^3*c^3*d - 42*a*b^2*c^2*d^2 + 105*a^2*b*c*d^3 + 10
5*a^3*d^4)*e - (16*b^3*c^4 - 72*a*b^2*c^3*d + 126*a^2*b*c^2*d^2 - 105*a^3*c
*d^3)*f - (3*(4*b^3*c^2*d^2 - 21*a*b^2*c*d^3 - 105*a^2*b*d^4)*e - (8*b^3*c^
3*d - 36*a*b^2*c^2*d^2 + 63*a^2*b*c*d^3 + 105*a^3*d^4)*f)*x)*sqrt(d*x + c))
/d^4, 2/315*(315*a^3*sqrt(-c)*d^4*e*arctan(sqrt(d*x + c)*sqrt(-c)/c) + (35*
b^3*d^4*f*x^4 + 5*(9*b^3*d^4*e + (b^3*c*d^3 + 27*a*b^2*d^4)*f)*x^3 + 3*(3*(
b^3*c*d^3 + 21*a*b^2*d^4)*e - (2*b^3*c^2*d^2 - 9*a*b^2*c*d^3 - 63*a^2*b*d^4
)*f)*x^2 + 3*(8*b^3*c^3*d - 42*a*b^2*c^2*d^2 + 105*a^2*b*c*d^3 + 105*a^3*d^
4)*e - (16*b^3*c^4 - 72*a*b^2*c^3*d + 126*a^2*b*c^2*d^2 - 105*a^3*c*d^3)*f
- (3*(4*b^3*c^2*d^2 - 21*a*b^2*c*d^3 - 105*a^2*b*d^4)*e - (8*b^3*c^3*d - 36
*a*b^2*c^2*d^2 + 63*a^2*b*c*d^3 + 105*a^3*d^4)*f)*x)*sqrt(d*x + c))/d^4]
```

giac [A] time = 1.41, size = 338, normalized size = 1.49

$$\frac{2\, a^3 c \arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right) e}{\sqrt{-c}}+\frac{2\left(35\,(dx+c)^{\frac{9}{2}} b^3 d^{32} f-135\,(dx+c)^{\frac{7}{2}} b^3 c d^{32} f+189\,(dx+c)^{\frac{5}{2}} b^3 c^2 d^{32} f-105\,(dx+c)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*a^3*c*arctan(sqrt(d*x + c)/sqrt(-c))*e/sqrt(-c) + 2/315*(35*(d*x + c)^(9/
2)*b^3*d^32*f - 135*(d*x + c)^(7/2)*b^3*c*d^32*f + 189*(d*x + c)^(5/2)*b^3*
c^2*d^32*f - 105*(d*x + c)^(3/2)*b^3*c^3*d^32*f + 135*(d*x + c)^(7/2)*a*b^2
*d^33*f - 378*(d*x + c)^(5/2)*a*b^2*c*d^33*f + 315*(d*x + c)^(3/2)*a*b^2*c^
2*d^33*f + 189*(d*x + c)^(5/2)*a^2*b*d^34*f - 315*(d*x + c)^(3/2)*a^2*b*c*d
^34*f + 105*(d*x + c)^(3/2)*a^3*d^35*f + 45*(d*x + c)^(7/2)*b^3*d^33*e - 12
6*(d*x + c)^(5/2)*b^3*c*d^33*e + 105*(d*x + c)^(3/2)*b^3*c^2*d^33*e + 189*(
d*x + c)^(5/2)*a*b^2*d^34*e - 315*(d*x + c)^(3/2)*a*b^2*c*d^34*e + 315*(d*x
 + c)^(3/2)*a^2*b*d^35*e + 315*sqrt(d*x + c)*a^3*d^36*e)/d^36
```

maple [A] time = 0.01, size = 301, normalized size = 1.33

$$\frac{-2a^3\sqrt{c}\, d^4 e \,\mathrm{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)+2\sqrt{dx+c}\, a^3 d^4 e+\frac{2(dx+c)^{\frac{3}{2}} a^3 d^3 f}{3}-2\,(dx+c)^{\frac{3}{2}} a^2 b c\, d^2 f+2\,(dx+c)^{\frac{3}{2}} a^2 b\, d^3 e+2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(f*x+e)*(d*x+c)^(1/2)/x,x)
```

```
[Out] 2/d^4*(1/9*f*b^3*(d*x+c)^(9/2)+3/7*(d*x+c)^(7/2)*a*b^2*d*f-3/7*(d*x+c)^(7/2
)*b^3*c*f+1/7*(d*x+c)^(7/2)*b^3*d*e+3/5*(d*x+c)^(5/2)*a^2*b*d^2*f-6/5*(d*x+
c)^(5/2)*a*b^2*c*d*f+3/5*(d*x+c)^(5/2)*a*b^2*d^2*e+3/5*(d*x+c)^(5/2)*b^3*c^
2*f-2/5*(d*x+c)^(5/2)*b^3*c*d*e+1/3*(d*x+c)^(3/2)*a^3*d^3*f-(d*x+c)^(3/2)*a
^2*b*c*d^2*f+(d*x+c)^(3/2)*a^2*b*d^3*e+(d*x+c)^(3/2)*a*b^2*c^2*d*f-(d*x+c)^
(3/2)*a*b^2*c*d^2*e-1/3*(d*x+c)^(3/2)*b^3*c^3*f+1/3*(d*x+c)^(3/2)*b^3*c^2*d
```

```
*e+a^3*d^4*e*(d*x+c)^(1/2)-a^3*c^(1/2)*d^4*e*arctanh((d*x+c)^(1/2)/c^(1/2))
)
```

maxima [A] time = 0.99, size = 239, normalized size = 1.05

$$a^3\sqrt{c}\,e\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)+\frac{2\left(315\sqrt{dx+c}\,a^3d^4e+35\,(dx+c)^{\frac{9}{2}}b^3f+45\left(b^3de-3\left(b^3c-ab^2d\right)f\right)(dx+c)^{\frac{7}{2}}-6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] a^3*sqrt(c)*e*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c))) + 2/
315*(315*sqrt(d*x + c)*a^3*d^4*e + 35*(d*x + c)^(9/2)*b^3*f + 45*(b^3*d*e -
 3*(b^3*c - a*b^2*d)*f)*(d*x + c)^(7/2) - 63*((2*b^3*c*d - 3*a*b^2*d^2)*e -
 3*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*f)*(d*x + c)^(5/2) + 105*((b^3*c^2*d
 - 3*a*b^2*c*d^2 + 3*a^2*b*d^3)*e - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^
2 - a^3*d^3)*f)*(d*x + c)^(3/2))/d^4
```

mupad [B] time = 0.16, size = 413, normalized size = 1.82

$$\left(c\left(c\left(c\left(\frac{2\,b^3\,d\,e-8\,b^3\,c\,f+6\,a\,b^2\,d\,f}{d^4}+\frac{2\,b^3\,c\,f}{d^4}\right)+\frac{6\,b\,(a\,d-b\,c)\,\left(a\,d\,f-2\,b\,c\,f+b\,d\,e\right)}{d^4}\right)+\frac{2\,(a\,d-b\,c)^2\,\left(a\,d\,f\right.}{d}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(a + b*x)^3*(c + d*x)^(1/2))/x,x)
```

```
[Out] (c*(c*(c*((2*b^3*d*e - 8*b^3*c*f + 6*a*b^2*d*f)/d^4 + (2*b^3*c*f)/d^4) + (6
*b*(a*d - b*c)*(a*d*f - 2*b*c*f + b*d*e))/d^4) + (2*(a*d - b*c)^2*(a*d*f -
4*b*c*f + 3*b*d*e))/d^4) - (2*(a*d - b*c)^3*(c*f - d*e))/d^4)*(c + d*x)^(1/
2) + ((c*(c*((2*b^3*d*e - 8*b^3*c*f + 6*a*b^2*d*f)/d^4 + (2*b^3*c*f)/d^4) +
 (6*b*(a*d - b*c)*(a*d*f - 2*b*c*f + b*d*e))/d^4))/3 + (2*(a*d - b*c)^2*(a*
d*f - 4*b*c*f + 3*b*d*e))/(3*d^4))*(c + d*x)^(3/2) + ((2*b^3*d*e - 8*b^3*c*
f + 6*a*b^2*d*f)/(7*d^4) + (2*b^3*c*f)/(7*d^4))*(c + d*x)^(7/2) + ((c*((2*b
^3*d*e - 8*b^3*c*f + 6*a*b^2*d*f)/d^4 + (2*b^3*c*f)/d^4))/5 + (6*b*(a*d - b
*c)*(a*d*f - 2*b*c*f + b*d*e))/(5*d^4))*(c + d*x)^(5/2) + a^3*c^(1/2)*e*ata
n(((c + d*x)^(1/2)*1i)/c^(1/2))*2i + (2*b^3*f*(c + d*x)^(9/2))/(9*d^4)
```

sympy [A] time = 37.79, size = 274, normalized size = 1.21

$$\frac{2a^3ce\,\mathrm{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}}+2a^3e\sqrt{c+dx}+\frac{2b^3f\,(c+dx)^{\frac{9}{2}}}{9d^4}+\frac{2\,(c+dx)^{\frac{7}{2}}\left(3ab^2df-3b^3cf+b^3de\right)}{7d^4}+\frac{2\,(c+dx)^{\frac{5}{2}}\left(3a^2bd^2f\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(f*x+e)*(d*x+c)**(1/2)/x,x)
```

```
[Out] 2*a**3*c*e*atan(sqrt(c + d*x)/sqrt(-c))/sqrt(-c) + 2*a**3*e*sqrt(c + d*x) +
 2*b**3*f*(c + d*x)**(9/2)/(9*d**4) + 2*(c + d*x)**(7/2)*(3*a*b**2*d*f - 3*
b**3*c*f + b**3*d*e)/(7*d**4) + 2*(c + d*x)**(5/2)*(3*a**2*b*d**2*f - 6*a*b
**2*c*d*f + 3*a*b**2*d**2*e + 3*b**3*c**2*f - 2*b**3*c*d*e)/(5*d**4) + 2*(c
 + d*x)**(3/2)*(a**3*d**3*f - 3*a**2*b*c*d**2*f + 3*a**2*b*d**3*e + 3*a*b**
2*c**2*d*f - 3*a*b**2*c*d**2*e - b**3*c**3*f + b**3*c**2*d*e)/(3*d**4)
```

3.9 $\int \frac{(a+bx)^2 \sqrt{c+dx} (e+fx)}{x} dx$

Optimal. Leaf size=146

$$\frac{2(c+dx)^{3/2}\left(2\left(10a^2d^2f+7abd(5de-2cf)+b^2(-c)(7de-4cf)\right)+3bdx(4adf-4bcf+7bde)\right)}{105d^3}+2a^2e\sqrt{c+dx}$$

```
[Out] 2/7*f*(b*x+a)^2*(d*x+c)^(3/2)/d+2/105*(d*x+c)^(3/2)*(20*a^2*d^2*f-2*b^2*c*(
-4*c*f+7*d*e)+14*a*b*d*(-2*c*f+5*d*e)+3*b*d*(4*a*d*f-4*b*c*f+7*b*d*e)*x)/d^
3-2*a^2*e*arctanh((d*x+c)^(1/2)/c^(1/2))*c^(1/2)+2*a^2*e*(d*x+c)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {153, 147, 50, 63, 208}

$$\frac{2(c+dx)^{3/2}\left(2\left(10a^2d^2f+7abd(5de-2cf)+b^2(-c)(7de-4cf)\right)+3bdx(4adf-4bcf+7bde)\right)}{105d^3}+2a^2e\sqrt{c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^2*Sqrt[c + d*x]*(e + f*x))/x,x]
```

```
[Out] 2*a^2*e*Sqrt[c + d*x] + (2*f*(a + b*x)^2*(c + d*x)^(3/2))/(7*d) + (2*(c + d
*x)^(3/2)*(2*(10*a^2*d^2*f - b^2*c*(7*d*e - 4*c*f) + 7*a*b*d*(5*d*e - 2*c*f
)) + 3*b*d*(7*b*d*e - 4*b*c*f + 4*a*d*f)*x))/(105*d^3) - 2*a^2*Sqrt[c]*e*Ar
cTanh[Sqrt[c + d*x]/Sqrt[c]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
```

```
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx)^2\sqrt{c+dx}\,(e+fx)}{x}\,dx = \frac{2f(a+bx)^2(c+dx)^{3/2}}{7d} + \frac{2\int \frac{(a+bx)\sqrt{c+dx}\left(\frac{7ade}{2}+\frac{1}{2}(7bde-4bcf+4adf)x\right)}{x}\,dx}{7d}$$

$$= \frac{2f(a+bx)^2(c+dx)^{3/2}}{7d} + \frac{2(c+dx)^{3/2}\left(2\left(10a^2d^2f-b^2c(7de-4cf)+7abd(5d\right.\right.}{105d^3}$$

$$= 2a^2e\sqrt{c+dx} + \frac{2f(a+bx)^2(c+dx)^{3/2}}{7d} + \frac{2(c+dx)^{3/2}\left(2\left(10a^2d^2f-b^2c(7de-\right.\right.}{}$$

$$= 2a^2e\sqrt{c+dx} + \frac{2f(a+bx)^2(c+dx)^{3/2}}{7d} + \frac{2(c+dx)^{3/2}\left(2\left(10a^2d^2f-b^2c(7de-\right.\right.}{}$$

$$= 2a^2e\sqrt{c+dx} + \frac{2f(a+bx)^2(c+dx)^{3/2}}{7d} + \frac{2(c+dx)^{3/2}\left(2\left(10a^2d^2f-b^2c(7de-\right.\right.}{}$$

Mathematica [A] time = 0.18, size = 145, normalized size = 0.99

$$\frac{2\left(7de\left(\sqrt{c+dx}\left(15a^2d^2+10abd(c+dx)+b^2\left(-2c^2+cdx+3d^2x^2\right)\right)-15a^2\sqrt{c}\,d^2\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)\right)+f(c+dx)^{3/}\right.}{105d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^2*Sqrt[c + d*x]*(e + f*x))/x,x]
```

```
[Out] (2*(f*(c + d*x)^(3/2)*(35*(b*c - a*d)^2 - 42*b*(b*c - a*d)*(c + d*x) + 15*b
^2*(c + d*x)^2) + 7*d*e*(Sqrt[c + d*x]*(15*a^2*d^2 + 10*a*b*d*(c + d*x) + b
^2*(-2*c^2 + c*d*x + 3*d^2*x^2)) - 15*a^2*Sqrt[c]*d^2*ArcTanh[Sqrt[c + d*x]
/Sqrt[c]])))/(105*d^3)
```

fricas [A] time = 1.15, size = 405, normalized size = 2.77

$$\left[\frac{105\,a^2\sqrt{c}\,d^3e\log\left(\frac{dx-2\sqrt{dx+c}\,\sqrt{c}+2c}{x}\right)+2\left(15\,b^2d^3fx^3+3\left(7\,b^2d^3e+\left(b^2cd^2+14\,abd^3\right)f\right)x^2-7\left(2\,b^2c^2d-10\,abcd\right.\right.}{1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/105*(105*a^2*sqrt(c)*d^3*e*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x)
+ 2*(15*b^2*d^3*f*x^3 + 3*(7*b^2*d^3*e + (b^2*c*d^2 + 14*a*b*d^3)*f)*x^2 -
7*(2*b^2*c^2*d - 10*a*b*c*d^2 - 15*a^2*d^3)*e + (8*b^2*c^3 - 28*a*b*c^2*d +
```

```
 35*a^2*c*d^2)*f + (7*(b^2*c*d^2 + 10*a*b*d^3)*e - (4*b^2*c^2*d - 14*a*b*c*
d^2 - 35*a^2*d^3)*f)*x)*sqrt(d*x + c))/d^3, 2/105*(105*a^2*sqrt(-c)*d^3*e*a
rctan(sqrt(d*x + c)*sqrt(-c)/c) + (15*b^2*d^3*f*x^3 + 3*(7*b^2*d^3*e + (b^2
*c*d^2 + 14*a*b*d^3)*f)*x^2 - 7*(2*b^2*c^2*d - 10*a*b*c*d^2 - 15*a^2*d^3)*e
 + (8*b^2*c^3 - 28*a*b*c^2*d + 35*a^2*c*d^2)*f + (7*(b^2*c*d^2 + 10*a*b*d^3
)*e - (4*b^2*c^2*d - 14*a*b*c*d^2 - 35*a^2*d^3)*f)*x)*sqrt(d*x + c))/d^3]
```

giac [A] time = 1.36, size = 201, normalized size = 1.38

$$\frac{2\,a^2c\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)e}{\sqrt{-c}}+\frac{2\left(15\,(dx+c)^{\frac{7}{2}}b^2d^{18}f-42\,(dx+c)^{\frac{5}{2}}b^2cd^{18}f+35\,(dx+c)^{\frac{3}{2}}b^2c^2d^{18}f+42\,(dx+c)^{\frac{5}{2}}ab\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*a^2*c*arctan(sqrt(d*x + c)/sqrt(-c))*e/sqrt(-c) + 2/105*(15*(d*x + c)^(7/
2)*b^2*d^18*f - 42*(d*x + c)^(5/2)*b^2*c*d^18*f + 35*(d*x + c)^(3/2)*b^2*c^
2*d^18*f + 42*(d*x + c)^(5/2)*a*b*d^19*f - 70*(d*x + c)^(3/2)*a*b*c*d^19*f
+ 35*(d*x + c)^(3/2)*a^2*d^20*f + 21*(d*x + c)^(5/2)*b^2*d^19*e - 35*(d*x +
 c)^(3/2)*b^2*c*d^19*e + 70*(d*x + c)^(3/2)*a*b*d^20*e + 105*sqrt(d*x + c)*
a^2*d^21*e)/d^21
```

maple [A] time = 0.01, size = 176, normalized size = 1.21

$$\frac{-2a^2\sqrt{c}\,d^3e\,\mathrm{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)+2\sqrt{dx+c}\,a^2d^3e+\frac{2(dx+c)^{\frac{3}{2}}a^2d^2f}{3}-\frac{4(dx+c)^{\frac{3}{2}}abcdf}{3}+\frac{4(dx+c)^{\frac{3}{2}}ab\,d^2e}{3}+\frac{2(dx+c)^{\frac{3}{2}}b^2c^2f}{3}-}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(f*x+e)*(d*x+c)^(1/2)/x,x)
```

```
[Out] 2/d^3*(1/7*b^2*f*(d*x+c)^(7/2)+2/5*(d*x+c)^(5/2)*a*b*d*f-2/5*(d*x+c)^(5/2)*
b^2*c*f+1/5*(d*x+c)^(5/2)*b^2*d*e+1/3*(d*x+c)^(3/2)*a^2*d^2*f-2/3*(d*x+c)^(
3/2)*a*b*c*d*f+2/3*(d*x+c)^(3/2)*a*b*d^2*e+1/3*(d*x+c)^(3/2)*b^2*c^2*f-1/3*
(d*x+c)^(3/2)*b^2*c*d*e+a^2*d^3*e*(d*x+c)^(1/2)-a^2*c^(1/2)*d^3*e*arctanh((
d*x+c)^(1/2)/c^(1/2)))
```

maxima [A] time = 0.96, size = 152, normalized size = 1.04

$$a^2\sqrt{c}\,e\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)+\frac{2\left(105\sqrt{dx+c}\,a^2d^3e+15\,(dx+c)^{\frac{7}{2}}b^2f+21\left(b^2de-2\left(b^2c-abd\right)f\right)(dx+c)^{\frac{5}{2}}-\right.}{105\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] a^2*sqrt(c)*e*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c))) + 2/
105*(105*sqrt(d*x + c)*a^2*d^3*e + 15*(d*x + c)^(7/2)*b^2*f + 21*(b^2*d*e -
 2*(b^2*c - a*b*d)*f)*(d*x + c)^(5/2) - 35*((b^2*c*d - 2*a*b*d^2)*e - (b^2*
c^2 - 2*a*b*c*d + a^2*d^2)*f)*(d*x + c)^(3/2))/d^3
```

mupad [B] time = 2.62, size = 263, normalized size = 1.80

$$\left(\frac{2\,b^2\,d\,e-6\,b^2\,c\,f+4\,a\,b\,d\,f}{5\,d^3}+\frac{2\,b^2\,c\,f}{5\,d^3}\right)\,(c+d\,x)^{5/2}+\left(c\,\left(c\,\left(\frac{2\,b^2\,d\,e-6\,b^2\,c\,f+4\,a\,b\,d\,f}{d^3}+\frac{2\,b^2\,c\,f}{d^3}\right)+\frac{2\,(a\,d\,-}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(a + b*x)^2*(c + d*x)^(1/2))/x,x)
```

```
[Out] ((2*b^2*d*e - 6*b^2*c*f + 4*a*b*d*f)/(5*d^3) + (2*b^2*c*f)/(5*d^3))*(c + d*
x)^(5/2) + (c*(c*((2*b^2*d*e - 6*b^2*c*f + 4*a*b*d*f)/d^3 + (2*b^2*c*f)/d^3
) + (2*(a*d - b*c)*(a*d*f - 3*b*c*f + 2*b*d*e))/d^3) - (2*(a*d - b*c)^2*(c*
f - d*e))/d^3)*(c + d*x)^(1/2) + ((c*((2*b^2*d*e - 6*b^2*c*f + 4*a*b*d*f)/d
^3 + (2*b^2*c*f)/d^3))/3 + (2*(a*d - b*c)*(a*d*f - 3*b*c*f + 2*b*d*e))/(3*d
^3))*(c + d*x)^(3/2) + a^2*c^(1/2)*e*atan(((c + d*x)^(1/2)*1i)/c^(1/2))*2i
+ (2*b^2*f*(c + d*x)^(7/2))/(7*d^3)
```

sympy [A] time = 27.60, size = 167, normalized size = 1.14

$$\frac{2a^2ce\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}}+2a^2e\sqrt{c+dx}+\frac{2b^2f\left(c+dx\right)^{\frac{7}{2}}}{7d^3}+\frac{2\left(c+dx\right)^{\frac{5}{2}}\left(2abdf-2b^2cf+b^2de\right)}{5d^3}+\frac{2\left(c+dx\right)^{\frac{3}{2}}\left(a^2d^2f-2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(f*x+e)*(d*x+c)**(1/2)/x,x)
```

```
[Out] 2*a**2*c*e*atan(sqrt(c + d*x)/sqrt(-c))/sqrt(-c) + 2*a**2*e*sqrt(c + d*x) +
 2*b**2*f*(c + d*x)**(7/2)/(7*d**3) + 2*(c + d*x)**(5/2)*(2*a*b*d*f - 2*b**
2*c*f + b**2*d*e)/(5*d**3) + 2*(c + d*x)**(3/2)*(a**2*d**2*f - 2*a*b*c*d*f
+ 2*a*b*d**2*e + b**2*c**2*f - b**2*c*d*e)/(3*d**3)
```

3.10 $\int \frac{(a+bx)\sqrt{c+dx}\,(e+fx)}{x}\,dx$

Optimal. Leaf size=77

$$-\frac{2(c+dx)^{3/2}(-5d(af+be)+2bcf-3bdfx)}{15d^2}+2ae\sqrt{c+dx}-2a\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)$$

```
[Out] -2/15*(d*x+c)^(3/2)*(2*b*c*f-5*d*(a*f+b*e)-3*b*d*f*x)/d^2-2*a*e*arctanh((d*
x+c)^(1/2)/c^(1/2))*c^(1/2)+2*a*e*(d*x+c)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {147, 50, 63, 208}

$$-\frac{2(c+dx)^{3/2}(-5d(af+be)+2bcf-3bdfx)}{15d^2}+2ae\sqrt{c+dx}-2a\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)*Sqrt[c + d*x]*(e + f*x))/x,x]
```

```
[Out] 2*a*e*Sqrt[c + d*x] - (2*(c + d*x)^(3/2)*(2*b*c*f - 5*d*(b*e + a*f) - 3*b*d
*f*x))/(15*d^2) - 2*a*Sqrt[c]*e*ArcTanh[Sqrt[c + d*x]/Sqrt[c]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx)\sqrt{c+dx}(e+fx)}{x}dx = -\frac{2(c+dx)^{3/2}(2bcf-5d(be+af)-3bdfx)}{15d^2}+(ae)\int \frac{\sqrt{c+dx}}{x}dx$$
$$= 2ae\sqrt{c+dx}-\frac{2(c+dx)^{3/2}(2bcf-5d(be+af)-3bdfx)}{15d^2}+(ace)\int \frac{1}{x\sqrt{c+dx}}$$
$$= 2ae\sqrt{c+dx}-\frac{2(c+dx)^{3/2}(2bcf-5d(be+af)-3bdfx)}{15d^2}+\frac{(2ace)\,\mathrm{Subst}\left(\int \right.}{}$$
$$= 2ae\sqrt{c+dx}-\frac{2(c+dx)^{3/2}(2bcf-5d(be+af)-3bdfx)}{15d^2}-2a\sqrt{c}\,e\tanh^{-1}\left(\right.$$

Mathematica [A] time = 0.16, size = 81, normalized size = 1.05

$$\frac{2\sqrt{c+dx}(5ad(cf+3de+dfx)-b(c+dx)(2cf-5de-3dfx))}{15d^2}-2a\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*Sqrt[c + d*x]*(e + f*x))/x,x]
[Out] (2*Sqrt[c + d*x]*(-(b*(c + d*x)*(-5*d*e + 2*c*f - 3*d*f*x)) + 5*a*d*(3*d*e
+ c*f + d*f*x)))/(15*d^2) - 2*a*Sqrt[c]*e*ArcTanh[Sqrt[c + d*x]/Sqrt[c]]
```

fricas [A] time = 1.29, size = 219, normalized size = 2.84

$$\left[\frac{15\,a\sqrt{c}\,d^2e\log\left(\frac{dx-2\,\sqrt{dx+c}\,\sqrt{c}+2\,c}{x}\right)+2\left(3\,bd^2fx^2+5\left(bcd+3\,ad^2\right)e-\left(2\,bc^2-5\,acd\right)f+\left(5\,bd^2e+\left(bcd+5\,ad^2\right)f\right.\right.}{15\,d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="fricas")
[Out] [1/15*(15*a*sqrt(c)*d^2*e*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 2*
(3*b*d^2*f*x^2 + 5*(b*c*d + 3*a*d^2)*e - (2*b*c^2 - 5*a*c*d)*f + (5*b*d^2*e
 + (b*c*d + 5*a*d^2)*f)*x)*sqrt(d*x + c))/d^2, 2/15*(15*a*sqrt(-c)*d^2*e*ar
ctan(sqrt(d*x + c)*sqrt(-c)/c) + (3*b*d^2*f*x^2 + 5*(b*c*d + 3*a*d^2)*e - (
2*b*c^2 - 5*a*c*d)*f + (5*b*d^2*e + (b*c*d + 5*a*d^2)*f)*x)*sqrt(d*x + c))/
d^2]
```

giac [A] time = 1.33, size = 105, normalized size = 1.36

$$\frac{2\,ac\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)e}{\sqrt{-c}}+\frac{2\left(3\,(dx+c)^{\frac{5}{2}}bd^8f-5\,(dx+c)^{\frac{3}{2}}bcd^8f+5\,(dx+c)^{\frac{3}{2}}ad^9f+5\,(dx+c)^{\frac{3}{2}}bd^9e+15\,\sqrt{dx+c}\right.}{15\,d^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="giac")
[Out] 2*a*c*arctan(sqrt(d*x + c)/sqrt(-c))*e/sqrt(-c) + 2/15*(3*(d*x + c)^(5/2)*b
*d^8*f - 5*(d*x + c)^(3/2)*b*c*d^8*f + 5*(d*x + c)^(3/2)*a*d^9*f + 5*(d*x +
 c)^(3/2)*b*d^9*e + 15*sqrt(d*x + c)*a*d^10*e)/d^10
```

maple [A] time = 0.01, size = 89, normalized size = 1.16

$$\frac{-2a\sqrt{c}\,d^2e\,\mathrm{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)+2\sqrt{dx+c}\,a\,d^2e+\frac{2(dx+c)^{\frac{3}{2}}adf}{3}-\frac{2(dx+c)^{\frac{3}{2}}bcf}{3}+\frac{2(dx+c)^{\frac{3}{2}}bde}{3}+\frac{2(dx+c)^{\frac{5}{2}}bf}{5}}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(f*x+e)*(d*x+c)^(1/2)/x,x)
```

```
[Out] 2/d^2*(1/5*f*b*(d*x+c)^(5/2)+1/3*(d*x+c)^(3/2)*a*d*f-1/3*(d*x+c)^(3/2)*b*c*
f+1/3*(d*x+c)^(3/2)*b*d*e+a*d^2*e*(d*x+c)^(1/2)-a*c^(1/2)*d^2*e*arctanh((d*
x+c)^(1/2)/c^(1/2)))
```

maxima [A] time = 0.98, size = 91, normalized size = 1.18

$$a\sqrt{c}\,e\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)+\frac{2\left(15\sqrt{dx+c}\,ad^2e+3\,(dx+c)^{\frac{5}{2}}bf+5\left(bde-(bc-ad)f\right)(dx+c)^{\frac{3}{2}}\right)}{15\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] a*sqrt(c)*e*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c))) + 2/15
*(15*sqrt(d*x + c)*a*d^2*e + 3*(d*x + c)^(5/2)*b*f + 5*(b*d*e - (b*c - a*d)
*f)*(d*x + c)^(3/2))/d^2
```

mupad [B] time = 0.09, size = 136, normalized size = 1.77

$$\left(c\left(\frac{2\,a\,d\,f-4\,b\,c\,f+2\,b\,d\,e}{d^2}+\frac{2\,b\,c\,f}{d^2}\right)-\frac{2\,(a\,d-b\,c)\,\left(c\,f-d\,e\right)}{d^2}\right)\sqrt{c+d\,x}+\left(\frac{2\,a\,d\,f-4\,b\,c\,f+2\,b\,d\,e}{3\,d^2}+\frac{2\,b\,c}{3\,d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(a + b*x)*(c + d*x)^(1/2))/x,x)
```

```
[Out] (c*((2*a*d*f - 4*b*c*f + 2*b*d*e)/d^2 + (2*b*c*f)/d^2) - (2*(a*d - b*c)*(c*
f - d*e))/d^2)*(c + d*x)^(1/2) + ((2*a*d*f - 4*b*c*f + 2*b*d*e)/(3*d^2) + (
2*b*c*f)/(3*d^2))*(c + d*x)^(3/2) + (2*b*f*(c + d*x)^(5/2))/(5*d^2) + a*c^(
1/2)*e*atan(((c + d*x)^(1/2)*1i)/c^(1/2))*2i
```

sympy [A] time = 25.99, size = 92, normalized size = 1.19

$$\frac{2ace\,\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}}+2ae\sqrt{c+dx}+\frac{2bf\,(c+dx)^{\frac{5}{2}}}{5d^2}+\frac{2\,(c+dx)^{\frac{3}{2}}\left(adf-bcf+bde\right)}{3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(f*x+e)*(d*x+c)**(1/2)/x,x)
```

```
[Out] 2*a*c*e*atan(sqrt(c + d*x)/sqrt(-c))/sqrt(-c) + 2*a*e*sqrt(c + d*x) + 2*b*f
*(c + d*x)**(5/2)/(5*d**2) + 2*(c + d*x)**(3/2)*(a*d*f - b*c*f + b*d*e)/(3*
d**2)
```

3.11 $\int \frac{\sqrt{c+dx}\,(e+fx)}{x}\,dx$

Optimal. Leaf size=54

$$2e\sqrt{c+dx} - 2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right) + \frac{2f(c+dx)^{3/2}}{3d}$$

```
[Out] 2/3*f*(d*x+c)^(3/2)/d-2*e*arctanh((d*x+c)^(1/2)/c^(1/2))*c^(1/2)+2*e*(d*x+c
)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {80, 50, 63, 208}

$$2e\sqrt{c+dx} - 2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right) + \frac{2f(c+dx)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*(e + f*x))/x,x]
```

```
[Out] 2*e*Sqrt[c + d*x] + (2*f*(c + d*x)^(3/2))/(3*d) - 2*Sqrt[c]*e*ArcTanh[Sqrt[
c + d*x]/Sqrt[c]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+dx}\,(e+fx)}{x}\,dx &= \frac{2f(c+dx)^{3/2}}{3d} + e\int \frac{\sqrt{c+dx}}{x}\,dx \\
&= 2e\sqrt{c+dx} + \frac{2f(c+dx)^{3/2}}{3d} + (ce)\int \frac{1}{x\sqrt{c+dx}}\,dx \\
&= 2e\sqrt{c+dx} + \frac{2f(c+dx)^{3/2}}{3d} + \frac{(2ce)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{d} \\
&= 2e\sqrt{c+dx} + \frac{2f(c+dx)^{3/2}}{3d} - 2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)
\end{aligned}$$

Mathematica [A] time = 0.05, size = 55, normalized size = 1.02

$$e\left(2\sqrt{c+dx} - 2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)\right) + \frac{2f(c+dx)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*(e + f*x))/x,x]
```

```
[Out] (2*f*(c + d*x)^(3/2))/(3*d) + e*(2*Sqrt[c + d*x] - 2*Sqrt[c]*ArcTanh[Sqrt[c
 + d*x]/Sqrt[c]])
```

fricas [A] time = 1.27, size = 111, normalized size = 2.06

$$\left[\frac{3\sqrt{c}\,de\log\left(\frac{dx-2\sqrt{dx+c}\sqrt{c}+2c}{x}\right) + 2\left(dfx+3de+cf\right)\sqrt{dx+c}}{3d}, \frac{2\left(3\sqrt{-c}\,de\arctan\left(\frac{\sqrt{dx+c}\sqrt{-c}}{c}\right) + \left(dfx+3de+\right.\right.}{3d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/3*(3*sqrt(c)*d*e*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 2*(d*f*x
 + 3*d*e + c*f)*sqrt(d*x + c))/d, 2/3*(3*sqrt(-c)*d*e*arctan(sqrt(d*x + c)*
sqrt(-c)/c) + (d*f*x + 3*d*e + c*f)*sqrt(d*x + c))/d]
```

giac [A] time = 1.24, size = 57, normalized size = 1.06

$$\frac{2\,c\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)e}{\sqrt{-c}} + \frac{2\left((dx+c)^{\frac{3}{2}}d^2f + 3\sqrt{dx+c}\,d^3e\right)}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*c*arctan(sqrt(d*x + c)/sqrt(-c))*e/sqrt(-c) + 2/3*((d*x + c)^(3/2)*d^2*f
+ 3*sqrt(d*x + c)*d^3*e)/d^3
```

maple [A] time = 0.01, size = 46, normalized size = 0.85

$$\frac{-2\sqrt{c}\,de\,\mathrm{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right) + 2\sqrt{dx+c}\,de + \frac{2(dx+c)^{\frac{3}{2}}f}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(d*x+c)^(1/2)/x,x)
```

```
[Out] 2/d*(1/3*f*(d*x+c)^(3/2)+d*e*(d*x+c)^(1/2)-c^(1/2)*d*e*arctanh((d*x+c)^(1/2
)/c^(1/2)))
```

maxima [A] time = 0.98, size = 60, normalized size = 1.11

$$\sqrt{c}\,e\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)+\frac{2\left(3\sqrt{dx+c}\,de+(dx+c)^{\frac{3}{2}}f\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] sqrt(c)*e*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c))) + 2/3*(3
*sqrt(d*x + c)*d*e + (d*x + c)^(3/2)*f)/d
```

mupad [B] time = 0.07, size = 45, normalized size = 0.83

$$2\,e\,\sqrt{c+d\,x}+\frac{2\,f\,{\left(c+d\,x\right)}^{3/2}}{3\,d}+\sqrt{c}\,e\,\mathrm{atan}\left(\frac{\sqrt{c+d\,x}\,1{}\mathrm{i}}{\sqrt{c}}\right)\,2{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(c + d*x)^(1/2))/x,x)
```

```
[Out] 2*e*(c + d*x)^(1/2) + c^(1/2)*e*atan(((c + d*x)^(1/2)*1i)/c^(1/2))*2i + (2*
f*(c + d*x)^(3/2))/(3*d)
```

sympy [A] time = 5.98, size = 54, normalized size = 1.00

$$\frac{2ce\,\mathrm{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}}+2e\sqrt{c+dx}+\frac{2f\,(c+dx)^{\frac{3}{2}}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)**(1/2)/x,x)
```

```
[Out] 2*c*e*atan(sqrt(c + d*x)/sqrt(-c))/sqrt(-c) + 2*e*sqrt(c + d*x) + 2*f*(c +
d*x)**(3/2)/(3*d)
```

3.12 $\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)}\,dx$

Optimal. Leaf size=101

$$\frac{2\sqrt{bc-ad}\,(be-af)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{ab^{3/2}}-\frac{2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a}+\frac{2f\sqrt{c+dx}}{b}$$

```
[Out] -2*e*arctanh((d*x+c)^(1/2)/c^(1/2))*c^(1/2)/a+2*(-a*f+b*e)*arctanh(b^(1/2)*
(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/a/b^(3/2)+2*f*(d*x+c)^(1/2
)/b
```

Rubi [A] time = 0.11, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {154, 156, 63, 208}

$$\frac{2\sqrt{bc-ad}\,(be-af)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{ab^{3/2}}-\frac{2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a}+\frac{2f\sqrt{c+dx}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*(e + f*x))/(x*(a + b*x)),x]
```

```
[Out] (2*f*Sqrt[c + d*x])/b - (2*Sqrt[c]*e*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/a + (2
*Sqrt[b*c - a*d]*(b*e - a*f)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d
]])/(a*b^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)}\,dx = \frac{2f\sqrt{c+dx}}{b} + \frac{2\int \frac{\frac{bce}{2}+\frac{1}{2}(bde+bcf-adf)x}{x(a+bx)\sqrt{c+dx}}\,dx}{b}$$
$$= \frac{2f\sqrt{c+dx}}{b} + \frac{(ce)\int \frac{1}{x\sqrt{c+dx}}\,dx}{a} - \frac{((bc-ad)(be-af))\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{ab}$$
$$= \frac{2f\sqrt{c+dx}}{b} + \frac{(2ce)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{ad} - \frac{(2(bc-ad)(be-af))\,\text{Subst}\left(\int -\right.}{abd}$$
$$= \frac{2f\sqrt{c+dx}}{b} - \frac{2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a} + \frac{2\sqrt{bc-ad}\,(be-af)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{ab^{3/2}}$$

Mathematica [A] time = 0.12, size = 101, normalized size = 1.00

$$\frac{2\sqrt{bc-ad}\,(be-af)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{ab^{3/2}} - \frac{2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a} + \frac{2f\sqrt{c+dx}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*(e + f*x))/(x*(a + b*x)),x]
```

```
[Out] (2*f*Sqrt[c + d*x])/b - (2*Sqrt[c]*e*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/a + (2
*Sqrt[b*c - a*d]*(b*e - a*f)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d
]])/(a*b^(3/2))
```

fricas [A] time = 1.65, size = 449, normalized size = 4.45

$$\left[\frac{b\sqrt{c}\,e\log\left(\frac{dx-2\sqrt{dx+c}\sqrt{c}+2c}{x}\right) + 2\sqrt{dx+c}\,af - (be-af)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx+2bc-ad-2\sqrt{dx+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx+a}\right)}{ab}, \frac{b\sqrt{c}\,e\log\left(\frac{dx-2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a),x, algorithm="fricas")
```

```
[Out] [(b*sqrt(c)*e*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 2*sqrt(d*x + c
)*a*f - (b*e - a*f)*sqrt((b*c - a*d)/b)*log((b*d*x + 2*b*c - a*d - 2*sqrt(d
*x + c)*b*sqrt((b*c - a*d)/b))/(b*x + a)))/(a*b), (b*sqrt(c)*e*log((d*x - 2
*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 2*sqrt(d*x + c)*a*f + 2*(b*e - a*f)*sqrt
(-(b*c - a*d)/b)*arctan(-sqrt(d*x + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)))
/(a*b), (2*b*sqrt(-c)*e*arctan(sqrt(d*x + c)*sqrt(-c)/c) + 2*sqrt(d*x + c)*
a*f - (b*e - a*f)*sqrt((b*c - a*d)/b)*log((b*d*x + 2*b*c - a*d - 2*sqrt(d*x
 + c)*b*sqrt((b*c - a*d)/b))/(b*x + a)))/(a*b), 2*(b*sqrt(-c)*e*arctan(sqrt
(d*x + c)*sqrt(-c)/c) + sqrt(d*x + c)*a*f + (b*e - a*f)*sqrt(-(b*c - a*d)/b
)*arctan(-sqrt(d*x + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)))/(a*b)]
```

giac [A] time = 1.35, size = 112, normalized size = 1.11

$$\frac{2\,c\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)e}{a\sqrt{-c}} + \frac{2\sqrt{dx+c}\,f}{b} + \frac{2\left(abcf-a^2df-b^2ce+abde\right)\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a),x, algorithm="giac")
```

```
[Out] 2*c*arctan(sqrt(d*x + c)/sqrt(-c))*e/(a*sqrt(-c)) + 2*sqrt(d*x + c)*f/b + 2
*(a*b*c*f - a^2*d*f - b^2*c*e + a*b*d*e)*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c
 + a*b*d))/(sqrt(-b^2*c + a*b*d)*a*b)
```

maple [**B**] time = 0.02, size = 196, normalized size = 1.94

$$-\frac{2adf\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)\,b}\;b}-\frac{2bce\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)\,b}\;a}+\frac{2cf\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)\,b}}+\frac{2de\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)\,b}}-\frac{2\sqrt{c}\;e\,\mathrm{ar}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a),x)
```

```
[Out] 2*f*(d*x+c)^(1/2)/b-2*a/b/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*
c)*b)^(1/2)*b)*d*f+2/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)
^(1/2)*b)*c*f+2/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2
)*b)*d*e-2/a*b/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)
*b)*c*e-2*e*arctanh((d*x+c)^(1/2)/c^(1/2))*c^(1/2)/a
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [**B**] time = 2.87, size = 2368, normalized size = 23.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(c + d*x)^(1/2))/(x*(a + b*x)),x)
```

```
[Out] (2*f*(c + d*x)^(1/2))/b - (c^(1/2)*e*atan(((c^(1/2)*e*((8*(c + d*x)^(1/2)*(
a^4*d^4*f^2 + a^2*b^2*d^4*e^2 + 2*b^4*c^2*d^2*e^2 - 2*a^3*b*d^4*e*f + a^2*b
^2*c^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a^3*b*c*d^3*f^2 - 2*a*b^3*c^2*d^2*e*
f + 4*a^2*b^2*c*d^3*e*f))/b + (c^(1/2)*e*((8*(a^3*b^2*c*d^3*f - a^2*b^3*c^2
*d^2*f))/b + (8*c^(1/2)*e*(a^3*b^3*d^3 - 2*a^2*b^4*c*d^2)*(c + d*x)^(1/2))/
(a*b)))/a)*1i)/a + (c^(1/2)*e*((8*(c + d*x)^(1/2)*(a^4*d^4*f^2 + a^2*b^2*d^
4*e^2 + 2*b^4*c^2*d^2*e^2 - 2*a^3*b*d^4*e*f + a^2*b^2*c^2*d^2*f^2 - 2*a*b^3
*c*d^3*e^2 - 2*a^3*b*c*d^3*f^2 - 2*a*b^3*c^2*d^2*e*f + 4*a^2*b^2*c*d^3*e*f)
)/b - (c^(1/2)*e*((8*(a^3*b^2*c*d^3*f - a^2*b^3*c^2*d^2*f))/b - (8*c^(1/2)*
e*(a^3*b^3*d^3 - 2*a^2*b^4*c*d^2)*(c + d*x)^(1/2))/(a*b)))/a)*1i)/a)/((16*(
b^3*c^2*d^3*e^3 - a*b^2*c*d^4*e^3 - a^3*c*d^4*e*f^2 + b^3*c^3*d^2*e^2*f - 3
*a*b^2*c^2*d^3*e^2*f - a*b^2*c^3*d^2*e*f^2 + 2*a^2*b*c^2*d^3*e*f^2 + 2*a^2*
b*c*d^4*e^2*f))/b - (c^(1/2)*e*((8*(c + d*x)^(1/2)*(a^4*d^4*f^2 + a^2*b^2*d
^4*e^2 + 2*b^4*c^2*d^2*e^2 - 2*a^3*b*d^4*e*f + a^2*b^2*c^2*d^2*f^2 - 2*a*b^
3*c*d^3*e^2 - 2*a^3*b*c*d^3*f^2 - 2*a*b^3*c^2*d^2*e*f + 4*a^2*b^2*c*d^3*e*f
))/b + (c^(1/2)*e*((8*(a^3*b^2*c*d^3*f - a^2*b^3*c^2*d^2*f))/b + (8*c^(1/2)
*e*(a^3*b^3*d^3 - 2*a^2*b^4*c*d^2)*(c + d*x)^(1/2))/(a*b)))/a))/a + (c^(1/2
)*e*((8*(c + d*x)^(1/2)*(a^4*d^4*f^2 + a^2*b^2*d^4*e^2 + 2*b^4*c^2*d^2*e^2
- 2*a^3*b*d^4*e*f + a^2*b^2*c^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a^3*b*c*d^3
*f^2 - 2*a*b^3*c^2*d^2*e*f + 4*a^2*b^2*c*d^3*e*f))/b - (c^(1/2)*e*((8*(a^3*
```

```
b^2*c*d^3*f - a^2*b^3*c^2*d^2*f))/b - (8*c^(1/2)*e*(a^3*b^3*d^3 - 2*a^2*b^4
*c*d^2)*(c + d*x)^(1/2))/(a*b)))/a))/a))*2i)/a - (atan(((((8*(c + d*x)^(1/2
)*(a^4*d^4*f^2 + a^2*b^2*d^4*e^2 + 2*b^4*c^2*d^2*e^2 - 2*a^3*b*d^4*e*f + a^
2*b^2*c^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a^3*b*c*d^3*f^2 - 2*a*b^3*c^2*d^2
*e*f + 4*a^2*b^2*c*d^3*e*f))/b + (((8*(a^3*b^2*c*d^3*f - a^2*b^3*c^2*d^2*f)
)/b + (8*(a^3*b^3*d^3 - 2*a^2*b^4*c*d^2)*(a*f - b*e)*(-b^3*(a*d - b*c))^(1/
2)*(c + d*x)^(1/2))/(a*b^4))*(a*f - b*e)*(-b^3*(a*d - b*c))^(1/2))/(a*b^3))
*(a*f - b*e)*(-b^3*(a*d - b*c))^(1/2)*1i)/(a*b^3) + (((8*(c + d*x)^(1/2)*(a
^4*d^4*f^2 + a^2*b^2*d^4*e^2 + 2*b^4*c^2*d^2*e^2 - 2*a^3*b*d^4*e*f + a^2*b^
2*c^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a^3*b*c*d^3*f^2 - 2*a*b^3*c^2*d^2*e*f
 + 4*a^2*b^2*c*d^3*e*f))/b - (((8*(a^3*b^2*c*d^3*f - a^2*b^3*c^2*d^2*f))/b
- (8*(a^3*b^3*d^3 - 2*a^2*b^4*c*d^2)*(a*f - b*e)*(-b^3*(a*d - b*c))^(1/2)*(
c + d*x)^(1/2))/(a*b^4))*(a*f - b*e)*(-b^3*(a*d - b*c))^(1/2))/(a*b^3))*(a*
f - b*e)*(-b^3*(a*d - b*c))^(1/2)*1i)/(a*b^3))/((16*(b^3*c^2*d^3*e^3 - a*b^
2*c*d^4*e^3 - a^3*c*d^4*e*f^2 + b^3*c^3*d^2*e^2*f - 3*a*b^2*c^2*d^3*e^2*f -
 a*b^2*c^3*d^2*e*f^2 + 2*a^2*b*c^2*d^3*e*f^2 + 2*a^2*b*c*d^4*e^2*f))/b - ((
(8*(c + d*x)^(1/2)*(a^4*d^4*f^2 + a^2*b^2*d^4*e^2 + 2*b^4*c^2*d^2*e^2 - 2*a
^3*b*d^4*e*f + a^2*b^2*c^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a^3*b*c*d^3*f^2
- 2*a*b^3*c^2*d^2*e*f + 4*a^2*b^2*c*d^3*e*f))/b + (((8*(a^3*b^2*c*d^3*f - a
^2*b^3*c^2*d^2*f))/b + (8*(a^3*b^3*d^3 - 2*a^2*b^4*c*d^2)*(a*f - b*e)*(-b^3
*(a*d - b*c))^(1/2)*(c + d*x)^(1/2))/(a*b^4))*(a*f - b*e)*(-b^3*(a*d - b*c)
)^(1/2))/(a*b^3))*(a*f - b*e)*(-b^3*(a*d - b*c))^(1/2))/(a*b^3) + (((8*(c +
 d*x)^(1/2)*(a^4*d^4*f^2 + a^2*b^2*d^4*e^2 + 2*b^4*c^2*d^2*e^2 - 2*a^3*b*d^
4*e*f + a^2*b^2*c^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a^3*b*c*d^3*f^2 - 2*a*b
^3*c^2*d^2*e*f + 4*a^2*b^2*c*d^3*e*f))/b - (((8*(a^3*b^2*c*d^3*f - a^2*b^3*
c^2*d^2*f))/b - (8*(a^3*b^3*d^3 - 2*a^2*b^4*c*d^2)*(a*f - b*e)*(-b^3*(a*d -
 b*c))^(1/2)*(c + d*x)^(1/2))/(a*b^4))*(a*f - b*e)*(-b^3*(a*d - b*c))^(1/2)
)/(a*b^3))*(a*f - b*e)*(-b^3*(a*d - b*c))^(1/2))/(a*b^3)))*(a*f - b*e)*(-b^
3*(a*d - b*c))^(1/2)*2i)/(a*b^3)
```

sympy [A] time = 27.33, size = 97, normalized size = 0.96

$$\frac{2f\sqrt{c+dx}}{b}+\frac{2ce\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{a\sqrt{-c}}-\frac{2\left(ad-bc\right)\left(af-be\right)\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{\frac{ad-bc}{b}}}\right)}{ab^{2}\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)**(1/2)/x/(b*x+a),x)
```

```
[Out] 2*f*sqrt(c + d*x)/b + 2*c*e*atan(sqrt(c + d*x)/sqrt(-c))/(a*sqrt(-c)) - 2*(
a*d - b*c)*(a*f - b*e)*atan(sqrt(c + d*x)/sqrt((a*d - b*c)/b))/(a*b**2*sqrt
((a*d - b*c)/b))
```

3.13 $\int \frac{\sqrt{c+dx}\,(e+fx)}{x(a+bx)^2}\,dx$

Optimal. Leaf size=127

$$\frac{\left(2b^2ce-ad(af+be)\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{a^2b^{3/2}\sqrt{bc-ad}}-\frac{2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2}+\frac{\sqrt{c+dx}\,(be-af)}{ab(a+bx)}$$

```
[Out] -2*e*arctanh((d*x+c)^(1/2)/c^(1/2))*c^(1/2)/a^2+(2*b^2*c*e-a*d*(a*f+b*e))*a
rctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/a^2/b^(3/2)/(-a*d+b*c)^(1/2)
+(-a*f+b*e)*(d*x+c)^(1/2)/a/b/(b*x+a)
```

Rubi [A] time = 0.11, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {149, 156, 63, 208}

$$\frac{\left(2b^2ce-ad(af+be)\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{a^2b^{3/2}\sqrt{bc-ad}}-\frac{2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2}+\frac{\sqrt{c+dx}\,(be-af)}{ab(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*(e + f*x))/(x*(a + b*x)^2),x]
```

```
[Out] ((b*e - a*f)*Sqrt[c + d*x])/(a*b*(a + b*x)) - (2*Sqrt[c]*e*ArcTanh[Sqrt[c +
 d*x]/Sqrt[c]])/a^2 + ((2*b^2*c*e - a*d*(b*e + a*f))*ArcTanh[(Sqrt[b]*Sqrt[
c + d*x])/Sqrt[b*c - a*d]])/(a^2*b^(3/2)*Sqrt[b*c - a*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}(e+fx)}{x(a+bx)^2}\,dx = \frac{(be-af)\sqrt{c+dx}}{ab(a+bx)} - \frac{\int \frac{-bce-\frac{1}{2}d(be+af)x}{x(a+bx)\sqrt{c+dx}}\,dx}{ab}$$

$$= \frac{(be-af)\sqrt{c+dx}}{ab(a+bx)} + \frac{(ce)\int \frac{1}{x\sqrt{c+dx}}\,dx}{a^2} + \frac{\left(-b^2ce+\frac{1}{2}ad(be+af)\right)\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{a^2b}$$

$$= \frac{(be-af)\sqrt{c+dx}}{ab(a+bx)} + \frac{(2ce)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{a^2d} + \frac{\left(2\left(-b^2ce+\frac{1}{2}ad(be+af)\right.\right.}{}$$

$$= \frac{(be-af)\sqrt{c+dx}}{ab(a+bx)} - \frac{2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2} + \frac{\left(2b^2ce-ad(be+af)\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{a^2b^{3/2}\sqrt{bc-ad}}$$

Mathematica [A] time = 0.44, size = 124, normalized size = 0.98

$$\frac{-\frac{\left(a^2df+abde-2b^2ce\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}} + \frac{a\sqrt{c+dx}(be-af)}{b(a+bx)} - 2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*(e + f*x))/(x*(a + b*x)^2),x]
```

```
[Out] ((a*(b*e - a*f)*Sqrt[c + d*x])/(b*(a + b*x)) - 2*Sqrt[c]*e*ArcTanh[Sqrt[c +
 d*x]/Sqrt[c]] - ((-2*b^2*c*e + a*b*d*e + a^2*d*f)*ArcTanh[(Sqrt[b]*Sqrt[c
+ d*x])/Sqrt[b*c - a*d]])/(b^(3/2)*Sqrt[b*c - a*d]))/a^2
```

fricas [B] time = 1.11, size = 1018, normalized size = 8.02

$$\left[\frac{\left(a^3df-\left(2\,ab^2c-a^2bd\right)e+\left(a^2bdf-\left(2\,b^3c-ab^2d\right)e\right)x\right)\sqrt{b^2c-abd}\,\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{b^2c-abd}\,\sqrt{dx+c}}{bx+a}\right)+2\left(\left(b^4c-\right.\right.}{2\left(a^3b^3c-a^4b^2d+\left(a^2b\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*((a^3*d*f - (2*a*b^2*c - a^2*b*d)*e + (a^2*b*d*f - (2*b^3*c - a*b^2*d)
*e)*x)*sqrt(b^2*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)
*sqrt(d*x + c))/(b*x + a)) + 2*((b^4*c - a*b^3*d)*e*x + (a*b^3*c - a^2*b^2*
d)*e)*sqrt(c)*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 2*((a*b^3*c -
a^2*b^2*d)*e - (a^2*b^2*c - a^3*b*d)*f)*sqrt(d*x + c))/(a^3*b^3*c - a^4*b^2
*d + (a^2*b^4*c - a^3*b^3*d)*x), ((a^3*d*f - (2*a*b^2*c - a^2*b*d)*e + (a^2
*b*d*f - (2*b^3*c - a*b^2*d)*e)*x)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(-b^2*c
+ a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) + ((b^4*c - a*b^3*d)*e*x + (a*b^3*c -
 a^2*b^2*d)*e)*sqrt(c)*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + ((a*b
^3*c - a^2*b^2*d)*e - (a^2*b^2*c - a^3*b*d)*f)*sqrt(d*x + c))/(a^3*b^3*c -
a^4*b^2*d + (a^2*b^4*c - a^3*b^3*d)*x), 1/2*(4*((b^4*c - a*b^3*d)*e*x + (a*
b^3*c - a^2*b^2*d)*e)*sqrt(-c)*arctan(sqrt(d*x + c)*sqrt(-c)/c) + (a^3*d*f
- (2*a*b^2*c - a^2*b*d)*e + (a^2*b*d*f - (2*b^3*c - a*b^2*d)*e)*x)*sqrt(b^2
*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))
/(b*x + a)) + 2*((a*b^3*c - a^2*b^2*d)*e - (a^2*b^2*c - a^3*b*d)*f)*sqrt(d*
x + c))/(a^3*b^3*c - a^4*b^2*d + (a^2*b^4*c - a^3*b^3*d)*x), ((a^3*d*f - (2
*a*b^2*c - a^2*b*d)*e + (a^2*b*d*f - (2*b^3*c - a*b^2*d)*e)*x)*sqrt(-b^2*c
+ a*b*d)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) + 2*((b^4
```

```
*c - a*b^3*d)*e*x + (a*b^3*c - a^2*b^2*d)*e)*sqrt(-c)*arctan(sqrt(d*x + c)*
sqrt(-c)/c) + ((a*b^3*c - a^2*b^2*d)*e - (a^2*b^2*c - a^3*b*d)*f)*sqrt(d*x
+ c))/(a^3*b^3*c - a^4*b^2*d + (a^2*b^4*c - a^3*b^3*d)*x)]
```

giac [A] time = 1.42, size = 142, normalized size = 1.12

$$\frac{2\,c\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)e}{a^2\sqrt{-c}}+\frac{\left(a^2df-2\,b^2ce+abde\right)\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,a^2b}-\frac{\sqrt{dx+c}\,adf-\sqrt{dx+c}\,bde}{((dx+c)b-bc+ad)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 2*c*arctan(sqrt(d*x + c)/sqrt(-c))*e/(a^2*sqrt(-c)) + (a^2*d*f - 2*b^2*c*e
+ a*b*d*e)*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*
d)*a^2*b) - (sqrt(d*x + c)*a*d*f - sqrt(d*x + c)*b*d*e)/(((d*x + c)*b - b*c
 + a*d)*a*b)
```

maple [A] time = 0.02, size = 192, normalized size = 1.51

$$\frac{de\arctan\left(\frac{\sqrt{dx+c}\;b}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)\,b}\;a}-\frac{2bce\arctan\left(\frac{\sqrt{dx+c}\;b}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)\,b}\;a^2}+\frac{df\arctan\left(\frac{\sqrt{dx+c}\;b}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)\,b}\;b}+\frac{\sqrt{dx+c}\;de}{(bdx+ad)\,a}-\frac{\sqrt{dx+c}\;df}{(bdx+ad)\,b}-\frac{2\sqrt{c}\;e\,\mathrm{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a)^2,x)
```

```
[Out] -d/b*(d*x+c)^(1/2)/(b*d*x+a*d)*f+d/a*(d*x+c)^(1/2)/(b*d*x+a*d)*e+d/b/((a*d-
b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)*f+d/a/((a*d-b*c)*
b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)*e-2/a^2*b/((a*d-b*c)*b
)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)*c*e-2*e*arctanh((d*x+c)
^(1/2)/c^(1/2))*c^(1/2)/a^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 0.60, size = 1827, normalized size = 14.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(c + d*x)^(1/2))/(x*(a + b*x)^2),x)
```

```
[Out] (atan((((((2*(2*a^4*b^3*c*d^3*e - 2*a^5*b^2*c*d^3*f))/(a^3*b) + ((4*a^5*b^
3*d^3 - 8*a^4*b^4*c*d^2)*(-b^3*(a*d - b*c))^(1/2)*(c + d*x)^(1/2)*(a^2*d*f
- 2*b^2*c*e + a*b*d*e))/(a^2*b*(a^2*b^4*c - a^3*b^3*d)))*(-b^3*(a*d - b*c))
^(1/2)*(a^2*d*f - 2*b^2*c*e + a*b*d*e))/(2*(a^2*b^4*c - a^3*b^3*d)) + (2*(c
 + d*x)^(1/2)*(a^4*d^4*f^2 + a^2*b^2*d^4*e^2 + 8*b^4*c^2*d^2*e^2 + 2*a^3*b*
d^4*e*f - 4*a*b^3*c*d^3*e^2 - 4*a^2*b^2*c*d^3*e*f))/(a^2*b))*(-b^3*(a*d - b
*c))^(1/2)*(a^2*d*f - 2*b^2*c*e + a*b*d*e)*1i)/(2*(a^2*b^4*c - a^3*b^3*d))
```

```
- (((((2*(2*a^4*b^3*c*d^3*e - 2*a^5*b^2*c*d^3*f))/(a^3*b) - ((4*a^5*b^3*d^3
 - 8*a^4*b^4*c*d^2)*(-b^3*(a*d - b*c))^(1/2)*(c + d*x)^(1/2)*(a^2*d*f - 2*b
^2*c*e + a*b*d*e))/(a^2*b*(a^2*b^4*c - a^3*b^3*d)))*(-b^3*(a*d - b*c))^(1/2
)*(a^2*d*f - 2*b^2*c*e + a*b*d*e))/(2*(a^2*b^4*c - a^3*b^3*d)) - (2*(c + d*
x)^(1/2)*(a^4*d^4*f^2 + a^2*b^2*d^4*e^2 + 8*b^4*c^2*d^2*e^2 + 2*a^3*b*d^4*e
*f - 4*a*b^3*c*d^3*e^2 - 4*a^2*b^2*c*d^3*e*f))/(a^2*b))*(-b^3*(a*d - b*c))^
(1/2)*(a^2*d*f - 2*b^2*c*e + a*b*d*e)*1i)/(2*(a^2*b^4*c - a^3*b^3*d)))/((4*
(a*b^2*c*d^4*e^3 - 2*b^3*c^2*d^3*e^3 + a^3*c*d^4*e*f^2 - 2*a*b^2*c^2*d^3*e^
2*f + 2*a^2*b*c*d^4*e^2*f))/(a^3*b) + (((((2*(2*a^4*b^3*c*d^3*e - 2*a^5*b^2
*c*d^3*f))/(a^3*b) + ((4*a^5*b^3*d^3 - 8*a^4*b^4*c*d^2)*(-b^3*(a*d - b*c))^
(1/2)*(c + d*x)^(1/2)*(a^2*d*f - 2*b^2*c*e + a*b*d*e))/(a^2*b*(a^2*b^4*c -
a^3*b^3*d)))*(-b^3*(a*d - b*c))^(1/2)*(a^2*d*f - 2*b^2*c*e + a*b*d*e))/(2*(
a^2*b^4*c - a^3*b^3*d)) + (2*(c + d*x)^(1/2)*(a^4*d^4*f^2 + a^2*b^2*d^4*e^2
 + 8*b^4*c^2*d^2*e^2 + 2*a^3*b*d^4*e*f - 4*a*b^3*c*d^3*e^2 - 4*a^2*b^2*c*d^
3*e*f))/(a^2*b))*(-b^3*(a*d - b*c))^(1/2)*(a^2*d*f - 2*b^2*c*e + a*b*d*e))/
(2*(a^2*b^4*c - a^3*b^3*d)) + (((((2*(2*a^4*b^3*c*d^3*e - 2*a^5*b^2*c*d^3*f
))/(a^3*b) - ((4*a^5*b^3*d^3 - 8*a^4*b^4*c*d^2)*(-b^3*(a*d - b*c))^(1/2)*(c
 + d*x)^(1/2)*(a^2*d*f - 2*b^2*c*e + a*b*d*e))/(a^2*b*(a^2*b^4*c - a^3*b^3*
d)))*(-b^3*(a*d - b*c))^(1/2)*(a^2*d*f - 2*b^2*c*e + a*b*d*e))/(2*(a^2*b^4*
c - a^3*b^3*d)) - (2*(c + d*x)^(1/2)*(a^4*d^4*f^2 + a^2*b^2*d^4*e^2 + 8*b^4
*c^2*d^2*e^2 + 2*a^3*b*d^4*e*f - 4*a*b^3*c*d^3*e^2 - 4*a^2*b^2*c*d^3*e*f))/
(a^2*b))*(-b^3*(a*d - b*c))^(1/2)*(a^2*d*f - 2*b^2*c*e + a*b*d*e))/(2*(a^2*
b^4*c - a^3*b^3*d))))*(-b^3*(a*d - b*c))^(1/2)*(a^2*d*f - 2*b^2*c*e + a*b*d
*e)*1i)/(a^2*b^4*c - a^3*b^3*d) - (2*c^(1/2)*e*atanh((4*c^(1/2)*d^4*e*f^2*(
c + d*x)^(1/2))/(4*c*d^4*e*f^2 + (4*b^2*c*d^4*e^3)/a^2 - (16*b^2*c^2*d^3*e^
2*f)/a^2 + (8*b*c*d^4*e^2*f)/a) + (8*c^(1/2)*d^4*e^2*f*(c + d*x)^(1/2))/(8*
c*d^4*e^2*f + (4*b*c*d^4*e^3)/a - (16*b*c^2*d^3*e^2*f)/a + (4*a*c*d^4*e*f^2
)/b) + (4*b*c^(1/2)*d^4*e^3*(c + d*x)^(1/2))/(4*b*c*d^4*e^3 + 8*a*c*d^4*e^2
*f - 16*b*c^2*d^3*e^2*f + (4*a^2*c*d^4*e*f^2)/b) - (16*b*c^(3/2)*d^3*e^2*f*
(c + d*x)^(1/2))/(4*b*c*d^4*e^3 + 8*a*c*d^4*e^2*f - 16*b*c^2*d^3*e^2*f + (4
*a^2*c*d^4*e*f^2)/b)))/a^2 - ((a*d*f - b*d*e)*(c + d*x)^(1/2))/(a*b*(a*d -
b*c + b*(c + d*x)))
```

sympy [B] time = 133.87, size = 1204, normalized size = 9.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)**(1/2)/x/(b*x+a)**2,x)
```

```
[Out] -2*a*d**2*f*sqrt(c + d*x)/(2*a**2*b*d**2 - 2*a*b**2*c*d + 2*a*b**2*d**2*x -
 2*b**3*c*d*x) + a*d**2*f*sqrt(-1/(b*(a*d - b*c)**3))*log(-a**2*d**2*sqrt(-
1/(b*(a*d - b*c)**3)) + 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) - b**2*c**2*s
qrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/(2*b) - a*d**2*f*sqrt(-1/(b*(a*
d - b*c)**3))*log(a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) - 2*a*b*c*d*sqrt(-1
/(b*(a*d - b*c)**3)) + b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x
))/(2*b) - 2*b*c*d*e*sqrt(c + d*x)/(2*a**3*d**2 - 2*a**2*b*c*d + 2*a**2*b*d
**2*x - 2*a*b**2*c*d*x) - c*d*f*sqrt(-1/(b*(a*d - b*c)**3))*log(-a**2*d**2*
sqrt(-1/(b*(a*d - b*c)**3)) + 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) - b**2*
c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/2 + c*d*f*sqrt(-1/(b*(a*d
 - b*c)**3))*log(a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) - 2*a*b*c*d*sqrt(-1/
(b*(a*d - b*c)**3)) + b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x)
)/2 + 2*c*d*f*sqrt(c + d*x)/(2*a**2*d**2 - 2*a*b*c*d + 2*a*b*d**2*x - 2*b**
2*c*d*x) - d**2*e*sqrt(-1/(b*(a*d - b*c)**3))*log(-a**2*d**2*sqrt(-1/(b*(a*
d - b*c)**3)) + 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) - b**2*c**2*sqrt(-1/(
b*(a*d - b*c)**3)) + sqrt(c + d*x))/2 + d**2*e*sqrt(-1/(b*(a*d - b*c)**3))*
log(a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) - 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c
)**3)) + b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/2 + 2*d**2*
e*sqrt(c + d*x)/(2*a**2*d**2 - 2*a*b*c*d + 2*a*b*d**2*x - 2*b**2*c*d*x) + 2
*d*f*atan(sqrt(c + d*x)/sqrt(a*d/b - c))/(b**2*sqrt(a*d/b - c)) + b*c*d*e*s
```

```
qrt(-1/(b*(a*d - b*c)**3))*log(-a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) + 2*a
*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) - b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3))
+ sqrt(c + d*x))/(2*a) - b*c*d*e*sqrt(-1/(b*(a*d - b*c)**3))*log(a**2*d**2*
sqrt(-1/(b*(a*d - b*c)**3)) - 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) + b**2*
c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/(2*a) - 2*c*e*atan(sqrt(c
 + d*x)/sqrt(a*d/b - c))/(a**2*sqrt(a*d/b - c)) + 2*c*e*atan(sqrt(c + d*x)/
sqrt(-c))/(a**2*sqrt(-c))
```

3.14 $\int \frac{\sqrt{c+dx}\,(e+fx)}{x(a+bx)^3}\,dx$

Optimal. Leaf size=208

$$-\frac{2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^3}+\frac{\sqrt{c+dx}\,\left(a^2(-d)f-3abde+4b^2ce\right)}{4a^2b(a+bx)(bc-ad)}+\frac{\left(a^3d^2f+3a^2bd^2e-12ab^2cde+8b^3c^2e\right)\tanh^{-1}\left(\ldots\right.}{4a^3b^{3/2}(bc-ad)^{3/2}}$$

```
[Out] 1/4*(a^3*d^2*f+3*a^2*b*d^2*e-12*a*b^2*c*d*e+8*b^3*c^2*e)*arctanh(b^(1/2)*(d
*x+c)^(1/2)/(-a*d+b*c)^(1/2))/a^3/b^(3/2)/(-a*d+b*c)^(3/2)-2*e*arctanh((d*x
+c)^(1/2)/c^(1/2))*c^(1/2)/a^3+1/2*(-a*f+b*e)*(d*x+c)^(1/2)/a/b/(b*x+a)^2+1
/4*(-a^2*d*f-3*a*b*d*e+4*b^2*c*e)*(d*x+c)^(1/2)/a^2/b/(-a*d+b*c)/(b*x+a)
```

Rubi [A] time = 0.27, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {149, 151, 156, 63, 208}

$$\frac{\left(3a^2bd^2e+a^3d^2f-12ab^2cde+8b^3c^2e\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4a^3b^{3/2}(bc-ad)^{3/2}}+\frac{\sqrt{c+dx}\,\left(a^2(-d)f-3abde+4b^2ce\right)}{4a^2b(a+bx)(bc-ad)}-\frac{2\sqrt{c}\,e\tanh^{-1}\left(\ldots\right.}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*(e + f*x))/(x*(a + b*x)^3),x]
```

```
[Out] ((b*e - a*f)*Sqrt[c + d*x])/(2*a*b*(a + b*x)^2) + ((4*b^2*c*e - 3*a*b*d*e -
 a^2*d*f)*Sqrt[c + d*x])/(4*a^2*b*(b*c - a*d)*(a + b*x)) - (2*Sqrt[c]*e*Arc
Tanh[Sqrt[c + d*x]/Sqrt[c]])/a^3 + ((8*b^3*c^2*e - 12*a*b^2*c*d*e + 3*a^2*b
*d^2*e + a^3*d^2*f)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(4*a^
3*b^(3/2)*(b*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\,(e+fx)}{x(a+bx)^3}\,dx = \frac{(be-af)\sqrt{c+dx}}{2ab(a+bx)^2} - \frac{\int \frac{-2bce-\frac{1}{2}d(3be+af)x}{x(a+bx)^2\sqrt{c+dx}}\,dx}{2ab}$$

$$= \frac{(be-af)\sqrt{c+dx}}{2ab(a+bx)^2} + \frac{\left(4b^2ce-3abde-a^2df\right)\sqrt{c+dx}}{4a^2b(bc-ad)(a+bx)} - \frac{\int \frac{-2bc(bc-ad)e-\frac{1}{4}d\left(4b^2ce-ad(3be+\right.}{x(a+bx)\sqrt{c+dx}}}{2a^2b(bc-ad)}$$

$$= \frac{(be-af)\sqrt{c+dx}}{2ab(a+bx)^2} + \frac{\left(4b^2ce-3abde-a^2df\right)\sqrt{c+dx}}{4a^2b(bc-ad)(a+bx)} + \frac{(ce)\int \frac{1}{x\sqrt{c+dx}}\,dx}{a^3} - \frac{\left(8b^3c^2e\right.}{}$$

$$= \frac{(be-af)\sqrt{c+dx}}{2ab(a+bx)^2} + \frac{\left(4b^2ce-3abde-a^2df\right)\sqrt{c+dx}}{4a^2b(bc-ad)(a+bx)} + \frac{(2ce)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x,\right.}{a^3d}$$

$$= \frac{(be-af)\sqrt{c+dx}}{2ab(a+bx)^2} + \frac{\left(4b^2ce-3abde-a^2df\right)\sqrt{c+dx}}{4a^2b(bc-ad)(a+bx)} - \frac{2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^3} + \frac{(8}{}$$

Mathematica [A] time = 0.60, size = 260, normalized size = 1.25

$$\frac{-\frac{(c+dx)^{3/2}\left(a^2df-5abde+4b^2ce\right)}{2a(a+bx)(ad-bc)} + \frac{4e(bc-ad)\left(\sqrt{c+dx}-\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)\right)}{a^2} + \frac{\left(a^3d^2f+3a^2bd^2e-12ab^2cde+8b^3c^2e\right)\left(\sqrt{b}\sqrt{c+dx}-\sqrt{bc-ad}\tanh^{-}\right.}{2a^2b^{3/2}(ad-bc)}}{2a(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*(e + f*x))/(x*(a + b*x)^3),x]
```

```
[Out] (((b*e - a*f)*(c + d*x)^(3/2))/(a + b*x)^2 - ((4*b^2*c*e - 5*a*b*d*e + a^2*
d*f)*(c + d*x)^(3/2))/(2*a*(-(b*c) + a*d)*(a + b*x)) + (4*(b*c - a*d)*e*(Sq
rt[c + d*x] - Sqrt[c]*ArcTanh[Sqrt[c + d*x]/Sqrt[c]]))/a^2 + ((8*b^3*c^2*e
- 12*a*b^2*c*d*e + 3*a^2*b*d^2*e + a^3*d^2*f)*(Sqrt[b]*Sqrt[c + d*x] - Sqrt
[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]]))/(2*a^2*b^(3/
2)*(-(b*c) + a*d)))/(2*a*(b*c - a*d))
```

fricas [B] time = 2.64, size = 2216, normalized size = 10.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/8*((a^5*d^2*f + (a^3*b^2*d^2*f + (8*b^5*c^2 - 12*a*b^4*c*d + 3*a^2*b^3*
d^2)*e)*x^2 + (8*a^2*b^3*c^2 - 12*a^3*b^2*c*d + 3*a^4*b*d^2)*e + 2*(a^4*b*d
```

```
^2*f + (8*a*b^4*c^2 - 12*a^2*b^3*c*d + 3*a^3*b^2*d^2)*e)*x)*sqrt(b^2*c - a*
b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x +
 a)) - 8*((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*e*x^2 + 2*(a*b^5*c^2 - 2*a^
2*b^4*c*d + a^3*b^3*d^2)*e*x + (a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*
e)*sqrt(c)*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) - 2*((6*a^2*b^4*c^2
 - 11*a^3*b^3*c*d + 5*a^4*b^2*d^2)*e - (2*a^3*b^3*c^2 - 3*a^4*b^2*c*d + a^5
*b*d^2)*f + ((4*a*b^5*c^2 - 7*a^2*b^4*c*d + 3*a^3*b^3*d^2)*e - (a^3*b^3*c*d
 - a^4*b^2*d^2)*f)*x)*sqrt(d*x + c))/(a^5*b^4*c^2 - 2*a^6*b^3*c*d + a^7*b^2
*d^2 + (a^3*b^6*c^2 - 2*a^4*b^5*c*d + a^5*b^4*d^2)*x^2 + 2*(a^4*b^5*c^2 - 2
*a^5*b^4*c*d + a^6*b^3*d^2)*x), -1/4*((a^5*d^2*f + (a^3*b^2*d^2*f + (8*b^5*
c^2 - 12*a*b^4*c*d + 3*a^2*b^3*d^2)*e)*x^2 + (8*a^2*b^3*c^2 - 12*a^3*b^2*c*
d + 3*a^4*b*d^2)*e + 2*(a^4*b*d^2*f + (8*a*b^4*c^2 - 12*a^2*b^3*c*d + 3*a^3
*b^2*d^2)*e)*x)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x +
 c)/(b*d*x + b*c)) - 4*((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*e*x^2 + 2*(a*
b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*e*x + (a^2*b^4*c^2 - 2*a^3*b^3*c*d +
 a^4*b^2*d^2)*e)*sqrt(c)*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) - ((6
*a^2*b^4*c^2 - 11*a^3*b^3*c*d + 5*a^4*b^2*d^2)*e - (2*a^3*b^3*c^2 - 3*a^4*b
^2*c*d + a^5*b*d^2)*f + ((4*a*b^5*c^2 - 7*a^2*b^4*c*d + 3*a^3*b^3*d^2)*e -
(a^3*b^3*c*d - a^4*b^2*d^2)*f)*x)*sqrt(d*x + c))/(a^5*b^4*c^2 - 2*a^6*b^3*c
*d + a^7*b^2*d^2 + (a^3*b^6*c^2 - 2*a^4*b^5*c*d + a^5*b^4*d^2)*x^2 + 2*(a^4
*b^5*c^2 - 2*a^5*b^4*c*d + a^6*b^3*d^2)*x), 1/8*(16*((b^6*c^2 - 2*a*b^5*c*d
 + a^2*b^4*d^2)*e*x^2 + 2*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*e*x + (
a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*e)*sqrt(-c)*arctan(sqrt(d*x + c)
*sqrt(-c)/c) - (a^5*d^2*f + (a^3*b^2*d^2*f + (8*b^5*c^2 - 12*a*b^4*c*d + 3*
a^2*b^3*d^2)*e)*x^2 + (8*a^2*b^3*c^2 - 12*a^3*b^2*c*d + 3*a^4*b*d^2)*e + 2*
(a^4*b*d^2*f + (8*a*b^4*c^2 - 12*a^2*b^3*c*d + 3*a^3*b^2*d^2)*e)*x)*sqrt(b^
2*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c)
)/(b*x + a)) + 2*((6*a^2*b^4*c^2 - 11*a^3*b^3*c*d + 5*a^4*b^2*d^2)*e - (2*a
^3*b^3*c^2 - 3*a^4*b^2*c*d + a^5*b*d^2)*f + ((4*a*b^5*c^2 - 7*a^2*b^4*c*d +
 3*a^3*b^3*d^2)*e - (a^3*b^3*c*d - a^4*b^2*d^2)*f)*x)*sqrt(d*x + c))/(a^5*b
^4*c^2 - 2*a^6*b^3*c*d + a^7*b^2*d^2 + (a^3*b^6*c^2 - 2*a^4*b^5*c*d + a^5*b
^4*d^2)*x^2 + 2*(a^4*b^5*c^2 - 2*a^5*b^4*c*d + a^6*b^3*d^2)*x), -1/4*((a^5*
d^2*f + (a^3*b^2*d^2*f + (8*b^5*c^2 - 12*a*b^4*c*d + 3*a^2*b^3*d^2)*e)*x^2
+ (8*a^2*b^3*c^2 - 12*a^3*b^2*c*d + 3*a^4*b*d^2)*e + 2*(a^4*b*d^2*f + (8*a*
b^4*c^2 - 12*a^2*b^3*c*d + 3*a^3*b^2*d^2)*e)*x)*sqrt(-b^2*c + a*b*d)*arctan
(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) - 8*((b^6*c^2 - 2*a*b^5*
c*d + a^2*b^4*d^2)*e*x^2 + 2*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*e*x
+ (a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*e)*sqrt(-c)*arctan(sqrt(d*x +
 c)*sqrt(-c)/c) - ((6*a^2*b^4*c^2 - 11*a^3*b^3*c*d + 5*a^4*b^2*d^2)*e - (2*
a^3*b^3*c^2 - 3*a^4*b^2*c*d + a^5*b*d^2)*f + ((4*a*b^5*c^2 - 7*a^2*b^4*c*d
+ 3*a^3*b^3*d^2)*e - (a^3*b^3*c*d - a^4*b^2*d^2)*f)*x)*sqrt(d*x + c))/(a^5*
b^4*c^2 - 2*a^6*b^3*c*d + a^7*b^2*d^2 + (a^3*b^6*c^2 - 2*a^4*b^5*c*d + a^5*
b^4*d^2)*x^2 + 2*(a^4*b^5*c^2 - 2*a^5*b^4*c*d + a^6*b^3*d^2)*x)]
```

giac [A] time = 1.39, size = 300, normalized size = 1.44

$$-\frac{\left(a^3d^2f+8\,b^3c^2e-12\,ab^2cde+3\,a^2bd^2e\right)\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{4\left(a^3b^2c-a^4bd\right)\sqrt{-b^2c+abd}}+\frac{2\,c\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)e}{a^3\sqrt{-c}}-\frac{(dx+c)^{\frac{3}{2}}a^2bd^2f+\sqrt{dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/4*(a^3*d^2*f + 8*b^3*c^2*e - 12*a*b^2*c*d*e + 3*a^2*b*d^2*e)*arctan(sqrt
(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((a^3*b^2*c - a^4*b*d)*sqrt(-b^2*c + a*b*
d)) + 2*c*arctan(sqrt(d*x + c)/sqrt(-c))*e/(a^3*sqrt(-c)) - 1/4*((d*x + c)^
(3/2)*a^2*b*d^2*f + sqrt(d*x + c)*a^2*b*c*d^2*f - sqrt(d*x + c)*a^3*d^3*f -
 4*(d*x + c)^(3/2)*b^3*c*d*e + 4*sqrt(d*x + c)*b^3*c^2*d*e + 3*(d*x + c)^(3
/2)*a*b^2*d^2*e - 9*sqrt(d*x + c)*a*b^2*c*d^2*e + 5*sqrt(d*x + c)*a^2*b*d^3
*e)/((a^2*b^2*c - a^3*b*d)*((d*x + c)*b - b*c + a*d)^2)
```

maple [B] time = 0.02, size = 424, normalized size = 2.04

$$\frac{3\,(dx+c)^{\frac{3}{2}}\,b\,d^2e}{4\,(bdx+ad)^2\,(ad-bc)\,a}+\frac{3d^2e\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{(ad-bc)b}}\right)}{4\,(ad-bc)\,\sqrt{(ad-bc)\,b}\,\,a}-\frac{(dx+c)^{\frac{3}{2}}\,b^2cde}{(bdx+ad)^2\,(ad-bc)\,a^2}-\frac{3bcde\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{(ad-bc)b}}\right)}{(ad-bc)\,\sqrt{(ad-bc)\,b}\,\,a^2}+\frac{2b^2\cdots}{(ad\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a)^3,x)
```

```
[Out] 1/4*d^2/(b*d*x+a*d)^2/(a*d-b*c)*(d*x+c)^(3/2)*f+3/4*d^2/a/(b*d*x+a*d)^2/(a*
d-b*c)*(d*x+c)^(3/2)*b*e-d/a^2/(b*d*x+a*d)^2/(a*d-b*c)*(d*x+c)^(3/2)*b^2*c*
e-1/4*d^2/(b*d*x+a*d)^2/b*(d*x+c)^(1/2)*f+5/4*d^2/a/(b*d*x+a*d)^2*(d*x+c)^(
1/2)*e-d/a^2/(b*d*x+a*d)^2*b*(d*x+c)^(1/2)*c*e+1/4*d^2/(a*d-b*c)/b/((a*d-b*
c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)*f+3/4*d^2/a/(a*d-b*
c)/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)*e-3*d/a^
2/(a*d-b*c)*b/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*
b)*c*e+2/a^3/(a*d-b*c)*b^2/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b
*c)*b)^(1/2)*b)*c^2*e-2*e*arctanh((d*x+c)^(1/2)/c^(1/2))*c^(1/2)/a^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 4.54, size = 4852, normalized size = 23.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(c + d*x)^(1/2))/(x*(a + b*x)^3),x)
```

```
[Out] (c^(1/2)*e*atan(((c^(1/2)*e*(((c + d*x)^(1/2)*(a^6*d^6*f^2 + 9*a^4*b^2*d^6*
e^2 + 128*b^6*c^4*d^2*e^2 + 6*a^5*b*d^6*e*f + 256*a^2*b^4*c^2*d^4*e^2 - 320
*a*b^5*c^3*d^3*e^2 - 72*a^3*b^3*c*d^5*e^2 + 16*a^3*b^3*c^2*d^4*e*f - 24*a^4
*b^2*c*d^5*e*f))/(8*(a^6*b*d^2 + a^4*b^3*c^2 - 2*a^5*b^2*c*d)) + (c^(1/2)*e
*((5*a^8*b^3*c*d^5*e - a^9*b^2*c*d^5*f + 4*a^6*b^5*c^3*d^3*e - 9*a^7*b^4*c^
2*d^4*e + a^8*b^3*c^2*d^4*f)/(a^8*b*d^2 + a^6*b^3*c^2 - 2*a^7*b^2*c*d) + (c
^(1/2)*e*(c + d*x)^(1/2)*(64*a^9*b^3*d^5 - 256*a^8*b^4*c*d^4 - 128*a^6*b^6*
c^3*d^2 + 320*a^7*b^5*c^2*d^3))/(8*a^3*(a^6*b*d^2 + a^4*b^3*c^2 - 2*a^5*b^2
*c*d))))/a^3)*1i)/a^3 + (c^(1/2)*e*(((c + d*x)^(1/2)*(a^6*d^6*f^2 + 9*a^4*b
^2*d^6*e^2 + 128*b^6*c^4*d^2*e^2 + 6*a^5*b*d^6*e*f + 256*a^2*b^4*c^2*d^4*e^
2 - 320*a*b^5*c^3*d^3*e^2 - 72*a^3*b^3*c*d^5*e^2 + 16*a^3*b^3*c^2*d^4*e*f -
 24*a^4*b^2*c*d^5*e*f))/(8*(a^6*b*d^2 + a^4*b^3*c^2 - 2*a^5*b^2*c*d)) - (c^
(1/2)*e*((5*a^8*b^3*c*d^5*e - a^9*b^2*c*d^5*f + 4*a^6*b^5*c^3*d^3*e - 9*a^7
*b^4*c^2*d^4*e + a^8*b^3*c^2*d^4*f)/(a^8*b*d^2 + a^6*b^3*c^2 - 2*a^7*b^2*c*
d) - (c^(1/2)*e*(c + d*x)^(1/2)*(64*a^9*b^3*d^5 - 256*a^8*b^4*c*d^4 - 128*a
^6*b^6*c^3*d^2 + 320*a^7*b^5*c^2*d^3))/(8*a^3*(a^6*b*d^2 + a^4*b^3*c^2 - 2*
a^5*b^2*c*d))))/a^3)*1i)/a^3)/(((a^5*c*d^6*e*f^2)/4 - 12*a^2*b^3*c^2*d^5*e^
3 - 8*b^5*c^4*d^3*e^3 + 18*a*b^4*c^3*d^4*e^3 + (9*a^3*b^2*c*d^6*e^3)/4 + 2*
a^2*b^3*c^3*d^4*e^2*f - 4*a^3*b^2*c^2*d^5*e^2*f + (3*a^4*b*c*d^6*e^2*f)/2)/
(a^8*b*d^2 + a^6*b^3*c^2 - 2*a^7*b^2*c*d) + (c^(1/2)*e*(((c + d*x)^(1/2)*(a
^6*d^6*f^2 + 9*a^4*b^2*d^6*e^2 + 128*b^6*c^4*d^2*e^2 + 6*a^5*b*d^6*e*f + 25
```

```
6*a^2*b^4*c^2*d^4*e^2 - 320*a*b^5*c^3*d^3*e^2 - 72*a^3*b^3*c*d^5*e^2 + 16*a
^3*b^3*c^2*d^4*e*f - 24*a^4*b^2*c*d^5*e*f))/(8*(a^6*b*d^2 + a^4*b^3*c^2 - 2
*a^5*b^2*c*d)) + (c^(1/2)*e*((5*a^8*b^3*c*d^5*e - a^9*b^2*c*d^5*f + 4*a^6*b
^5*c^3*d^3*e - 9*a^7*b^4*c^2*d^4*e + a^8*b^3*c^2*d^4*f)/(a^8*b*d^2 + a^6*b^
3*c^2 - 2*a^7*b^2*c*d) + (c^(1/2)*e*(c + d*x)^(1/2)*(64*a^9*b^3*d^5 - 256*a
^8*b^4*c*d^4 - 128*a^6*b^6*c^3*d^2 + 320*a^7*b^5*c^2*d^3))/(8*a^3*(a^6*b*d^
2 + a^4*b^3*c^2 - 2*a^5*b^2*c*d))))/a^3))/a^3 - (c^(1/2)*e*(((c + d*x)^(1/2
)*(a^6*d^6*f^2 + 9*a^4*b^2*d^6*e^2 + 128*b^6*c^4*d^2*e^2 + 6*a^5*b*d^6*e*f
+ 256*a^2*b^4*c^2*d^4*e^2 - 320*a*b^5*c^3*d^3*e^2 - 72*a^3*b^3*c*d^5*e^2 +
16*a^3*b^3*c^2*d^4*e*f - 24*a^4*b^2*c*d^5*e*f))/(8*(a^6*b*d^2 + a^4*b^3*c^2
 - 2*a^5*b^2*c*d)) - (c^(1/2)*e*((5*a^8*b^3*c*d^5*e - a^9*b^2*c*d^5*f + 4*a
^6*b^5*c^3*d^3*e - 9*a^7*b^4*c^2*d^4*e + a^8*b^3*c^2*d^4*f)/(a^8*b*d^2 + a^
6*b^3*c^2 - 2*a^7*b^2*c*d) - (c^(1/2)*e*(c + d*x)^(1/2)*(64*a^9*b^3*d^5 - 2
56*a^8*b^4*c*d^4 - 128*a^6*b^6*c^3*d^2 + 320*a^7*b^5*c^2*d^3))/(8*a^3*(a^6*
b*d^2 + a^4*b^3*c^2 - 2*a^5*b^2*c*d))))/a^3))/a^3))*2i)/a^3 - (((c + d*x)^(
1/2)*(a^2*d^2*f - 5*a*b*d^2*e + 4*b^2*c*d*e))/(4*a^2*b) - ((c + d*x)^(3/2)*
(a^2*d^2*f + 3*a*b*d^2*e - 4*b^2*c*d*e))/(4*a^2*(a*d - b*c)))/(b^2*(c + d*x
)^2 - (2*b^2*c - 2*a*b*d)*(c + d*x) + a^2*d^2 + b^2*c^2 - 2*a*b*c*d) + (ata
n((((-b^3*(a*d - b*c)^3)^(1/2)*(((c + d*x)^(1/2)*(a^6*d^6*f^2 + 9*a^4*b^2*d
^6*e^2 + 128*b^6*c^4*d^2*e^2 + 6*a^5*b*d^6*e*f + 256*a^2*b^4*c^2*d^4*e^2 -
320*a*b^5*c^3*d^3*e^2 - 72*a^3*b^3*c*d^5*e^2 + 16*a^3*b^3*c^2*d^4*e*f - 24*
a^4*b^2*c*d^5*e*f))/(8*(a^6*b*d^2 + a^4*b^3*c^2 - 2*a^5*b^2*c*d)) - ((-b^3*
(a*d - b*c)^3)^(1/2)*((5*a^8*b^3*c*d^5*e - a^9*b^2*c*d^5*f + 4*a^6*b^5*c^3*
d^3*e - 9*a^7*b^4*c^2*d^4*e + a^8*b^3*c^2*d^4*f)/(a^8*b*d^2 + a^6*b^3*c^2 -
 2*a^7*b^2*c*d) - ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x)^(1/2)*(8*b^3*c^2*e
+ a^3*d^2*f + 3*a^2*b*d^2*e - 12*a*b^2*c*d*e)*(64*a^9*b^3*d^5 - 256*a^8*b^4
*c*d^4 - 128*a^6*b^6*c^3*d^2 + 320*a^7*b^5*c^2*d^3))/(64*(a^6*b*d^2 + a^4*b
^3*c^2 - 2*a^5*b^2*c*d)*(a^3*b^6*c^3 - a^6*b^3*d^3 - 3*a^4*b^5*c^2*d + 3*a^
5*b^4*c*d^2)))*(8*b^3*c^2*e + a^3*d^2*f + 3*a^2*b*d^2*e - 12*a*b^2*c*d*e))/
(8*(a^3*b^6*c^3 - a^6*b^3*d^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*d^2)))*(8*b^3
*c^2*e + a^3*d^2*f + 3*a^2*b*d^2*e - 12*a*b^2*c*d*e)*1i)/(8*(a^3*b^6*c^3 -
a^6*b^3*d^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*d^2)) + ((-b^3*(a*d - b*c)^3)^(
1/2)*(((c + d*x)^(1/2)*(a^6*d^6*f^2 + 9*a^4*b^2*d^6*e^2 + 128*b^6*c^4*d^2*e
^2 + 6*a^5*b*d^6*e*f + 256*a^2*b^4*c^2*d^4*e^2 - 320*a*b^5*c^3*d^3*e^2 - 72
*a^3*b^3*c*d^5*e^2 + 16*a^3*b^3*c^2*d^4*e*f - 24*a^4*b^2*c*d^5*e*f))/(8*(a^
6*b*d^2 + a^4*b^3*c^2 - 2*a^5*b^2*c*d)) + ((-b^3*(a*d - b*c)^3)^(1/2)*((5*a
^8*b^3*c*d^5*e - a^9*b^2*c*d^5*f + 4*a^6*b^5*c^3*d^3*e - 9*a^7*b^4*c^2*d^4*
e + a^8*b^3*c^2*d^4*f)/(a^8*b*d^2 + a^6*b^3*c^2 - 2*a^7*b^2*c*d) + ((-b^3*(
a*d - b*c)^3)^(1/2)*(c + d*x)^(1/2)*(8*b^3*c^2*e + a^3*d^2*f + 3*a^2*b*d^2*
e - 12*a*b^2*c*d*e)*(64*a^9*b^3*d^5 - 256*a^8*b^4*c*d^4 - 128*a^6*b^6*c^3*d
^2 + 320*a^7*b^5*c^2*d^3))/(64*(a^6*b*d^2 + a^4*b^3*c^2 - 2*a^5*b^2*c*d)*(a
^3*b^6*c^3 - a^6*b^3*d^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*d^2)))*(8*b^3*c^2*
e + a^3*d^2*f + 3*a^2*b*d^2*e - 12*a*b^2*c*d*e))/(8*(a^3*b^6*c^3 - a^6*b^3*
d^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*d^2)))*(8*b^3*c^2*e + a^3*d^2*f + 3*a^2
*b*d^2*e - 12*a*b^2*c*d*e)*1i)/(8*(a^3*b^6*c^3 - a^6*b^3*d^3 - 3*a^4*b^5*c^
2*d + 3*a^5*b^4*c*d^2)))/(((a^5*c*d^6*e*f^2)/4 - 12*a^2*b^3*c^2*d^5*e^3 - 8
*b^5*c^4*d^3*e^3 + 18*a*b^4*c^3*d^4*e^3 + (9*a^3*b^2*c*d^6*e^3)/4 + 2*a^2*b
^3*c^3*d^4*e^2*f - 4*a^3*b^2*c^2*d^5*e^2*f + (3*a^4*b*c*d^6*e^2*f)/2)/(a^8*
b*d^2 + a^6*b^3*c^2 - 2*a^7*b^2*c*d) - ((-b^3*(a*d - b*c)^3)^(1/2)*(((c + d
*x)^(1/2)*(a^6*d^6*f^2 + 9*a^4*b^2*d^6*e^2 + 128*b^6*c^4*d^2*e^2 + 6*a^5*b*
d^6*e*f + 256*a^2*b^4*c^2*d^4*e^2 - 320*a*b^5*c^3*d^3*e^2 - 72*a^3*b^3*c*d^
5*e^2 + 16*a^3*b^3*c^2*d^4*e*f - 24*a^4*b^2*c*d^5*e*f))/(8*(a^6*b*d^2 + a^4
*b^3*c^2 - 2*a^5*b^2*c*d)) - ((-b^3*(a*d - b*c)^3)^(1/2)*((5*a^8*b^3*c*d^5*
e - a^9*b^2*c*d^5*f + 4*a^6*b^5*c^3*d^3*e - 9*a^7*b^4*c^2*d^4*e + a^8*b^3*c
^2*d^4*f)/(a^8*b*d^2 + a^6*b^3*c^2 - 2*a^7*b^2*c*d) - ((-b^3*(a*d - b*c)^3)
^(1/2)*(c + d*x)^(1/2)*(8*b^3*c^2*e + a^3*d^2*f + 3*a^2*b*d^2*e - 12*a*b^2*
c*d*e)*(64*a^9*b^3*d^5 - 256*a^8*b^4*c*d^4 - 128*a^6*b^6*c^3*d^2 + 320*a^7*
b^5*c^2*d^3))/(64*(a^6*b*d^2 + a^4*b^3*c^2 - 2*a^5*b^2*c*d)*(a^3*b^6*c^3 -
a^6*b^3*d^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*d^2)))*(8*b^3*c^2*e + a^3*d^2*f
```

```
 + 3*a^2*b*d^2*e - 12*a*b^2*c*d*e))/(8*(a^3*b^6*c^3 - a^6*b^3*d^3 - 3*a^4*b
^5*c^2*d + 3*a^5*b^4*c*d^2)))*(8*b^3*c^2*e + a^3*d^2*f + 3*a^2*b*d^2*e - 12
*a*b^2*c*d*e))/(8*(a^3*b^6*c^3 - a^6*b^3*d^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*
c*d^2)) + ((-b^3*(a*d - b*c)^3)^(1/2)*(((c + d*x)^(1/2)*(a^6*d^6*f^2 + 9*a^
4*b^2*d^6*e^2 + 128*b^6*c^4*d^2*e^2 + 6*a^5*b*d^6*e*f + 256*a^2*b^4*c^2*d^4
*e^2 - 320*a*b^5*c^3*d^3*e^2 - 72*a^3*b^3*c*d^5*e^2 + 16*a^3*b^3*c^2*d^4*e*
f - 24*a^4*b^2*c*d^5*e*f))/(8*(a^6*b*d^2 + a^4*b^3*c^2 - 2*a^5*b^2*c*d)) +
((-b^3*(a*d - b*c)^3)^(1/2)*((5*a^8*b^3*c*d^5*e - a^9*b^2*c*d^5*f + 4*a^6*b
^5*c^3*d^3*e - 9*a^7*b^4*c^2*d^4*e + a^8*b^3*c^2*d^4*f)/(a^8*b*d^2 + a^6*b^
3*c^2 - 2*a^7*b^2*c*d) + ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x)^(1/2)*(8*b^3
*c^2*e + a^3*d^2*f + 3*a^2*b*d^2*e - 12*a*b^2*c*d*e)*(64*a^9*b^3*d^5 - 256*
a^8*b^4*c*d^4 - 128*a^6*b^6*c^3*d^2 + 320*a^7*b^5*c^2*d^3))/(64*(a^6*b*d^2
+ a^4*b^3*c^2 - 2*a^5*b^2*c*d)*(a^3*b^6*c^3 - a^6*b^3*d^3 - 3*a^4*b^5*c^2*d
 + 3*a^5*b^4*c*d^2)))*(8*b^3*c^2*e + a^3*d^2*f + 3*a^2*b*d^2*e - 12*a*b^2*c
*d*e))/(8*(a^3*b^6*c^3 - a^6*b^3*d^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*d^2)))
*(8*b^3*c^2*e + a^3*d^2*f + 3*a^2*b*d^2*e - 12*a*b^2*c*d*e))/(8*(a^3*b^6*c^
3 - a^6*b^3*d^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*d^2))))*(-b^3*(a*d - b*c)^3
)^(1/2)*(8*b^3*c^2*e + a^3*d^2*f + 3*a^2*b*d^2*e - 12*a*b^2*c*d*e)*1i)/(4*(
a^3*b^6*c^3 - a^6*b^3*d^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*d^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)**(1/2)/x/(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.15 $\int \frac{\sqrt{a+bx}\,(c+dx)^3(e+fx)}{x}\,dx$

Optimal. Leaf size=226

$$-\frac{2(a+bx)^{3/2}\left(2\left(8a^3d^3f-12a^2bd^2(3cf+de)+3ab^2cd(16cf+21de)-5b^3c^2(4cf+27de)\right)-3bdx\left(4(bc-ad)(-2\right.\right.}{315b^4}$$

```
[Out] 2/21*(-2*a*d*f+2*b*c*f+3*b*d*e)*(b*x+a)^(3/2)*(d*x+c)^2/b^2+2/9*f*(b*x+a)^(
3/2)*(d*x+c)^3/b-2/315*(b*x+a)^(3/2)*(16*a^3*d^3*f-24*a^2*b*d^2*(3*c*f+d*e)
-10*b^3*c^2*(4*c*f+27*d*e)+6*a*b^2*c*d*(16*c*f+21*d*e)-3*b*d*(21*b^2*c*d*e+
4*(-a*d+b*c)*(-2*a*d*f+2*b*c*f+3*b*d*e))*x)/b^4-2*c^3*e*arctanh((b*x+a)^(1/
2)/a^(1/2))*a^(1/2)+2*c^3*e*(b*x+a)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {153, 147, 50, 63, 208}

$$-\frac{2(a+bx)^{3/2}\left(2\left(-12a^2bd^2(3cf+de)+8a^3d^3f+3ab^2cd(16cf+21de)-5b^3c^2(4cf+27de)\right)-3bdx\left(4(bc-ad)(\cdot\right.\right.}{315b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x]*(c + d*x)^3*(e + f*x))/x,x]
```

```
[Out] 2*c^3*e*Sqrt[a + b*x] + (2*(3*b*d*e + 2*b*c*f - 2*a*d*f)*(a + b*x)^(3/2)*(c
 + d*x)^2)/(21*b^2) + (2*f*(a + b*x)^(3/2)*(c + d*x)^3)/(9*b) - (2*(a + b*x
)^(3/2)*(2*(8*a^3*d^3*f - 12*a^2*b*d^2*(d*e + 3*c*f) - 5*b^3*c^2*(27*d*e +
4*c*f) + 3*a*b^2*c*d*(21*d*e + 16*c*f)) - 3*b*d*(21*b^2*c*d*e + 4*(b*c - a*
d)*(3*b*d*e + 2*b*c*f - 2*a*d*f))*x))/(315*b^4) - 2*Sqrt[a]*c^3*e*ArcTanh[S
qrt[a + b*x]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
```

```
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}\,(c+dx)^3(e+fx)}{x}\,dx = \frac{2f(a+bx)^{3/2}(c+dx)^3}{9b} + \frac{2\int \frac{\sqrt{a+bx}\,(c+dx)^2\left(\frac{9bce}{2}+\frac{3}{2}(3bde+2bcf-2adf)x\right)}{x}\,dx}{9b}$$

$$= \frac{2(3bde+2bcf-2adf)(a+bx)^{3/2}(c+dx)^2}{21b^2} + \frac{2f(a+bx)^{3/2}(c+dx)^3}{9b} + \frac{4\int$$

$$= \frac{2(3bde+2bcf-2adf)(a+bx)^{3/2}(c+dx)^2}{21b^2} + \frac{2f(a+bx)^{3/2}(c+dx)^3}{9b} - \frac{2(a$$

$$= 2c^3e\sqrt{a+bx} + \frac{2(3bde+2bcf-2adf)(a+bx)^{3/2}(c+dx)^2}{21b^2} + \frac{2f(a+bx)^{3/2}}{9b}$$

$$= 2c^3e\sqrt{a+bx} + \frac{2(3bde+2bcf-2adf)(a+bx)^{3/2}(c+dx)^2}{21b^2} + \frac{2f(a+bx)^{3/2}}{9b}$$

$$= 2c^3e\sqrt{a+bx} + \frac{2(3bde+2bcf-2adf)(a+bx)^{3/2}(c+dx)^2}{21b^2} + \frac{2f(a+bx)^{3/2}}{9b}$$

Mathematica [A] time = 0.28, size = 204, normalized size = 0.90

$$2\left(3be\left(35d(a+bx)^{3/2}\left(a^2d^2-3abcd+3b^2c^2\right)+105b^3c^3\sqrt{a+bx}-105\sqrt{a}\,b^3c^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+21d^2(a+bx)\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x]*(c + d*x)^3*(e + f*x))/x,x]
```

```
[Out] (2*(f*(a + b*x)^(3/2)*(105*(b*c - a*d)^3 + 189*d*(b*c - a*d)^2*(a + b*x) +
135*d^2*(b*c - a*d)*(a + b*x)^2 + 35*d^3*(a + b*x)^3) + 3*b*e*(105*b^3*c^3*
Sqrt[a + b*x] + 35*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*(a + b*x)^(3/2) + 21
*d^2*(3*b*c - 2*a*d)*(a + b*x)^(5/2) + 15*d^3*(a + b*x)^(7/2) - 105*Sqrt[a]
*b^3*c^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])))/(315*b^4)
```

fricas [A] time = 0.79, size = 641, normalized size = 2.84

$$\left[\frac{315\sqrt{a}\,b^4c^3e\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\left(35\,b^4d^3fx^4+5\left(9\,b^4d^3e+\left(27\,b^4cd^2+ab^3d^3\right)f\right)x^3+3\left(3\left(21\,b^4cd^2+ab\right.\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/315*(315*sqrt(a)*b^4*c^3*e*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)
+ 2*(35*b^4*d^3*f*x^4 + 5*(9*b^4*d^3*e + (27*b^4*c*d^2 + a*b^3*d^3)*f)*x^3
+ 3*(3*(21*b^4*c*d^2 + a*b^3*d^3)*e + (63*b^4*c^2*d + 9*a*b^3*c*d^2 - 2*a^2
*b^2*d^3)*f)*x^2 + 3*(105*b^4*c^3 + 105*a*b^3*c^2*d - 42*a^2*b^2*c*d^2 + 8*
a^3*b*d^3)*e + (105*a*b^3*c^3 - 126*a^2*b^2*c^2*d + 72*a^3*b*c*d^2 - 16*a^4
*d^3)*f + (3*(105*b^4*c^2*d + 21*a*b^3*c*d^2 - 4*a^2*b^2*d^3)*e + (105*b^4*
c^3 + 63*a*b^3*c^2*d - 36*a^2*b^2*c*d^2 + 8*a^3*b*d^3)*f)*x)*sqrt(b*x + a))
/b^4, 2/315*(315*sqrt(-a)*b^4*c^3*e*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (35*
b^4*d^3*f*x^4 + 5*(9*b^4*d^3*e + (27*b^4*c*d^2 + a*b^3*d^3)*f)*x^3 + 3*(3*(
21*b^4*c*d^2 + a*b^3*d^3)*e + (63*b^4*c^2*d + 9*a*b^3*c*d^2 - 2*a^2*b^2*d^3
)*f)*x^2 + 3*(105*b^4*c^3 + 105*a*b^3*c^2*d - 42*a^2*b^2*c*d^2 + 8*a^3*b*d^
3)*e + (105*a*b^3*c^3 - 126*a^2*b^2*c^2*d + 72*a^3*b*c*d^2 - 16*a^4*d^3)*f
+ (3*(105*b^4*c^2*d + 21*a*b^3*c*d^2 - 4*a^2*b^2*d^3)*e + (105*b^4*c^3 + 63
*a*b^3*c^2*d - 36*a^2*b^2*c*d^2 + 8*a^3*b*d^3)*f)*x)*sqrt(b*x + a))/b^4]
```

giac [A] time = 1.39, size = 338, normalized size = 1.50

$$\frac{2\,ac^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)e}{\sqrt{-a}}+\frac{2\left(105\,(bx+a)^{\frac{3}{2}}b^{35}c^3f+189\,(bx+a)^{\frac{5}{2}}b^{34}c^2df-315\,(bx+a)^{\frac{3}{2}}ab^{34}c^2df+135\,(bx+a)^{\frac{7}{2}}b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*a*c^3*arctan(sqrt(b*x + a)/sqrt(-a))*e/sqrt(-a) + 2/315*(105*(b*x + a)^(3
/2)*b^35*c^3*f + 189*(b*x + a)^(5/2)*b^34*c^2*d*f - 315*(b*x + a)^(3/2)*a*b
^34*c^2*d*f + 135*(b*x + a)^(7/2)*b^33*c*d^2*f - 378*(b*x + a)^(5/2)*a*b^33
*c*d^2*f + 315*(b*x + a)^(3/2)*a^2*b^33*c*d^2*f + 35*(b*x + a)^(9/2)*b^32*d
^3*f - 135*(b*x + a)^(7/2)*a*b^32*d^3*f + 189*(b*x + a)^(5/2)*a^2*b^32*d^3*
f - 105*(b*x + a)^(3/2)*a^3*b^32*d^3*f + 315*sqrt(b*x + a)*b^36*c^3*e + 315
*(b*x + a)^(3/2)*b^35*c^2*d*e + 189*(b*x + a)^(5/2)*b^34*c*d^2*e - 315*(b*x
 + a)^(3/2)*a*b^34*c*d^2*e + 45*(b*x + a)^(7/2)*b^33*d^3*e - 126*(b*x + a)^
(5/2)*a*b^33*d^3*e + 105*(b*x + a)^(3/2)*a^2*b^33*d^3*e)/b^36
```

maple [A] time = 0.01, size = 301, normalized size = 1.33

$$\frac{-2\sqrt{a}\;b^4c^3e\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+2\sqrt{bx+a}\;b^4c^3e-\frac{2(bx+a)^{\frac{3}{2}}a^3d^3f}{3}+2\,(bx+a)^{\frac{3}{2}}\;a^2bc\,d^2f+\frac{2(bx+a)^{\frac{3}{2}}a^2b\,d^3e}{3}-2\,(bx+a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(f*x+e)*(b*x+a)^(1/2)/x,x)
```

```
[Out] 2/b^4*(1/9*f*d^3*(b*x+a)^(9/2)-3/7*(b*x+a)^(7/2)*a*d^3*f+3/7*(b*x+a)^(7/2)*
b*c*d^2*f+1/7*(b*x+a)^(7/2)*b*d^3*e+3/5*(b*x+a)^(5/2)*a^2*d^3*f-6/5*(b*x+a)
^(5/2)*a*b*c*d^2*f-2/5*(b*x+a)^(5/2)*a*b*d^3*e+3/5*(b*x+a)^(5/2)*b^2*c^2*d*
f+3/5*(b*x+a)^(5/2)*b^2*c*d^2*e-1/3*(b*x+a)^(3/2)*a^3*d^3*f+(b*x+a)^(3/2)*a
^2*b*c*d^2*f+1/3*(b*x+a)^(3/2)*a^2*b*d^3*e-(b*x+a)^(3/2)*a*b^2*c^2*d*f-(b*x
+a)^(3/2)*a*b^2*c*d^2*e+1/3*(b*x+a)^(3/2)*b^3*c^3*f+(b*x+a)^(3/2)*b^3*c^2*d
```

```
*e+b^4*c^3*e*(b*x+a)^(1/2)-a^(1/2)*b^4*c^3*e*arctanh((b*x+a)^(1/2)/a^(1/2))
)
```

maxima [A] time = 0.97, size = 238, normalized size = 1.05

$$\sqrt{a}\, c^3 e \log\left(\frac{\sqrt{bx+a} - \sqrt{a}}{\sqrt{bx+a} + \sqrt{a}}\right) + \frac{2\left(315\sqrt{bx+a}\, b^4 c^3 e + 35\,(bx+a)^{\frac{9}{2}} d^3 f + 45\left(bd^3 e + 3\left(bcd^2 - ad^3\right)f\right)(bx+a)^{\frac{7}{2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] sqrt(a)*c^3*e*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2/
315*(315*sqrt(b*x + a)*b^4*c^3*e + 35*(b*x + a)^(9/2)*d^3*f + 45*(b*d^3*e +
 3*(b*c*d^2 - a*d^3)*f)*(b*x + a)^(7/2) + 63*((3*b^2*c*d^2 - 2*a*b*d^3)*e +
 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*f)*(b*x + a)^(5/2) + 105*((3*b^3*c^2
*d - 3*a*b^2*c*d^2 + a^2*b*d^3)*e + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^
2 - a^3*d^3)*f)*(b*x + a)^(3/2))/b^4
```

mupad [B] time = 2.53, size = 413, normalized size = 1.83

$$\left(\frac{2\,b\,d^3\,e - 8\,a\,d^3\,f + 6\,b\,c\,d^2\,f}{7\,b^4} + \frac{2\,a\,d^3\,f}{7\,b^4}\right)(a+b\,x)^{7/2} + \left(\frac{a\left(\frac{2\,b\,d^3\,e - 8\,a\,d^3\,f + 6\,b\,c\,d^2\,f}{b^4} + \frac{2\,a\,d^3\,f}{b^4}\right)}{5} - \frac{6\,d\,(a\,d - b\,c)\,(b\,c}{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(a + b*x)^(1/2)*(c + d*x)^3)/x,x)
```

```
[Out] ((2*b*d^3*e - 8*a*d^3*f + 6*b*c*d^2*f)/(7*b^4) + (2*a*d^3*f)/(7*b^4))*(a +
b*x)^(7/2) + ((a*((2*b*d^3*e - 8*a*d^3*f + 6*b*c*d^2*f)/b^4 + (2*a*d^3*f)/b
^4))/5 - (6*d*(a*d - b*c)*(b*c*f - 2*a*d*f + b*d*e))/(5*b^4))*(a + b*x)^(5/
2) + (a*(a*(a*((2*b*d^3*e - 8*a*d^3*f + 6*b*c*d^2*f)/b^4 + (2*a*d^3*f)/b^4)
 - (6*d*(a*d - b*c)*(b*c*f - 2*a*d*f + b*d*e))/b^4) + (2*(a*d - b*c)^2*(b*c
*f - 4*a*d*f + 3*b*d*e))/b^4) + (2*(a*d - b*c)^3*(a*f - b*e))/b^4)*(a + b*x
)^(1/2) + ((a*(a*((2*b*d^3*e - 8*a*d^3*f + 6*b*c*d^2*f)/b^4 + (2*a*d^3*f)/b
^4) - (6*d*(a*d - b*c)*(b*c*f - 2*a*d*f + b*d*e))/b^4))/3 + (2*(a*d - b*c)^
2*(b*c*f - 4*a*d*f + 3*b*d*e))/(3*b^4))*(a + b*x)^(3/2) + a^(1/2)*c^3*e*ata
n(((a + b*x)^(1/2)*1i)/a^(1/2))*2i + (2*d^3*f*(a + b*x)^(9/2))/(9*b^4)
```

sympy [A] time = 37.95, size = 274, normalized size = 1.21

$$\frac{2ac^3 e \operatorname{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2c^3 e\sqrt{a+bx} + \frac{2d^3 f\,(a+bx)^{\frac{9}{2}}}{9b^4} + \frac{2\,(a+bx)^{\frac{7}{2}}\left(-3ad^3 f + 3bcd^2 f + bd^3 e\right)}{7b^4} + \frac{2\,(a+bx)^{\frac{5}{2}}\left(3a^2 d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(f*x+e)*(b*x+a)**(1/2)/x,x)
```

```
[Out] 2*a*c**3*e*atan(sqrt(a + b*x)/sqrt(-a))/sqrt(-a) + 2*c**3*e*sqrt(a + b*x) +
 2*d**3*f*(a + b*x)**(9/2)/(9*b**4) + 2*(a + b*x)**(7/2)*(-3*a*d**3*f + 3*b
*c*d**2*f + b*d**3*e)/(7*b**4) + 2*(a + b*x)**(5/2)*(3*a**2*d**3*f - 6*a*b*
c*d**2*f - 2*a*b*d**3*e + 3*b**2*c**2*d*f + 3*b**2*c*d**2*e)/(5*b**4) + 2*(
a + b*x)**(3/2)*(-a**3*d**3*f + 3*a**2*b*c*d**2*f + a**2*b*d**3*e - 3*a*b**
2*c**2*d*f - 3*a*b**2*c*d**2*e + b**3*c**3*f + 3*b**3*c**2*d*e)/(3*b**4)
```

3.16 $\int \frac{\sqrt{a+bx}\,(c+dx)^2(e+fx)}{x}\,dx$

Optimal. Leaf size=145

$$\frac{2(a+bx)^{3/2}\left(2\left(4a^2d^2f-7abd(2cf+de)+5b^2c(2cf+7de)\right)+3bdx(-4adf+4bcf+7bde)\right)}{105b^3}+2c^2e\sqrt{a+bx}-2\sqrt{a}$$

```
[Out] 2/7*f*(b*x+a)^(3/2)*(d*x+c)^2/b+2/105*(b*x+a)^(3/2)*(8*a^2*d^2*f-14*a*b*d*(
2*c*f+d*e)+10*b^2*c*(2*c*f+7*d*e)+3*b*d*(-4*a*d*f+4*b*c*f+7*b*d*e)*x)/b^3-2
*c^2*e*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2)+2*c^2*e*(b*x+a)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {153, 147, 50, 63, 208}

$$\frac{2(a+bx)^{3/2}\left(2\left(4a^2d^2f-7abd(2cf+de)+5b^2c(2cf+7de)\right)+3bdx(-4adf+4bcf+7bde)\right)}{105b^3}+2c^2e\sqrt{a+bx}-2\sqrt{a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x]*(c + d*x)^2*(e + f*x))/x,x]
```

```
[Out] 2*c^2*e*Sqrt[a + b*x] + (2*f*(a + b*x)^(3/2)*(c + d*x)^2)/(7*b) + (2*(a + b
*x)^(3/2)*(2*(4*a^2*d^2*f - 7*a*b*d*(d*e + 2*c*f) + 5*b^2*c*(7*d*e + 2*c*f)
) + 3*b*d*(7*b*d*e + 4*b*c*f - 4*a*d*f)*x))/(105*b^3) - 2*Sqrt[a]*c^2*e*Arc
Tanh[Sqrt[a + b*x]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
```

```
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}\,(c+dx)^2(e+fx)}{x}\,dx = \frac{2f(a+bx)^{3/2}(c+dx)^2}{7b} + \frac{2\int \frac{\sqrt{a+bx}\,(c+dx)\left(\frac{7bce}{2}+\frac{1}{2}(7bde+4bcf-4adf)x\right)}{x}\,dx}{7b}$$

$$= \frac{2f(a+bx)^{3/2}(c+dx)^2}{7b} + \frac{2(a+bx)^{3/2}\left(2\left(4a^2d^2f-7abd(de+2cf)+5b^2c(\right.\right.}{105b^3}$$

$$= 2c^2e\sqrt{a+bx} + \frac{2f(a+bx)^{3/2}(c+dx)^2}{7b} + \frac{2(a+bx)^{3/2}\left(2\left(4a^2d^2f-7abd(de\right.\right.}{}$$

$$= 2c^2e\sqrt{a+bx} + \frac{2f(a+bx)^{3/2}(c+dx)^2}{7b} + \frac{2(a+bx)^{3/2}\left(2\left(4a^2d^2f-7abd(de\right.\right.}{}$$

$$= 2c^2e\sqrt{a+bx} + \frac{2f(a+bx)^{3/2}(c+dx)^2}{7b} + \frac{2(a+bx)^{3/2}\left(2\left(4a^2d^2f-7abd(de\right.\right.}{}$$

Mathematica [A] time = 0.18, size = 146, normalized size = 1.01

$$\frac{2\left(7be\left(15b^2c^2\sqrt{a+bx}-15\sqrt{a}\,b^2c^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+5d(a+bx)^{3/2}(2bc-ad)+3d^2(a+bx)^{5/2}\right)+f(a+bx)^{3/2}\right.}{105b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x]*(c + d*x)^2*(e + f*x))/x,x]
```

```
[Out] (2*(f*(a + b*x)^(3/2)*(35*(b*c - a*d)^2 + 42*d*(b*c - a*d)*(a + b*x) + 15*d
^2*(a + b*x)^2) + 7*b*e*(15*b^2*c^2*Sqrt[a + b*x] + 5*d*(2*b*c - a*d)*(a +
b*x)^(3/2) + 3*d^2*(a + b*x)^(5/2) - 15*Sqrt[a]*b^2*c^2*ArcTanh[Sqrt[a + b*
x]/Sqrt[a]])))/(105*b^3)
```

fricas [A] time = 0.96, size = 403, normalized size = 2.78

$$\left[\frac{105\sqrt{a}\,b^3c^2e\log\left(\frac{bx-2\sqrt{bx+a}\,\sqrt{a}+2a}{x}\right)+2\left(15b^3d^2fx^3+3\left(7b^3d^2e+\left(14b^3cd+ab^2d^2\right)f\right)x^2+7\left(15b^3c^2+10a\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/105*(105*sqrt(a)*b^3*c^2*e*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)
+ 2*(15*b^3*d^2*f*x^3 + 3*(7*b^3*d^2*e + (14*b^3*c*d + a*b^2*d^2)*f)*x^2 +
7*(15*b^3*c^2 + 10*a*b^2*c*d - 2*a^2*b*d^2)*e + (35*a*b^2*c^2 - 28*a^2*b*c*
```

```
d + 8*a^3*d^2)*f + (7*(10*b^3*c*d + a*b^2*d^2)*e + (35*b^3*c^2 + 14*a*b^2*c
*d - 4*a^2*b*d^2)*f)*x)*sqrt(b*x + a))/b^3, 2/105*(105*sqrt(-a)*b^3*c^2*e*a
rctan(sqrt(b*x + a)*sqrt(-a)/a) + (15*b^3*d^2*f*x^3 + 3*(7*b^3*d^2*e + (14*
b^3*c*d + a*b^2*d^2)*f)*x^2 + 7*(15*b^3*c^2 + 10*a*b^2*c*d - 2*a^2*b*d^2)*e
 + (35*a*b^2*c^2 - 28*a^2*b*c*d + 8*a^3*d^2)*f + (7*(10*b^3*c*d + a*b^2*d^2
)*e + (35*b^3*c^2 + 14*a*b^2*c*d - 4*a^2*b*d^2)*f)*x)*sqrt(b*x + a))/b^3]
```

giac [A] time = 1.37, size = 201, normalized size = 1.39

$$\frac{2\,ac^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)e}{\sqrt{-a}}+\frac{2\left(35\,(bx+a)^{\frac{3}{2}}b^{20}c^2f+42\,(bx+a)^{\frac{5}{2}}b^{19}cdf-70\,(bx+a)^{\frac{3}{2}}ab^{19}cdf+15\,(bx+a)^{\frac{7}{2}}b^{18}d^2f\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*a*c^2*arctan(sqrt(b*x + a)/sqrt(-a))*e/sqrt(-a) + 2/105*(35*(b*x + a)^(3/
2)*b^20*c^2*f + 42*(b*x + a)^(5/2)*b^19*c*d*f - 70*(b*x + a)^(3/2)*a*b^19*c
*d*f + 15*(b*x + a)^(7/2)*b^18*d^2*f - 42*(b*x + a)^(5/2)*a*b^18*d^2*f + 35
*(b*x + a)^(3/2)*a^2*b^18*d^2*f + 105*sqrt(b*x + a)*b^21*c^2*e + 70*(b*x +
a)^(3/2)*b^20*c*d*e + 21*(b*x + a)^(5/2)*b^19*d^2*e - 35*(b*x + a)^(3/2)*a*
b^19*d^2*e)/b^21
```

maple [A] time = 0.01, size = 176, normalized size = 1.21

$$\frac{-2\sqrt{a}\;b^3c^2e\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+2\sqrt{bx+a}\;b^3c^2e+\frac{2(bx+a)^{\frac{3}{2}}a^2d^2f}{3}-\frac{4(bx+a)^{\frac{3}{2}}abcdf}{3}-\frac{2(bx+a)^{\frac{3}{2}}abd^2e}{3}+\frac{2(bx+a)^{\frac{3}{2}}b^2c^2f}{3}+\frac{4(b\cdots}{}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(f*x+e)*(b*x+a)^(1/2)/x,x)
```

```
[Out] 2/b^3*(1/7*d^2*f*(b*x+a)^(7/2)-2/5*(b*x+a)^(5/2)*a*d^2*f+2/5*(b*x+a)^(5/2)*
b*c*d*f+1/5*(b*x+a)^(5/2)*b*d^2*e+1/3*(b*x+a)^(3/2)*a^2*d^2*f-2/3*(b*x+a)^(
3/2)*a*b*c*d*f-1/3*(b*x+a)^(3/2)*a*b*d^2*e+1/3*(b*x+a)^(3/2)*b^2*c^2*f+2/3*
(b*x+a)^(3/2)*b^2*c*d*e+b^3*c^2*e*(b*x+a)^(1/2)-a^(1/2)*b^3*c^2*e*arctanh((
b*x+a)^(1/2)/a^(1/2)))
```

maxima [A] time = 0.98, size = 152, normalized size = 1.05

$$\sqrt{a}\,c^2e\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+\frac{2\left(105\sqrt{bx+a}\,b^3c^2e+15\,(bx+a)^{\frac{7}{2}}d^2f+21\left(bd^2e+2\left(bcd-ad^2\right)f\right)(bx+a)^{\frac{5}{2}}+35\cdots\right.}{105\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] sqrt(a)*c^2*e*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2/
105*(105*sqrt(b*x + a)*b^3*c^2*e + 15*(b*x + a)^(7/2)*d^2*f + 21*(b*d^2*e +
 2*(b*c*d - a*d^2)*f)*(b*x + a)^(5/2) + 35*((2*b^2*c*d - a*b*d^2)*e + (b^2*
c^2 - 2*a*b*c*d + a^2*d^2)*f)*(b*x + a)^(3/2))/b^3
```

mupad [B] time = 0.09, size = 263, normalized size = 1.81

$$\left(\frac{2\,b\,d^2\,e-6\,a\,d^2\,f+4\,b\,c\,d\,f}{5\,b^3}+\frac{2\,a\,d^2\,f}{5\,b^3}\right)\,(a+b\,x)^{5/2}+\left(a\,\left(a\,\left(\frac{2\,b\,d^2\,e-6\,a\,d^2\,f+4\,b\,c\,d\,f}{b^3}+\frac{2\,a\,d^2\,f}{b^3}\right)-\frac{2\,(a\,d-b\cdots}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(a + b*x)^(1/2)*(c + d*x)^2)/x,x)
```

```
[Out] ((2*b*d^2*e - 6*a*d^2*f + 4*b*c*d*f)/(5*b^3) + (2*a*d^2*f)/(5*b^3))*(a + b*
x)^(5/2) + (a*(a*((2*b*d^2*e - 6*a*d^2*f + 4*b*c*d*f)/b^3 + (2*a*d^2*f)/b^3
) - (2*(a*d - b*c)*(b*c*f - 3*a*d*f + 2*b*d*e))/b^3) - (2*(a*d - b*c)^2*(a*
f - b*e))/b^3)*(a + b*x)^(1/2) + ((a*((2*b*d^2*e - 6*a*d^2*f + 4*b*c*d*f)/b
^3 + (2*a*d^2*f)/b^3))/3 - (2*(a*d - b*c)*(b*c*f - 3*a*d*f + 2*b*d*e))/(3*b
^3))*(a + b*x)^(3/2) + a^(1/2)*c^2*e*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*2i
+ (2*d^2*f*(a + b*x)^(7/2))/(7*b^3)
```

sympy [A] time = 26.17, size = 167, normalized size = 1.15

$$\frac{2ac^2e\,\mathrm{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2c^2e\sqrt{a+bx}+\frac{2d^2f\,(a+bx)^{\frac{7}{2}}}{7b^3}+\frac{2\,(a+bx)^{\frac{5}{2}}\left(-2ad^2f+2bcdf+bd^2e\right)}{5b^3}+\frac{2\,(a+bx)^{\frac{3}{2}}\left(a^2d^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(f*x+e)*(b*x+a)**(1/2)/x,x)
```

```
[Out] 2*a*c**2*e*atan(sqrt(a + b*x)/sqrt(-a))/sqrt(-a) + 2*c**2*e*sqrt(a + b*x) +
 2*d**2*f*(a + b*x)**(7/2)/(7*b**3) + 2*(a + b*x)**(5/2)*(-2*a*d**2*f + 2*b
*c*d*f + b*d**2*e)/(5*b**3) + 2*(a + b*x)**(3/2)*(a**2*d**2*f - 2*a*b*c*d*f
 - a*b*d**2*e + b**2*c**2*f + 2*b**2*c*d*e)/(3*b**3)
```

3.17 $\int \frac{\sqrt{a+bx}\,(c+dx)(e+fx)}{x}\,dx$

Optimal. Leaf size=77

$$-\frac{2(a+bx)^{3/2}(2adf-5b(cf+de)-3bdfx)}{15b^2}+2ce\sqrt{a+bx}-2\sqrt{a}\,ce\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

```
[Out] -2/15*(b*x+a)^(3/2)*(2*a*d*f-5*b*(c*f+d*e)-3*b*d*f*x)/b^2-2*c*e*arctanh((b*
x+a)^(1/2)/a^(1/2))*a^(1/2)+2*c*e*(b*x+a)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {147, 50, 63, 208}

$$-\frac{2(a+bx)^{3/2}(2adf-5b(cf+de)-3bdfx)}{15b^2}+2ce\sqrt{a+bx}-2\sqrt{a}\,ce\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x]*(c + d*x)*(e + f*x))/x,x]
```

```
[Out] 2*c*e*Sqrt[a + b*x] - (2*(a + b*x)^(3/2)*(2*a*d*f - 5*b*(d*e + c*f) - 3*b*d
*f*x))/(15*b^2) - 2*Sqrt[a]*c*e*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}\,(c+dx)(e+fx)}{x}\,dx = -\frac{2(a+bx)^{3/2}(2adf-5b(de+cf)-3bdfx)}{15b^2} + (ce)\int \frac{\sqrt{a+bx}}{x}\,dx$$
$$= 2ce\sqrt{a+bx} - \frac{2(a+bx)^{3/2}(2adf-5b(de+cf)-3bdfx)}{15b^2} + (ace)\int \frac{1}{x\sqrt{a+b}}$$
$$= 2ce\sqrt{a+bx} - \frac{2(a+bx)^{3/2}(2adf-5b(de+cf)-3bdfx)}{15b^2} + \frac{(2ace)\,\text{Subst}\Big(\int}{}$$
$$= 2ce\sqrt{a+bx} - \frac{2(a+bx)^{3/2}(2adf-5b(de+cf)-3bdfx)}{15b^2} - 2\sqrt{a}\,ce\tanh^{-1}\Big($$

Mathematica [A] time = 0.13, size = 87, normalized size = 1.13

$$\frac{2(a+bx)^{3/2}(-adf+bcf+bde)}{3b^2} + \frac{2df(a+bx)^{5/2}}{5b^2} + 2ce\sqrt{a+bx} - 2\sqrt{a}\,ce\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x]*(c + d*x)*(e + f*x))/x,x]
```

```
[Out] 2*c*e*Sqrt[a + b*x] + (2*(b*d*e + b*c*f - a*d*f)*(a + b*x)^(3/2))/(3*b^2) +
 (2*d*f*(a + b*x)^(5/2))/(5*b^2) - 2*Sqrt[a]*c*e*ArcTanh[Sqrt[a + b*x]/Sqrt
[a]]
```

fricas [A] time = 1.08, size = 217, normalized size = 2.82

$$\Bigg[\frac{15\sqrt{a}\,b^2ce\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right) + 2\left(3b^2dfx^2 + 5\left(3b^2c+abd\right)e + \left(5abc-2a^2d\right)f + \left(5b^2de + \left(5b^2c+ab\right.\right.\right.}{15b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/15*(15*sqrt(a)*b^2*c*e*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*
(3*b^2*d*f*x^2 + 5*(3*b^2*c + a*b*d)*e + (5*a*b*c - 2*a^2*d)*f + (5*b^2*d*e
 + (5*b^2*c + a*b*d)*f)*x)*sqrt(b*x + a))/b^2, 2/15*(15*sqrt(-a)*b^2*c*e*ar
ctan(sqrt(b*x + a)*sqrt(-a)/a) + (3*b^2*d*f*x^2 + 5*(3*b^2*c + a*b*d)*e + (
5*a*b*c - 2*a^2*d)*f + (5*b^2*d*e + (5*b^2*c + a*b*d)*f)*x)*sqrt(b*x + a))/
b^2]
```

giac [A] time = 1.26, size = 105, normalized size = 1.36

$$\frac{2ac\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)e}{\sqrt{-a}} + \frac{2\left(5(bx+a)^{\frac{3}{2}}b^9cf + 3(bx+a)^{\frac{5}{2}}b^8df - 5(bx+a)^{\frac{3}{2}}ab^8df + 15\sqrt{bx+a}\,b^{10}ce + 5(bx+\right.}{15b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*a*c*arctan(sqrt(b*x + a)/sqrt(-a))*e/sqrt(-a) + 2/15*(5*(b*x + a)^(3/2)*b
^9*c*f + 3*(b*x + a)^(5/2)*b^8*d*f - 5*(b*x + a)^(3/2)*a*b^8*d*f + 15*sqrt(
b*x + a)*b^10*c*e + 5*(b*x + a)^(3/2)*b^9*d*e)/b^10
```

maple [A] time = 0.01, size = 89, normalized size = 1.16

$$\frac{-2\sqrt{a}\, b^2 ce \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) + 2\sqrt{bx+a}\, b^2 ce - \frac{2(bx+a)^{\frac{3}{2}} adf}{3} + \frac{2(bx+a)^{\frac{3}{2}} bcf}{3} + \frac{2(bx+a)^{\frac{3}{2}} bde}{3} + \frac{2(bx+a)^{\frac{5}{2}} df}{5}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(f*x+e)*(b*x+a)^(1/2)/x,x)
```

```
[Out] 2/b^2*(1/5*d*f*(b*x+a)^(5/2)-1/3*(b*x+a)^(3/2)*a*d*f+1/3*(b*x+a)^(3/2)*b*c*
f+1/3*(b*x+a)^(3/2)*b*d*e+b^2*c*e*(b*x+a)^(1/2)-a^(1/2)*b^2*c*e*arctanh((b*
x+a)^(1/2)/a^(1/2)))
```

maxima [A] time = 0.98, size = 90, normalized size = 1.17

$$\sqrt{a}\, ce \log\left(\frac{\sqrt{bx+a} - \sqrt{a}}{\sqrt{bx+a} + \sqrt{a}}\right) + \frac{2\left(15\sqrt{bx+a}\, b^2 ce + 3(bx+a)^{\frac{5}{2}} df + 5\left(bde + (bc - ad)f\right)(bx+a)^{\frac{3}{2}}\right)}{15\, b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] sqrt(a)*c*e*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2/15
*(15*sqrt(b*x + a)*b^2*c*e + 3*(b*x + a)^(5/2)*d*f + 5*(b*d*e + (b*c - a*d)
*f)*(b*x + a)^(3/2))/b^2
```

mupad [B] time = 2.49, size = 136, normalized size = 1.77

$$\left(a\left(\frac{2\,b\,c\,f - 4\,a\,d\,f + 2\,b\,d\,e}{b^2} + \frac{2\,a\,d\,f}{b^2}\right) + \frac{2\,(a\,d - b\,c)\,\left(a\,f - b\,e\right)}{b^2}\right)\sqrt{a+b\,x} + \left(\frac{2\,b\,c\,f - 4\,a\,d\,f + 2\,b\,d\,e}{3\,b^2} + \frac{2\,a\,d\,f}{3\,b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(a + b*x)^(1/2)*(c + d*x))/x,x)
```

```
[Out] (a*((2*b*c*f - 4*a*d*f + 2*b*d*e)/b^2 + (2*a*d*f)/b^2) + (2*(a*d - b*c)*(a*
f - b*e))/b^2)*(a + b*x)^(1/2) + ((2*b*c*f - 4*a*d*f + 2*b*d*e)/(3*b^2) + (
2*a*d*f)/(3*b^2))*(a + b*x)^(3/2) + (2*d*f*(a + b*x)^(5/2))/(5*b^2) + a^(1/
2)*c*e*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*2i
```

sympy [A] time = 25.97, size = 92, normalized size = 1.19

$$\frac{2ace \operatorname{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2ce\sqrt{a+bx} + \frac{2df\,(a+bx)^{\frac{5}{2}}}{5b^2} + \frac{2\,(a+bx)^{\frac{3}{2}}\left(-adf + bcf + bde\right)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(f*x+e)*(b*x+a)**(1/2)/x,x)
```

```
[Out] 2*a*c*e*atan(sqrt(a + b*x)/sqrt(-a))/sqrt(-a) + 2*c*e*sqrt(a + b*x) + 2*d*f
*(a + b*x)**(5/2)/(5*b**2) + 2*(a + b*x)**(3/2)*(-a*d*f + b*c*f + b*d*e)/(3
*b**2)
```

3.18 $\int \frac{\sqrt{a+bx}(e+fx)}{x} dx$

Optimal. Leaf size=54

$$2e\sqrt{a+bx} - 2\sqrt{a}\, e \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right) + \frac{2f(a+bx)^{3/2}}{3b}$$

```
[Out] 2/3*f*(b*x+a)^(3/2)/b-2*e*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2)+2*e*(b*x+a
)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {80, 50, 63, 208}

$$2e\sqrt{a+bx} - 2\sqrt{a}\, e \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right) + \frac{2f(a+bx)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x]*(e + f*x))/x,x]
```

```
[Out] 2*e*Sqrt[a + b*x] + (2*f*(a + b*x)^(3/2))/(3*b) - 2*Sqrt[a]*e*ArcTanh[Sqrt[
a + b*x]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}\,(e+fx)}{x}\,dx = \frac{2f(a+bx)^{3/2}}{3b} + e\int \frac{\sqrt{a+bx}}{x}\,dx$$
$$= 2e\sqrt{a+bx} + \frac{2f(a+bx)^{3/2}}{3b} + (ae)\int \frac{1}{x\sqrt{a+bx}}\,dx$$
$$= 2e\sqrt{a+bx} + \frac{2f(a+bx)^{3/2}}{3b} + \frac{(2ae)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{b}$$
$$= 2e\sqrt{a+bx} + \frac{2f(a+bx)^{3/2}}{3b} - 2\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Mathematica [A] time = 0.05, size = 55, normalized size = 1.02

$$e\left(2\sqrt{a+bx} - 2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right) + \frac{2f(a+bx)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x]*(e + f*x))/x,x]
```

```
[Out] (2*f*(a + b*x)^(3/2))/(3*b) + e*(2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqrt[a
 + b*x]/Sqrt[a]])
```

fricas [A] time = 0.93, size = 111, normalized size = 2.06

$$\left[\frac{3\sqrt{a}\,be\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\left(bfx+3be+af\right)\sqrt{bx+a}}{3b}, \frac{2\left(3\sqrt{-a}\,be\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\left(bfx+3be+a\right.\right.}{3b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/3*(3*sqrt(a)*b*e*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(b*f*x
 + 3*b*e + a*f)*sqrt(b*x + a))/b, 2/3*(3*sqrt(-a)*b*e*arctan(sqrt(b*x + a)*
sqrt(-a)/a) + (b*f*x + 3*b*e + a*f)*sqrt(b*x + a))/b]
```

giac [A] time = 1.20, size = 57, normalized size = 1.06

$$\frac{2\,a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)e}{\sqrt{-a}} + \frac{2\left((bx+a)^{\frac{3}{2}}b^2f + 3\sqrt{bx+a}\,b^3e\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*a*arctan(sqrt(b*x + a)/sqrt(-a))*e/sqrt(-a) + 2/3*((b*x + a)^(3/2)*b^2*f
+ 3*sqrt(b*x + a)*b^3*e)/b^3
```

maple [A] time = 0.01, size = 46, normalized size = 0.85

$$\frac{-2\sqrt{a}\;be\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) + 2\sqrt{bx+a}\;be + \frac{2(bx+a)^{\frac{3}{2}}f}{3}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(b*x+a)^(1/2)/x,x)
```

```
[Out] 2/b*(1/3*f*(b*x+a)^(3/2)+(b*x+a)^(1/2)*b*e-a^(1/2)*b*e*arctanh((b*x+a)^(1/2
)/a^(1/2)))
```

maxima [A] time = 0.97, size = 60, normalized size = 1.11

$$\sqrt{a}\, e \log\left(\frac{\sqrt{bx+a} - \sqrt{a}}{\sqrt{bx+a} + \sqrt{a}}\right) + \frac{2\left(3\sqrt{bx+a}\, be + (bx+a)^{\frac{3}{2}} f\right)}{3\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] sqrt(a)*e*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2/3*(3
*sqrt(b*x + a)*b*e + (b*x + a)^(3/2)*f)/b
```

mupad [B] time = 0.07, size = 45, normalized size = 0.83

$$2\, e\, \sqrt{a+b\, x} + \frac{2\, f\, (a+b\, x)^{3/2}}{3\, b} + \sqrt{a}\, e\, \mathrm{atan}\left(\frac{\sqrt{a+b\, x}\, 1\mathrm{i}}{\sqrt{a}}\right)\, 2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(a + b*x)^(1/2))/x,x)
```

```
[Out] 2*e*(a + b*x)^(1/2) + a^(1/2)*e*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*2i + (2*
f*(a + b*x)^(3/2))/(3*b)
```

sympy [A] time = 6.08, size = 54, normalized size = 1.00

$$\frac{2ae\, \mathrm{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2e\sqrt{a+bx} + \frac{2f\,(a+bx)^{\frac{3}{2}}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)**(1/2)/x,x)
```

```
[Out] 2*a*e*atan(sqrt(a + b*x)/sqrt(-a))/sqrt(-a) + 2*e*sqrt(a + b*x) + 2*f*(a +
b*x)**(3/2)/(3*b)
```

3.19 $\int \frac{\sqrt{a+bx}\,(e+fx)}{x(c+dx)}\,dx$

Optimal. Leaf size=101

$$\frac{2\sqrt{bc-ad}\,(de-cf)\tan^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{cd^{3/2}}-\frac{2\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c}+\frac{2f\sqrt{a+bx}}{d}$$

```
[Out] -2*e*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2)/c+2*(-c*f+d*e)*arctan(d^(1/2)*(
b*x+a)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/c/d^(3/2)+2*f*(b*x+a)^(1/2)
/d
```

Rubi [A] time = 0.12, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {154, 156, 63, 208, 205}

$$\frac{2\sqrt{bc-ad}\,(de-cf)\tan^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{cd^{3/2}}-\frac{2\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c}+\frac{2f\sqrt{a+bx}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x]*(e + f*x))/(x*(c + d*x)),x]
```

```
[Out] (2*f*Sqrt[a + b*x])/d + (2*Sqrt[b*c - a*d]*(d*e - c*f)*ArcTan[(Sqrt[d]*Sqrt
[a + b*x])/Sqrt[b*c - a*d]])/(c*d^(3/2)) - (2*Sqrt[a]*e*ArcTanh[Sqrt[a + b*
x]/Sqrt[a]])/c
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)}\,dx = \frac{2f\sqrt{a+bx}}{d} + \frac{2\int \frac{\frac{ade}{2}+\frac{1}{2}(bde-bcf+adf)x}{x\sqrt{a+bx}(c+dx)}\,dx}{d}$$
$$= \frac{2f\sqrt{a+bx}}{d} + \frac{(ae)\int \frac{1}{x\sqrt{a+bx}}\,dx}{c} + \frac{((bc-ad)(de-cf))\int \frac{1}{\sqrt{a+bx}(c+dx)}\,dx}{cd}$$
$$= \frac{2f\sqrt{a+bx}}{d} + \frac{(2ae)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{bc} + \frac{(2(bc-ad)(de-cf))\,\text{Subst}\Big(}{bc}$$
$$= \frac{2f\sqrt{a+bx}}{d} + \frac{2\sqrt{bc-ad}\,(de-cf)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{cd^{3/2}} - \frac{2\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c}$$

Mathematica [A] time = 0.23, size = 100, normalized size = 0.99

$$\frac{-\frac{2\sqrt{bc-ad}\,(cf-de)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{d^{3/2}} + \frac{2cf\sqrt{a+bx}}{d} - 2\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x]*(e + f*x))/(x*(c + d*x)),x]
```

```
[Out] ((2*c*f*Sqrt[a + b*x])/d - (2*Sqrt[b*c - a*d]*(-(d*e) + c*f)*ArcTan[(Sqrt[d
]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/d^(3/2) - 2*Sqrt[a]*e*ArcTanh[Sqrt[a + b
*x]/Sqrt[a]])/c
```

fricas [A] time = 1.12, size = 450, normalized size = 4.46

$$\left[\frac{\sqrt{a}\,de\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right) + 2\sqrt{bx+a}\,cf - \left(de-cf\right)\sqrt{-\frac{bc-ad}{d}}\log\left(\frac{bdx-bc+2ad-2\sqrt{bx+a}\,d\sqrt{-\frac{bc-ad}{d}}}{dx+c}\right)}{cd}, \frac{2\sqrt{-a}\,de\,\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c),x, algorithm="fricas")
```

```
[Out] [(sqrt(a)*d*e*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*sqrt(b*x + a
)*c*f - (d*e - c*f)*sqrt(-(b*c - a*d)/d)*log((b*d*x - b*c + 2*a*d - 2*sqrt(
b*x + a)*d*sqrt(-(b*c - a*d)/d))/(d*x + c)))/(c*d), (2*sqrt(-a)*d*e*arctan(
sqrt(b*x + a)*sqrt(-a)/a) + 2*sqrt(b*x + a)*c*f - (d*e - c*f)*sqrt(-(b*c -
a*d)/d)*log((b*d*x - b*c + 2*a*d - 2*sqrt(b*x + a)*d*sqrt(-(b*c - a*d)/d))/
(d*x + c)))/(c*d), (sqrt(a)*d*e*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x
) + 2*sqrt(b*x + a)*c*f - 2*(d*e - c*f)*sqrt((b*c - a*d)/d)*arctan(-sqrt(b*
x + a)*d*sqrt((b*c - a*d)/d)/(b*c - a*d)))/(c*d), 2*(sqrt(-a)*d*e*arctan(sq
rt(b*x + a)*sqrt(-a)/a) + sqrt(b*x + a)*c*f - (d*e - c*f)*sqrt((b*c - a*d)/
d)*arctan(-sqrt(b*x + a)*d*sqrt((b*c - a*d)/d)/(b*c - a*d)))/(c*d)]
```

giac [A] time = 1.22, size = 112, normalized size = 1.11

$$\frac{2a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)e}{\sqrt{-a}\,c} + \frac{2\sqrt{bx+a}\,f}{d} - \frac{2\left(bc^2f-acdf-bcde+ad^2e\right)\arctan\left(\frac{\sqrt{bx+a}\,d}{\sqrt{bcd-ad^2}}\right)}{\sqrt{bcd-ad^2}\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c),x, algorithm="giac")
```

```
[Out] 2*a*arctan(sqrt(b*x + a)/sqrt(-a))*e/(sqrt(-a)*c) + 2*sqrt(b*x + a)*f/d - 2
*(b*c^2*f - a*c*d*f - b*c*d*e + a*d^2*e)*arctan(sqrt(b*x + a)*d/sqrt(b*c*d
- a*d^2))/(sqrt(b*c*d - a*d^2)*c*d)
```

maple [A] time = 0.02, size = 103, normalized size = 1.02

$$-\frac{2\sqrt{a}\,e\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{c}+\frac{2\sqrt{bx+a}\,f}{d}-\frac{2\left(acdf-a\,d^{2}e-b\,c^{2}f+bcde\right)\mathrm{arctanh}\left(\frac{\sqrt{bx+a}\,d}{\sqrt{(ad-bc)d}}\right)}{\sqrt{(ad-bc)\,d}\;cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c),x)
```

```
[Out] 2*f*(b*x+a)^(1/2)/d-2*e*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2)/c-2/d*(a*c*d
*f-a*d^2*e-b*c^2*f+b*c*d*e)/c/((a*d-b*c)*d)^(1/2)*arctanh((b*x+a)^(1/2)*d/(
(a*d-b*c)*d)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 2.82, size = 2355, normalized size = 23.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(a + b*x)^(1/2))/(x*(c + d*x)),x)
```

```
[Out] (2*f*(a + b*x)^(1/2))/d - (a^(1/2)*e*atan(((a^(1/2)*e*((8*(a + b*x)^(1/2)*(
b^4*c^4*f^2 + 2*a^2*b^2*d^4*e^2 + b^4*c^2*d^2*e^2 - 2*b^4*c^3*d*e*f + a^2*b
^2*c^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a*b^3*c^3*d*f^2 + 4*a*b^3*c^2*d^2*e*
f - 2*a^2*b^2*c*d^3*e*f))/d + (a^(1/2)*e*((8*(a*b^3*c^3*d^2*f - a^2*b^2*c^2
*d^3*f))/d + (8*a^(1/2)*e*(b^3*c^3*d^3 - 2*a*b^2*c^2*d^4)*(a + b*x)^(1/2))/
(c*d)))/c)*1i)/c + (a^(1/2)*e*((8*(a + b*x)^(1/2)*(b^4*c^4*f^2 + 2*a^2*b^2*
d^4*e^2 + b^4*c^2*d^2*e^2 - 2*b^4*c^3*d*e*f + a^2*b^2*c^2*d^2*f^2 - 2*a*b^3
*c*d^3*e^2 - 2*a*b^3*c^3*d*f^2 + 4*a*b^3*c^2*d^2*e*f - 2*a^2*b^2*c*d^3*e*f)
)/d - (a^(1/2)*e*((8*(a*b^3*c^3*d^2*f - a^2*b^2*c^2*d^3*f))/d - (8*a^(1/2)*
e*(b^3*c^3*d^3 - 2*a*b^2*c^2*d^4)*(a + b*x)^(1/2))/(c*d)))/c)*1i)/c)/((16*(
a^2*b^3*d^3*e^3 - a*b^4*c*d^2*e^3 - a*b^4*c^3*e*f^2 + a^3*b^2*d^3*e^2*f - 3
*a^2*b^3*c*d^2*e^2*f + 2*a^2*b^3*c^2*d*e*f^2 - a^3*b^2*c*d^2*e*f^2 + 2*a*b^
4*c^2*d*e^2*f))/d - (a^(1/2)*e*((8*(a + b*x)^(1/2)*(b^4*c^4*f^2 + 2*a^2*b^2
*d^4*e^2 + b^4*c^2*d^2*e^2 - 2*b^4*c^3*d*e*f + a^2*b^2*c^2*d^2*f^2 - 2*a*b^
3*c*d^3*e^2 - 2*a*b^3*c^3*d*f^2 + 4*a*b^3*c^2*d^2*e*f - 2*a^2*b^2*c*d^3*e*f
))/d + (a^(1/2)*e*((8*(a*b^3*c^3*d^2*f - a^2*b^2*c^2*d^3*f))/d + (8*a^(1/2)
*e*(b^3*c^3*d^3 - 2*a*b^2*c^2*d^4)*(a + b*x)^(1/2))/(c*d)))/c))/c + (a^(1/2
)*e*((8*(a + b*x)^(1/2)*(b^4*c^4*f^2 + 2*a^2*b^2*d^4*e^2 + b^4*c^2*d^2*e^2
- 2*b^4*c^3*d*e*f + a^2*b^2*c^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a*b^3*c^3*d
*f^2 + 4*a*b^3*c^2*d^2*e*f - 2*a^2*b^2*c*d^3*e*f))/d - (a^(1/2)*e*((8*(a*b^
```

```
3*c^3*d^2*f - a^2*b^2*c^2*d^3*f))/d - (8*a^(1/2)*e*(b^3*c^3*d^3 - 2*a*b^2*c
^2*d^4)*(a + b*x)^(1/2))/(c*d)))/c))/c))*2i)/c - (atan(((((8*(a + b*x)^(1/2
)*(b^4*c^4*f^2 + 2*a^2*b^2*d^4*e^2 + b^4*c^2*d^2*e^2 - 2*b^4*c^3*d*e*f + a^
2*b^2*c^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a*b^3*c^3*d*f^2 + 4*a*b^3*c^2*d^2
*e*f - 2*a^2*b^2*c*d^3*e*f))/d + (((8*(a*b^3*c^3*d^2*f - a^2*b^2*c^2*d^3*f)
)/d + (8*(b^3*c^3*d^3 - 2*a*b^2*c^2*d^4)*(c*f - d*e)*(d^3*(a*d - b*c))^(1/2
)*(a + b*x)^(1/2))/(c*d^4))*(c*f - d*e)*(d^3*(a*d - b*c))^(1/2))/(c*d^3))*(
c*f - d*e)*(d^3*(a*d - b*c))^(1/2)*1i)/(c*d^3) + (((8*(a + b*x)^(1/2)*(b^4*
c^4*f^2 + 2*a^2*b^2*d^4*e^2 + b^4*c^2*d^2*e^2 - 2*b^4*c^3*d*e*f + a^2*b^2*c
^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a*b^3*c^3*d*f^2 + 4*a*b^3*c^2*d^2*e*f -
2*a^2*b^2*c*d^3*e*f))/d - (((8*(a*b^3*c^3*d^2*f - a^2*b^2*c^2*d^3*f))/d - (
8*(b^3*c^3*d^3 - 2*a*b^2*c^2*d^4)*(c*f - d*e)*(d^3*(a*d - b*c))^(1/2)*(a +
b*x)^(1/2))/(c*d^4))*(c*f - d*e)*(d^3*(a*d - b*c))^(1/2))/(c*d^3))*(c*f - d
*e)*(d^3*(a*d - b*c))^(1/2)*1i)/(c*d^3))/((16*(a^2*b^3*d^3*e^3 - a*b^4*c*d^
2*e^3 - a*b^4*c^3*e*f^2 + a^3*b^2*d^3*e^2*f - 3*a^2*b^3*c*d^2*e^2*f + 2*a^2
*b^3*c^2*d*e*f^2 - a^3*b^2*c*d^2*e*f^2 + 2*a*b^4*c^2*d*e^2*f))/d - (((8*(a
+ b*x)^(1/2)*(b^4*c^4*f^2 + 2*a^2*b^2*d^4*e^2 + b^4*c^2*d^2*e^2 - 2*b^4*c^3
*d*e*f + a^2*b^2*c^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a*b^3*c^3*d*f^2 + 4*a*
b^3*c^2*d^2*e*f - 2*a^2*b^2*c*d^3*e*f))/d + (((8*(a*b^3*c^3*d^2*f - a^2*b^2
*c^2*d^3*f))/d + (8*(b^3*c^3*d^3 - 2*a*b^2*c^2*d^4)*(c*f - d*e)*(d^3*(a*d -
 b*c))^(1/2)*(a + b*x)^(1/2))/(c*d^4))*(c*f - d*e)*(d^3*(a*d - b*c))^(1/2))
/(c*d^3))*(c*f - d*e)*(d^3*(a*d - b*c))^(1/2))/(c*d^3) + (((8*(a + b*x)^(1/
2)*(b^4*c^4*f^2 + 2*a^2*b^2*d^4*e^2 + b^4*c^2*d^2*e^2 - 2*b^4*c^3*d*e*f + a
^2*b^2*c^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a*b^3*c^3*d*f^2 + 4*a*b^3*c^2*d^
2*e*f - 2*a^2*b^2*c*d^3*e*f))/d - (((8*(a*b^3*c^3*d^2*f - a^2*b^2*c^2*d^3*f
))/d - (8*(b^3*c^3*d^3 - 2*a*b^2*c^2*d^4)*(c*f - d*e)*(d^3*(a*d - b*c))^(1/
2)*(a + b*x)^(1/2))/(c*d^4))*(c*f - d*e)*(d^3*(a*d - b*c))^(1/2))/(c*d^3))*
(c*f - d*e)*(d^3*(a*d - b*c))^(1/2))/(c*d^3)))*(c*f - d*e)*(d^3*(a*d - b*c)
)^(1/2)*2i)/(c*d^3)
```

sympy [A] time = 24.25, size = 100, normalized size = 0.99

$$\frac{2ae\operatorname{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{c\sqrt{-a}}+\frac{2f\sqrt{a+bx}}{d}+\frac{2\left(ad-bc\right)\left(cf-de\right)\operatorname{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-\frac{ad-bc}{d}}}\right)}{cd^{2}\sqrt{-\frac{ad-bc}{d}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)**(1/2)/x/(d*x+c),x)
```

```
[Out] 2*a*e*atan(sqrt(a + b*x)/sqrt(-a))/(c*sqrt(-a)) + 2*f*sqrt(a + b*x)/d + 2*(
a*d - b*c)*(c*f - d*e)*atan(sqrt(a + b*x)/sqrt(-(a*d - b*c)/d))/(c*d**2*sqr
t(-(a*d - b*c)/d))
```

3.20 $\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)^2} dx$

Optimal. Leaf size=128

$$-\frac{\left(2ad^2e-bc(cf+de)\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{c^2d^{3/2}\sqrt{bc-ad}}-\frac{2\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c^2}+\frac{\sqrt{a+bx}\,(de-cf)}{cd(c+dx)}$$

```
[Out] -2*e*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2)/c^2-(2*a*d^2*e-b*c*(c*f+d*e))*a
rctan(d^(1/2)*(b*x+a)^(1/2)/(-a*d+b*c)^(1/2))/c^2/d^(3/2)/(-a*d+b*c)^(1/2)+
(-c*f+d*e)*(b*x+a)^(1/2)/c/d/(d*x+c)
```

Rubi [A] time = 0.12, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{149, 156, 63, 208, 205\}$

$$-\frac{\left(2ad^2e-bc(cf+de)\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{c^2d^{3/2}\sqrt{bc-ad}}-\frac{2\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c^2}+\frac{\sqrt{a+bx}\,(de-cf)}{cd(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x]*(e + f*x))/(x*(c + d*x)^2),x]
```

```
[Out] ((d*e - c*f)*Sqrt[a + b*x])/(c*d*(c + d*x)) - ((2*a*d^2*e - b*c*(d*e + c*f)
)*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(c^2*d^(3/2)*Sqrt[b*c -
a*d]) - (2*Sqrt[a]*e*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/c^2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}\,(e+fx)}{x(c+dx)^2}\,dx = \frac{(de-cf)\sqrt{a+bx}}{cd(c+dx)} - \frac{\int \frac{-ade-\frac{1}{2}b(de+cf)x}{x\sqrt{a+bx}\,(c+dx)}\,dx}{cd}$$

$$= \frac{(de-cf)\sqrt{a+bx}}{cd(c+dx)} + \frac{(ae)\int \frac{1}{x\sqrt{a+bx}}\,dx}{c^2} - \frac{\left(2ad^2e-bc(de+cf)\right)\int \frac{1}{\sqrt{a+bx}\,(c+dx)}\,dx}{2c^2d}$$

$$= \frac{(de-cf)\sqrt{a+bx}}{cd(c+dx)} + \frac{(2ae)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{bc^2} - \frac{\left(2ad^2e-bc(de+cf)\right)}{}$$

$$= \frac{(de-cf)\sqrt{a+bx}}{cd(c+dx)} - \frac{\left(2ad^2e-bc(de+cf)\right)\tan^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{c^2d^{3/2}\sqrt{bc-ad}} - \frac{2\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c^2}$$

Mathematica [A] time = 0.20, size = 122, normalized size = 0.95

$$\frac{\frac{\left(bc(cf+de)-2ad^2e\right)\tan^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{d^{3/2}\sqrt{bc-ad}} + \frac{c\sqrt{a+bx}\,(de-cf)}{d(c+dx)} - 2\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x]*(e + f*x))/(x*(c + d*x)^2),x]
```

```
[Out] ((c*(d*e - c*f)*Sqrt[a + b*x])/(d*(c + d*x)) + ((-2*a*d^2*e + b*c*(d*e + c*
f))*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(d^(3/2)*Sqrt[b*c - a*
d]) - 2*Sqrt[a]*e*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/c^2
```

fricas [B] time = 1.26, size = 1008, normalized size = 7.88

$$\left[-\frac{\left(bc^3f+\left(bc^2d-2\,acd^2\right)e+\left(bc^2df+\left(bcd^2-2\,ad^3\right)e\right)x\right)\sqrt{-bcd+ad^2}\,\log\left(\frac{bdx-bc+2\,ad-2\,\sqrt{-bcd+ad^2}\,\sqrt{bx+a}}{dx+c}\right)-2}{2\left(bc^4d^2-ac^3d^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((b*c^3*f + (b*c^2*d - 2*a*c*d^2)*e + (b*c^2*d*f + (b*c*d^2 - 2*a*d^3
)*e)*x)*sqrt(-b*c*d + a*d^2)*log((b*d*x - b*c + 2*a*d - 2*sqrt(-b*c*d + a*d
^2)*sqrt(b*x + a))/(d*x + c)) - 2*((b*c*d^3 - a*d^4)*e*x + (b*c^2*d^2 - a*c
*d^3)*e)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*((b*c^2*d
^2 - a*c*d^3)*e - (b*c^3*d - a*c^2*d^2)*f)*sqrt(b*x + a))/(b*c^4*d^2 - a*c^
3*d^3 + (b*c^3*d^3 - a*c^2*d^4)*x), 1/2*(4*((b*c*d^3 - a*d^4)*e*x + (b*c^2*
d^2 - a*c*d^3)*e)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) - (b*c^3*f + (b
*c^2*d - 2*a*c*d^2)*e + (b*c^2*d*f + (b*c*d^2 - 2*a*d^3)*e)*x)*sqrt(-b*c*d
+ a*d^2)*log((b*d*x - b*c + 2*a*d - 2*sqrt(-b*c*d + a*d^2)*sqrt(b*x + a))/(
d*x + c)) + 2*((b*c^2*d^2 - a*c*d^3)*e - (b*c^3*d - a*c^2*d^2)*f)*sqrt(b*x
+ a))/(b*c^4*d^2 - a*c^3*d^3 + (b*c^3*d^3 - a*c^2*d^4)*x), -((b*c^3*f + (b*
c^2*d - 2*a*c*d^2)*e + (b*c^2*d*f + (b*c*d^2 - 2*a*d^3)*e)*x)*sqrt(b*c*d -
a*d^2)*arctan(sqrt(b*c*d - a*d^2)*sqrt(b*x + a)/(b*d*x + a*d)) - ((b*c*d^3
```

```
- a*d^4)*e*x + (b*c^2*d^2 - a*c*d^3)*e)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*
sqrt(a) + 2*a)/x) - ((b*c^2*d^2 - a*c*d^3)*e - (b*c^3*d - a*c^2*d^2)*f)*sqr
t(b*x + a))/(b*c^4*d^2 - a*c^3*d^3 + (b*c^3*d^3 - a*c^2*d^4)*x), -((b*c^3*f
 + (b*c^2*d - 2*a*c*d^2)*e + (b*c^2*d*f + (b*c*d^2 - 2*a*d^3)*e)*x)*sqrt(b*
c*d - a*d^2)*arctan(sqrt(b*c*d - a*d^2)*sqrt(b*x + a)/(b*d*x + a*d)) - 2*((
b*c*d^3 - a*d^4)*e*x + (b*c^2*d^2 - a*c*d^3)*e)*sqrt(-a)*arctan(sqrt(b*x +
a)*sqrt(-a)/a) - ((b*c^2*d^2 - a*c*d^3)*e - (b*c^3*d - a*c^2*d^2)*f)*sqrt(b
*x + a))/(b*c^4*d^2 - a*c^3*d^3 + (b*c^3*d^3 - a*c^2*d^4)*x)]
```

giac [A] time = 1.32, size = 142, normalized size = 1.11

$$\frac{2\,a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)e}{\sqrt{-a}\,c^2}+\frac{\left(bc^2f+bcde-2\,ad^2e\right)\arctan\left(\frac{\sqrt{bx+a}\,d}{\sqrt{bcd-ad^2}}\right)}{\sqrt{bcd-ad^2}\,c^2d}-\frac{\sqrt{bx+a}\,bcf-\sqrt{bx+a}\,bde}{(bc+(bx+a)d-ad)cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 2*a*arctan(sqrt(b*x + a)/sqrt(-a))*e/(sqrt(-a)*c^2) + (b*c^2*f + b*c*d*e -
2*a*d^2*e)*arctan(sqrt(b*x + a)*d/sqrt(b*c*d - a*d^2))/(sqrt(b*c*d - a*d^2)
*c^2*d) - (sqrt(b*x + a)*b*c*f - sqrt(b*x + a)*b*d*e)/((b*c + (b*x + a)*d -
 a*d)*c*d)
```

maple [A] time = 0.02, size = 137, normalized size = 1.07

$$2\left(-\frac{\sqrt{a}\;e\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{b\,c^2}-\frac{\frac{\left(cf-de\right)\sqrt{bx+a}\;bc}{2(-ad+bc+(bx+a)d)d}-\frac{\left(2a\,d^2e-b\,c^2f-bcde\right)\mathrm{arctanh}\left(\frac{\sqrt{bx+a}\;d}{\sqrt{(ad-bc)d}}\right)}{2\sqrt{(ad-bc)d}\;d}}{b\,c^2}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c)^2,x)
```

```
[Out] 2*b*(-a^(1/2)/b*e/c^2*arctanh((b*x+a)^(1/2)/a^(1/2))-1/c^2/b*(1/2*b*c*(c*f-
d*e)/d*(b*x+a)^(1/2)/((b*x+a)*d-a*d+b*c)-1/2*(2*a*d^2*e-b*c^2*f-b*c*d*e)/d/
((a*d-b*c)*d)^(1/2)*arctanh((b*x+a)^(1/2)/((a*d-b*c)*d)^(1/2)*d)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 2.95, size = 1814, normalized size = 14.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(a + b*x)^(1/2))/(x*(c + d*x)^2),x)
```

```
[Out] (atan((((((((2*(2*a*b^3*c^4*d^3*e - 2*a*b^3*c^5*d^2*f))/(c^3*d) + ((4*b^3*c^
5*d^3 - 8*a*b^2*c^4*d^4)*(d^3*(a*d - b*c))^(1/2)*(a + b*x)^(1/2)*(b*c^2*f -
```

```
 2*a*d^2*e + b*c*d*e))/(c^2*d*(a*c^2*d^4 - b*c^3*d^3)))*(d^3*(a*d - b*c))^(
1/2)*(b*c^2*f - 2*a*d^2*e + b*c*d*e))/(2*(a*c^2*d^4 - b*c^3*d^3)) + (2*(a +
 b*x)^(1/2)*(b^4*c^4*f^2 + 8*a^2*b^2*d^4*e^2 + b^4*c^2*d^2*e^2 + 2*b^4*c^3*
d*e*f - 4*a*b^3*c*d^3*e^2 - 4*a*b^3*c^2*d^2*e*f))/(c^2*d))*(d^3*(a*d - b*c)
)^(1/2)*(b*c^2*f - 2*a*d^2*e + b*c*d*e)*1i)/(2*(a*c^2*d^4 - b*c^3*d^3)) - (
((((2*(2*a*b^3*c^4*d^3*e - 2*a*b^3*c^5*d^2*f))/(c^3*d) - ((4*b^3*c^5*d^3 -
8*a*b^2*c^4*d^4)*(d^3*(a*d - b*c))^(1/2)*(a + b*x)^(1/2)*(b*c^2*f - 2*a*d^2
*e + b*c*d*e))/(c^2*d*(a*c^2*d^4 - b*c^3*d^3)))*(d^3*(a*d - b*c))^(1/2)*(b*
c^2*f - 2*a*d^2*e + b*c*d*e))/(2*(a*c^2*d^4 - b*c^3*d^3)) - (2*(a + b*x)^(1
/2)*(b^4*c^4*f^2 + 8*a^2*b^2*d^4*e^2 + b^4*c^2*d^2*e^2 + 2*b^4*c^3*d*e*f -
4*a*b^3*c*d^3*e^2 - 4*a*b^3*c^2*d^2*e*f))/(c^2*d))*(d^3*(a*d - b*c))^(1/2)*
(b*c^2*f - 2*a*d^2*e + b*c*d*e)*1i)/(2*(a*c^2*d^4 - b*c^3*d^3)))/((4*(a*b^4
*c*d^2*e^3 - 2*a^2*b^3*d^3*e^3 + a*b^4*c^3*e*f^2 - 2*a^2*b^3*c*d^2*e^2*f +
2*a*b^4*c^2*d*e^2*f))/(c^3*d) + (((((2*(2*a*b^3*c^4*d^3*e - 2*a*b^3*c^5*d^2
*f))/(c^3*d) + ((4*b^3*c^5*d^3 - 8*a*b^2*c^4*d^4)*(d^3*(a*d - b*c))^(1/2)*(
a + b*x)^(1/2)*(b*c^2*f - 2*a*d^2*e + b*c*d*e))/(c^2*d*(a*c^2*d^4 - b*c^3*d
^3)))*(d^3*(a*d - b*c))^(1/2)*(b*c^2*f - 2*a*d^2*e + b*c*d*e))/(2*(a*c^2*d^
4 - b*c^3*d^3)) + (2*(a + b*x)^(1/2)*(b^4*c^4*f^2 + 8*a^2*b^2*d^4*e^2 + b^4
*c^2*d^2*e^2 + 2*b^4*c^3*d*e*f - 4*a*b^3*c*d^3*e^2 - 4*a*b^3*c^2*d^2*e*f))/
(c^2*d))*(d^3*(a*d - b*c))^(1/2)*(b*c^2*f - 2*a*d^2*e + b*c*d*e))/(2*(a*c^2
*d^4 - b*c^3*d^3)) + (((((2*(2*a*b^3*c^4*d^3*e - 2*a*b^3*c^5*d^2*f))/(c^3*d
) - ((4*b^3*c^5*d^3 - 8*a*b^2*c^4*d^4)*(d^3*(a*d - b*c))^(1/2)*(a + b*x)^(1
/2)*(b*c^2*f - 2*a*d^2*e + b*c*d*e))/(c^2*d*(a*c^2*d^4 - b*c^3*d^3)))*(d^3*
(a*d - b*c))^(1/2)*(b*c^2*f - 2*a*d^2*e + b*c*d*e))/(2*(a*c^2*d^4 - b*c^3*d
^3)) - (2*(a + b*x)^(1/2)*(b^4*c^4*f^2 + 8*a^2*b^2*d^4*e^2 + b^4*c^2*d^2*e^
2 + 2*b^4*c^3*d*e*f - 4*a*b^3*c*d^3*e^2 - 4*a*b^3*c^2*d^2*e*f))/(c^2*d))*(d
^3*(a*d - b*c))^(1/2)*(b*c^2*f - 2*a*d^2*e + b*c*d*e))/(2*(a*c^2*d^4 - b*c^
3*d^3))))*(d^3*(a*d - b*c))^(1/2)*(b*c^2*f - 2*a*d^2*e + b*c*d*e)*1i)/(a*c^
2*d^4 - b*c^3*d^3) - (2*a^(1/2)*e*atanh((4*a^(1/2)*b^4*e*f^2*(a + b*x)^(1/2
))/(4*a*b^4*e*f^2 + (4*a*b^4*d^2*e^3)/c^2 - (16*a^2*b^3*d^2*e^2*f)/c^2 + (8
*a*b^4*d*e^2*f)/c) + (8*a^(1/2)*b^4*e^2*f*(a + b*x)^(1/2))/(8*a*b^4*e^2*f +
 (4*a*b^4*d*e^3)/c - (16*a^2*b^3*d*e^2*f)/c + (4*a*b^4*c*e*f^2)/d) + (4*a^(
1/2)*b^4*d*e^3*(a + b*x)^(1/2))/(4*a*b^4*d*e^3 + 8*a*b^4*c*e^2*f - 16*a^2*b
^3*d*e^2*f + (4*a*b^4*c^2*e*f^2)/d) - (16*a^(3/2)*b^3*d*e^2*f*(a + b*x)^(1/
2))/(4*a*b^4*d*e^3 + 8*a*b^4*c*e^2*f - 16*a^2*b^3*d*e^2*f + (4*a*b^4*c^2*e*
f^2)/d)))/c^2 - ((b*c*f - b*d*e)*(a + b*x)^(1/2))/(c*d*(b*c - a*d + d*(a +
b*x)))
```

sympy [B] time = 142.24, size = 1149, normalized size = 8.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)**(1/2)/x/(d*x+c)**2,x)
```

```
[Out] 2*a*b*d*e*sqrt(a + b*x)/(2*a*b*c**2*d + 2*a*b*c*d**2*x - 2*b**2*c**3 - 2*b*
*2*c**2*d*x) - a*b*f*sqrt(1/(d*(a*d - b*c)**3))*log(-a**2*d**2*sqrt(1/(d*(a
*d - b*c)**3)) + 2*a*b*c*d*sqrt(1/(d*(a*d - b*c)**3)) - b**2*c**2*sqrt(1/(d
*(a*d - b*c)**3)) + sqrt(a + b*x))/2 + a*b*f*sqrt(1/(d*(a*d - b*c)**3))*log
(a**2*d**2*sqrt(1/(d*(a*d - b*c)**3)) - 2*a*b*c*d*sqrt(1/(d*(a*d - b*c)**3)
) + b**2*c**2*sqrt(1/(d*(a*d - b*c)**3)) + sqrt(a + b*x))/2 - 2*a*b*f*sqrt(
a + b*x)/(2*a*b*c*d + 2*a*b*d**2*x - 2*b**2*c**2 - 2*b**2*c*d*x) + a*b*d*e*
sqrt(1/(d*(a*d - b*c)**3))*log(-a**2*d**2*sqrt(1/(d*(a*d - b*c)**3)) + 2*a*
b*c*d*sqrt(1/(d*(a*d - b*c)**3)) - b**2*c**2*sqrt(1/(d*(a*d - b*c)**3)) + s
qrt(a + b*x))/(2*c) - a*b*d*e*sqrt(1/(d*(a*d - b*c)**3))*log(a**2*d**2*sqrt
(1/(d*(a*d - b*c)**3)) - 2*a*b*c*d*sqrt(1/(d*(a*d - b*c)**3)) + b**2*c**2*s
qrt(1/(d*(a*d - b*c)**3)) + sqrt(a + b*x))/(2*c) - 2*a*e*atan(sqrt(a + b*x)
/sqrt(-a + b*c/d))/(c**2*sqrt(-a + b*c/d)) + 2*a*e*atan(sqrt(a + b*x)/sqrt(
-a))/(c**2*sqrt(-a)) + 2*b**2*c*f*sqrt(a + b*x)/(2*a*b*c*d**2 + 2*a*b*d**3*
x - 2*b**2*c**2*d - 2*b**2*c*d**2*x) + b**2*c*f*sqrt(1/(d*(a*d - b*c)**3))*
```

```
log(-a**2*d**2*sqrt(1/(d*(a*d - b*c)**3)) + 2*a*b*c*d*sqrt(1/(d*(a*d - b*c)
**3)) - b**2*c**2*sqrt(1/(d*(a*d - b*c)**3)) + sqrt(a + b*x))/(2*d) - b**2*
c*f*sqrt(1/(d*(a*d - b*c)**3))*log(a**2*d**2*sqrt(1/(d*(a*d - b*c)**3)) - 2
*a*b*c*d*sqrt(1/(d*(a*d - b*c)**3)) + b**2*c**2*sqrt(1/(d*(a*d - b*c)**3))
+ sqrt(a + b*x))/(2*d) - b**2*e*sqrt(1/(d*(a*d - b*c)**3))*log(-a**2*d**2*s
qrt(1/(d*(a*d - b*c)**3)) + 2*a*b*c*d*sqrt(1/(d*(a*d - b*c)**3)) - b**2*c**
2*sqrt(1/(d*(a*d - b*c)**3)) + sqrt(a + b*x))/2 + b**2*e*sqrt(1/(d*(a*d - b
*c)**3))*log(a**2*d**2*sqrt(1/(d*(a*d - b*c)**3)) - 2*a*b*c*d*sqrt(1/(d*(a*
d - b*c)**3)) + b**2*c**2*sqrt(1/(d*(a*d - b*c)**3)) + sqrt(a + b*x))/2 - 2
*b**2*e*sqrt(a + b*x)/(2*a*b*c*d + 2*a*b*d**2*x - 2*b**2*c**2 - 2*b**2*c*d*
x) + 2*b*f*atan(sqrt(a + b*x)/sqrt(-a + b*c/d))/(d**2*sqrt(-a + b*c/d))
```

3.21 $\int \frac{\sqrt{a+bx}\,(e+fx)}{x(c+dx)^3}\,dx$

Optimal. Leaf size=205

$$-\frac{\left(-8a^2d^3e+12abcd^2e-b^2c^2(cf+3de)\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{4c^3d^{3/2}(bc-ad)^{3/2}}-\frac{2\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c^3}-\frac{\sqrt{a+bx}\,\left(4ad^2e-bc(cf+\right.}{4c^2d(c+dx)(bc-ad}$$

```
[Out] -1/4*(12*a*b*c*d^2*e-8*a^2*d^3*e-b^2*c^2*(c*f+3*d*e))*arctan(d^(1/2)*(b*x+a
)^(1/2)/(-a*d+b*c)^(1/2))/c^3/d^(3/2)/(-a*d+b*c)^(3/2)-2*e*arctanh((b*x+a)^
(1/2)/a^(1/2))*a^(1/2)/c^3+1/2*(-c*f+d*e)*(b*x+a)^(1/2)/c/d/(d*x+c)^2-1/4*(
4*a*d^2*e-b*c*(c*f+3*d*e))*(b*x+a)^(1/2)/c^2/d/(-a*d+b*c)/(d*x+c)
```

Rubi [A] time = 0.28, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {149, 151, 156, 63, 208, 205}

$$-\frac{\left(-8a^2d^3e+12abcd^2e-b^2c^2(cf+3de)\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{4c^3d^{3/2}(bc-ad)^{3/2}}-\frac{\sqrt{a+bx}\,\left(4ad^2e-bc(cf+3de)\right)}{4c^2d(c+dx)(bc-ad)}-\frac{2\sqrt{a}\,e\tanh^{-1}\left(\right.}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x]*(e + f*x))/(x*(c + d*x)^3),x]
```

```
[Out] ((d*e - c*f)*Sqrt[a + b*x])/(2*c*d*(c + d*x)^2) - ((4*a*d^2*e - b*c*(3*d*e
+ c*f))*Sqrt[a + b*x])/(4*c^2*d*(b*c - a*d)*(c + d*x)) - ((12*a*b*c*d^2*e -
 8*a^2*d^3*e - b^2*c^2*(3*d*e + c*f))*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b
*c - a*d]])/(4*c^3*d^(3/2)*(b*c - a*d)^(3/2)) - (2*Sqrt[a]*e*ArcTanh[Sqrt[a
 + b*x]/Sqrt[a]])/c^3
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}\,(e+fx)}{x(c+dx)^3}\,dx = \frac{(de-cf)\sqrt{a+bx}}{2cd(c+dx)^2} - \frac{\int \frac{-2ade-\frac{1}{2}b(3de+cf)x}{x\sqrt{a+bx}\,(c+dx)^2}\,dx}{2cd}$$

$$= \frac{(de-cf)\sqrt{a+bx}}{2cd(c+dx)^2} - \frac{\left(4ad^2e-bc(3de+cf)\right)\sqrt{a+bx}}{4c^2d(bc-ad)(c+dx)} + \frac{\int \frac{2ad(bc-ad)e-\frac{1}{4}b\left(4ad^2e-bc(3de+cf)\right)x}{x\sqrt{a+bx}\,(c+dx)}}{2c^2d(bc-ad)}$$

$$= \frac{(de-cf)\sqrt{a+bx}}{2cd(c+dx)^2} - \frac{\left(4ad^2e-bc(3de+cf)\right)\sqrt{a+bx}}{4c^2d(bc-ad)(c+dx)} + \frac{(ae)\int \frac{1}{x\sqrt{a+bx}}\,dx}{c^3} - \frac{\left(12abcd^2e\right.}{}$$

$$= \frac{(de-cf)\sqrt{a+bx}}{2cd(c+dx)^2} - \frac{\left(4ad^2e-bc(3de+cf)\right)\sqrt{a+bx}}{4c^2d(bc-ad)(c+dx)} + \frac{(2ae)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a}\right.}{bc^3}$$

$$= \frac{(de-cf)\sqrt{a+bx}}{2cd(c+dx)^2} - \frac{\left(4ad^2e-bc(3de+cf)\right)\sqrt{a+bx}}{4c^2d(bc-ad)(c+dx)} - \frac{\left(12abcd^2e-8a^2d^3e-b^2c^2(3de\right.}{4c^3d^{3/2}(bc-}$$

Mathematica [A] time = 0.53, size = 259, normalized size = 1.26

$$\frac{\frac{2\left(\frac{\left(8a^2d^3e-12abcd^2e+b^2c^2(cf+3de)\right)\left(\sqrt{d}\sqrt{a+bx}-\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)\right)}{4d^{3/2}}+2e(bc-ad)^2\left(\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-\sqrt{a+bx}\right)\right)}{c^2(bc-ad)} - \frac{(a+bx)^{3/2}\left(4ad^2e+bc(cf-5de)\right)}{2c(c+dx)(bc-ad)} + }{2c(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x]*(e + f*x))/(x*(c + d*x)^3),x]
```

```
[Out] (((d*e - c*f)*(a + b*x)^(3/2))/(c + d*x)^2 - ((4*a*d^2*e + b*c*(-5*d*e + c*
f))*(a + b*x)^(3/2))/(2*c*(b*c - a*d)*(c + d*x)) + (2*(((-12*a*b*c*d^2*e +
8*a^2*d^3*e + b^2*c^2*(3*d*e + c*f))*(Sqrt[d]*Sqrt[a + b*x] - Sqrt[b*c - a*
d]*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]]))/(4*d^(3/2)) + 2*(b*c -
 a*d)^2*e*(-Sqrt[a + b*x] + Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])))/(c^2*
(b*c - a*d)))/(2*c*(-(b*c) + a*d))
```

fricas [B] time = 2.48, size = 2211, normalized size = 10.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] [1/8*((b^2*c^5*f + (b^2*c^3*d^2*f + (3*b^2*c^2*d^3 - 12*a*b*c*d^4 + 8*a^2*d
^5)*e)*x^2 + (3*b^2*c^4*d - 12*a*b*c^3*d^2 + 8*a^2*c^2*d^3)*e + 2*(b^2*c^4*
d*f + (3*b^2*c^3*d^2 - 12*a*b*c^2*d^3 + 8*a^2*c*d^4)*e)*x)*sqrt(-b*c*d + a*
d^2)*log((b*d*x - b*c + 2*a*d + 2*sqrt(-b*c*d + a*d^2)*sqrt(b*x + a))/(d*x
+ c)) + 8*((b^2*c^2*d^4 - 2*a*b*c*d^5 + a^2*d^6)*e*x^2 + 2*(b^2*c^3*d^3 - 2
*a*b*c^2*d^4 + a^2*c*d^5)*e*x + (b^2*c^4*d^2 - 2*a*b*c^3*d^3 + a^2*c^2*d^4)
*e)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*((5*b^2*c^4*d^
2 - 11*a*b*c^3*d^3 + 6*a^2*c^2*d^4)*e - (b^2*c^5*d - 3*a*b*c^4*d^2 + 2*a^2*
c^3*d^3)*f + ((3*b^2*c^3*d^3 - 7*a*b*c^2*d^4 + 4*a^2*c*d^5)*e + (b^2*c^4*d^
2 - a*b*c^3*d^3)*f)*x)*sqrt(b*x + a))/(b^2*c^7*d^2 - 2*a*b*c^6*d^3 + a^2*c^
5*d^4 + (b^2*c^5*d^4 - 2*a*b*c^4*d^5 + a^2*c^3*d^6)*x^2 + 2*(b^2*c^6*d^3 -
2*a*b*c^5*d^4 + a^2*c^4*d^5)*x), 1/8*(16*((b^2*c^2*d^4 - 2*a*b*c*d^5 + a^2*
d^6)*e*x^2 + 2*(b^2*c^3*d^3 - 2*a*b*c^2*d^4 + a^2*c*d^5)*e*x + (b^2*c^4*d^2
 - 2*a*b*c^3*d^3 + a^2*c^2*d^4)*e)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a
) + (b^2*c^5*f + (b^2*c^3*d^2*f + (3*b^2*c^2*d^3 - 12*a*b*c*d^4 + 8*a^2*d^5
)*e)*x^2 + (3*b^2*c^4*d - 12*a*b*c^3*d^2 + 8*a^2*c^2*d^3)*e + 2*(b^2*c^4*d*
f + (3*b^2*c^3*d^2 - 12*a*b*c^2*d^3 + 8*a^2*c*d^4)*e)*x)*sqrt(-b*c*d + a*d^
2)*log((b*d*x - b*c + 2*a*d + 2*sqrt(-b*c*d + a*d^2)*sqrt(b*x + a))/(d*x +
c)) + 2*((5*b^2*c^4*d^2 - 11*a*b*c^3*d^3 + 6*a^2*c^2*d^4)*e - (b^2*c^5*d -
3*a*b*c^4*d^2 + 2*a^2*c^3*d^3)*f + ((3*b^2*c^3*d^3 - 7*a*b*c^2*d^4 + 4*a^2*
c*d^5)*e + (b^2*c^4*d^2 - a*b*c^3*d^3)*f)*x)*sqrt(b*x + a))/(b^2*c^7*d^2 -
2*a*b*c^6*d^3 + a^2*c^5*d^4 + (b^2*c^5*d^4 - 2*a*b*c^4*d^5 + a^2*c^3*d^6)*x
^2 + 2*(b^2*c^6*d^3 - 2*a*b*c^5*d^4 + a^2*c^4*d^5)*x), -1/4*((b^2*c^5*f + (
b^2*c^3*d^2*f + (3*b^2*c^2*d^3 - 12*a*b*c*d^4 + 8*a^2*d^5)*e)*x^2 + (3*b^2*
c^4*d - 12*a*b*c^3*d^2 + 8*a^2*c^2*d^3)*e + 2*(b^2*c^4*d*f + (3*b^2*c^3*d^2
 - 12*a*b*c^2*d^3 + 8*a^2*c*d^4)*e)*x)*sqrt(b*c*d - a*d^2)*arctan(sqrt(b*c*
d - a*d^2)*sqrt(b*x + a)/(b*d*x + a*d)) - 4*((b^2*c^2*d^4 - 2*a*b*c*d^5 + a
^2*d^6)*e*x^2 + 2*(b^2*c^3*d^3 - 2*a*b*c^2*d^4 + a^2*c*d^5)*e*x + (b^2*c^4*
d^2 - 2*a*b*c^3*d^3 + a^2*c^2*d^4)*e)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sq
rt(a) + 2*a)/x) - ((5*b^2*c^4*d^2 - 11*a*b*c^3*d^3 + 6*a^2*c^2*d^4)*e - (b^
2*c^5*d - 3*a*b*c^4*d^2 + 2*a^2*c^3*d^3)*f + ((3*b^2*c^3*d^3 - 7*a*b*c^2*d^
4 + 4*a^2*c*d^5)*e + (b^2*c^4*d^2 - a*b*c^3*d^3)*f)*x)*sqrt(b*x + a))/(b^2*
c^7*d^2 - 2*a*b*c^6*d^3 + a^2*c^5*d^4 + (b^2*c^5*d^4 - 2*a*b*c^4*d^5 + a^2*
c^3*d^6)*x^2 + 2*(b^2*c^6*d^3 - 2*a*b*c^5*d^4 + a^2*c^4*d^5)*x), -1/4*((b^2
*c^5*f + (b^2*c^3*d^2*f + (3*b^2*c^2*d^3 - 12*a*b*c*d^4 + 8*a^2*d^5)*e)*x^2
 + (3*b^2*c^4*d - 12*a*b*c^3*d^2 + 8*a^2*c^2*d^3)*e + 2*(b^2*c^4*d*f + (3*b
^2*c^3*d^2 - 12*a*b*c^2*d^3 + 8*a^2*c*d^4)*e)*x)*sqrt(b*c*d - a*d^2)*arctan
(sqrt(b*c*d - a*d^2)*sqrt(b*x + a)/(b*d*x + a*d)) - 8*((b^2*c^2*d^4 - 2*a*b
*c*d^5 + a^2*d^6)*e*x^2 + 2*(b^2*c^3*d^3 - 2*a*b*c^2*d^4 + a^2*c*d^5)*e*x +
 (b^2*c^4*d^2 - 2*a*b*c^3*d^3 + a^2*c^2*d^4)*e)*sqrt(-a)*arctan(sqrt(b*x +
a)*sqrt(-a)/a) - ((5*b^2*c^4*d^2 - 11*a*b*c^3*d^3 + 6*a^2*c^2*d^4)*e - (b^2
*c^5*d - 3*a*b*c^4*d^2 + 2*a^2*c^3*d^3)*f + ((3*b^2*c^3*d^3 - 7*a*b*c^2*d^4
 + 4*a^2*c*d^5)*e + (b^2*c^4*d^2 - a*b*c^3*d^3)*f)*x)*sqrt(b*x + a))/(b^2*c
^7*d^2 - 2*a*b*c^6*d^3 + a^2*c^5*d^4 + (b^2*c^5*d^4 - 2*a*b*c^4*d^5 + a^2*c
^3*d^6)*x^2 + 2*(b^2*c^6*d^3 - 2*a*b*c^5*d^4 + a^2*c^4*d^5)*x)]
```

giac [A] time = 1.40, size = 301, normalized size = 1.47

$$\frac{\left(b^2c^3f+3\,b^2c^2de-12\,abcd^2e+8\,a^2d^3e\right)\arctan\left(\frac{\sqrt{bx+a}\,d}{\sqrt{bcd-ad^2}}\right)}{4\left(bc^4d-ac^3d^2\right)\sqrt{bcd-ad^2}}+\frac{2\,a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)e}{\sqrt{-a}\,c^3}-\frac{\sqrt{bx+a}\,b^3c^3f-(bx+a)^{\frac{3}{2}}\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/4*(b^2*c^3*f + 3*b^2*c^2*d*e - 12*a*b*c*d^2*e + 8*a^2*d^3*e)*arctan(sqrt(
```

```
b*x + a)*d/sqrt(b*c*d - a*d^2))/((b*c^4*d - a*c^3*d^2)*sqrt(b*c*d - a*d^2))
 + 2*a*arctan(sqrt(b*x + a)/sqrt(-a))*e/(sqrt(-a)*c^3) - 1/4*(sqrt(b*x + a)
*b^3*c^3*f - (b*x + a)^(3/2)*b^2*c^2*d*f - sqrt(b*x + a)*a*b^2*c^2*d*f - 5*
sqrt(b*x + a)*b^3*c^2*d*e - 3*(b*x + a)^(3/2)*b^2*c*d^2*e + 9*sqrt(b*x + a)
*a*b^2*c*d^2*e + 4*(b*x + a)^(3/2)*a*b*d^3*e - 4*sqrt(b*x + a)*a^2*b*d^3*e)
/((b*c^3*d - a*c^2*d^2)*(b*c + (b*x + a)*d - a*d)^2)
```

maple [A] time = 0.02, size = 221, normalized size = 1.08

$$2\left(-\frac{\sqrt{a}\, e \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{b^2c^3}-\frac{-\frac{\left(8a^2d^3e-12abc\,d^2e+c^3b^2f+3b^2c^2de\right)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}\,d}{\sqrt{(ad-bc)d}}\right)}{8(ad-bc)\sqrt{(ad-bc)d}\,d}+\frac{-\frac{\left(4a\,d^2e-b\,c^2f-3bcde\right)(bx+a)^{\frac{3}{2}}bc}{8(ad-bc)}+\frac{\left(4a\,d^2e+b\,c^2f-5bcc\right.}{8d}}{(-ad+bc+(bx+a)d)^2}}{b^2c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c)^3,x)
```

```
[Out] 2*b^2*(-a^(1/2)/b^2*e/c^3*arctanh((b*x+a)^(1/2)/a^(1/2))-1/c^3/b^2*((-1/8*b
*c*(4*a*d^2*e-b*c^2*f-3*b*c*d*e)/(a*d-b*c)*(b*x+a)^(3/2)+1/8*(4*a*d^2*e+b*c
^2*f-5*b*c*d*e)*b*c/d*(b*x+a)^(1/2))/(-a*d+b*c+(b*x+a)*d)^2-1/8*(8*a^2*d^3*
e-12*a*b*c*d^2*e+b^2*c^3*f+3*b^2*c^2*d*e)/(a*d-b*c)/d/((a*d-b*c)*d)^(1/2)*a
rctanh((b*x+a)^(1/2)/((a*d-b*c)*d)^(1/2)*d)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 4.63, size = 4839, normalized size = 23.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(a + b*x)^(1/2))/(x*(c + d*x)^3),x)
```

```
[Out] (atan((((d^3*(a*d - b*c)^3)^(1/2)*(((a + b*x)^(1/2)*(b^6*c^6*f^2 + 128*a^4*
b^2*d^6*e^2 + 9*b^6*c^4*d^2*e^2 + 6*b^6*c^5*d*e*f + 256*a^2*b^4*c^2*d^4*e^2
 - 72*a*b^5*c^3*d^3*e^2 - 320*a^3*b^3*c*d^5*e^2 + 16*a^2*b^4*c^3*d^3*e*f -
24*a*b^5*c^4*d^2*e*f))/(8*(b^2*c^6*d + a^2*c^4*d^3 - 2*a*b*c^5*d^2)) - ((d^
3*(a*d - b*c)^3)^(1/2)*((5*a*b^5*c^8*d^3*e - a*b^5*c^9*d^2*f - 9*a^2*b^4*c^
7*d^4*e + 4*a^3*b^3*c^6*d^5*e + a^2*b^4*c^8*d^3*f)/(b^2*c^8*d + a^2*c^6*d^3
 - 2*a*b*c^7*d^2) - ((d^3*(a*d - b*c)^3)^(1/2)*(a + b*x)^(1/2)*(8*a^2*d^3*e
 + b^2*c^3*f + 3*b^2*c^2*d*e - 12*a*b*c*d^2*e)*(64*b^5*c^9*d^3 - 256*a*b^4*
c^8*d^4 + 320*a^2*b^3*c^7*d^5 - 128*a^3*b^2*c^6*d^6))/(64*(b^2*c^6*d + a^2*
c^4*d^3 - 2*a*b*c^5*d^2)*(a^3*c^3*d^6 - b^3*c^6*d^3 + 3*a*b^2*c^5*d^4 - 3*a
^2*b*c^4*d^5)))*(8*a^2*d^3*e + b^2*c^3*f + 3*b^2*c^2*d*e - 12*a*b*c*d^2*e))
/(8*(a^3*c^3*d^6 - b^3*c^6*d^3 + 3*a*b^2*c^5*d^4 - 3*a^2*b*c^4*d^5)))*(8*a^
2*d^3*e + b^2*c^3*f + 3*b^2*c^2*d*e - 12*a*b*c*d^2*e)*1i)/(8*(a^3*c^3*d^6 -
 b^3*c^6*d^3 + 3*a*b^2*c^5*d^4 - 3*a^2*b*c^4*d^5)) + ((d^3*(a*d - b*c)^3)^(
1/2)*(((a + b*x)^(1/2)*(b^6*c^6*f^2 + 128*a^4*b^2*d^6*e^2 + 9*b^6*c^4*d^2*e
```

```
^2 + 6*b^6*c^5*d*e*f + 256*a^2*b^4*c^2*d^4*e^2 - 72*a*b^5*c^3*d^3*e^2 - 320
*a^3*b^3*c*d^5*e^2 + 16*a^2*b^4*c^3*d^3*e*f - 24*a*b^5*c^4*d^2*e*f))/(8*(b^
2*c^6*d + a^2*c^4*d^3 - 2*a*b*c^5*d^2)) + ((d^3*(a*d - b*c)^3)^(1/2)*((5*a*
b^5*c^8*d^3*e - a*b^5*c^9*d^2*f - 9*a^2*b^4*c^7*d^4*e + 4*a^3*b^3*c^6*d^5*e
 + a^2*b^4*c^8*d^3*f)/(b^2*c^8*d + a^2*c^6*d^3 - 2*a*b*c^7*d^2) + ((d^3*(a*
d - b*c)^3)^(1/2)*(a + b*x)^(1/2)*(8*a^2*d^3*e + b^2*c^3*f + 3*b^2*c^2*d*e
- 12*a*b*c*d^2*e)*(64*b^5*c^9*d^3 - 256*a*b^4*c^8*d^4 + 320*a^2*b^3*c^7*d^5
 - 128*a^3*b^2*c^6*d^6))/(64*(b^2*c^6*d + a^2*c^4*d^3 - 2*a*b*c^5*d^2)*(a^3
*c^3*d^6 - b^3*c^6*d^3 + 3*a*b^2*c^5*d^4 - 3*a^2*b*c^4*d^5)))*(8*a^2*d^3*e
+ b^2*c^3*f + 3*b^2*c^2*d*e - 12*a*b*c*d^2*e))/(8*(a^3*c^3*d^6 - b^3*c^6*d^
3 + 3*a*b^2*c^5*d^4 - 3*a^2*b*c^4*d^5)))*(8*a^2*d^3*e + b^2*c^3*f + 3*b^2*c
^2*d*e - 12*a*b*c*d^2*e)*1i)/(8*(a^3*c^3*d^6 - b^3*c^6*d^3 + 3*a*b^2*c^5*d^
4 - 3*a^2*b*c^4*d^5)))/(((a*b^6*c^5*e*f^2)/4 - 12*a^2*b^5*c^2*d^3*e^3 - 8*a
^4*b^3*d^5*e^3 + (9*a*b^6*c^3*d^2*e^3)/4 + 18*a^3*b^4*c*d^4*e^3 - 4*a^2*b^5
*c^3*d^2*e^2*f + 2*a^3*b^4*c^2*d^3*e^2*f + (3*a*b^6*c^4*d*e^2*f)/2)/(b^2*c^
8*d + a^2*c^6*d^3 - 2*a*b*c^7*d^2) - ((d^3*(a*d - b*c)^3)^(1/2)*(((a + b*x)
^(1/2)*(b^6*c^6*f^2 + 128*a^4*b^2*d^6*e^2 + 9*b^6*c^4*d^2*e^2 + 6*b^6*c^5*d
*e*f + 256*a^2*b^4*c^2*d^4*e^2 - 72*a*b^5*c^3*d^3*e^2 - 320*a^3*b^3*c*d^5*e
^2 + 16*a^2*b^4*c^3*d^3*e*f - 24*a*b^5*c^4*d^2*e*f))/(8*(b^2*c^6*d + a^2*c^
4*d^3 - 2*a*b*c^5*d^2)) - ((d^3*(a*d - b*c)^3)^(1/2)*((5*a*b^5*c^8*d^3*e -
a*b^5*c^9*d^2*f - 9*a^2*b^4*c^7*d^4*e + 4*a^3*b^3*c^6*d^5*e + a^2*b^4*c^8*d
^3*f)/(b^2*c^8*d + a^2*c^6*d^3 - 2*a*b*c^7*d^2) - ((d^3*(a*d - b*c)^3)^(1/2
)*(a + b*x)^(1/2)*(8*a^2*d^3*e + b^2*c^3*f + 3*b^2*c^2*d*e - 12*a*b*c*d^2*e
)*(64*b^5*c^9*d^3 - 256*a*b^4*c^8*d^4 + 320*a^2*b^3*c^7*d^5 - 128*a^3*b^2*c
^6*d^6))/(64*(b^2*c^6*d + a^2*c^4*d^3 - 2*a*b*c^5*d^2)*(a^3*c^3*d^6 - b^3*c
^6*d^3 + 3*a*b^2*c^5*d^4 - 3*a^2*b*c^4*d^5)))*(8*a^2*d^3*e + b^2*c^3*f + 3*
b^2*c^2*d*e - 12*a*b*c*d^2*e))/(8*(a^3*c^3*d^6 - b^3*c^6*d^3 + 3*a*b^2*c^5*
d^4 - 3*a^2*b*c^4*d^5)))*(8*a^2*d^3*e + b^2*c^3*f + 3*b^2*c^2*d*e - 12*a*b*
c*d^2*e))/(8*(a^3*c^3*d^6 - b^3*c^6*d^3 + 3*a*b^2*c^5*d^4 - 3*a^2*b*c^4*d^5
)) + ((d^3*(a*d - b*c)^3)^(1/2)*(((a + b*x)^(1/2)*(b^6*c^6*f^2 + 128*a^4*b^
2*d^6*e^2 + 9*b^6*c^4*d^2*e^2 + 6*b^6*c^5*d*e*f + 256*a^2*b^4*c^2*d^4*e^2 -
 72*a*b^5*c^3*d^3*e^2 - 320*a^3*b^3*c*d^5*e^2 + 16*a^2*b^4*c^3*d^3*e*f - 24
*a*b^5*c^4*d^2*e*f))/(8*(b^2*c^6*d + a^2*c^4*d^3 - 2*a*b*c^5*d^2)) + ((d^3*
(a*d - b*c)^3)^(1/2)*((5*a*b^5*c^8*d^3*e - a*b^5*c^9*d^2*f - 9*a^2*b^4*c^7*
d^4*e + 4*a^3*b^3*c^6*d^5*e + a^2*b^4*c^8*d^3*f)/(b^2*c^8*d + a^2*c^6*d^3 -
 2*a*b*c^7*d^2) + ((d^3*(a*d - b*c)^3)^(1/2)*(a + b*x)^(1/2)*(8*a^2*d^3*e +
 b^2*c^3*f + 3*b^2*c^2*d*e - 12*a*b*c*d^2*e)*(64*b^5*c^9*d^3 - 256*a*b^4*c^
8*d^4 + 320*a^2*b^3*c^7*d^5 - 128*a^3*b^2*c^6*d^6))/(64*(b^2*c^6*d + a^2*c^
4*d^3 - 2*a*b*c^5*d^2)*(a^3*c^3*d^6 - b^3*c^6*d^3 + 3*a*b^2*c^5*d^4 - 3*a^2
*b*c^4*d^5)))*(8*a^2*d^3*e + b^2*c^3*f + 3*b^2*c^2*d*e - 12*a*b*c*d^2*e))/(
8*(a^3*c^3*d^6 - b^3*c^6*d^3 + 3*a*b^2*c^5*d^4 - 3*a^2*b*c^4*d^5)))*(8*a^2*
d^3*e + b^2*c^3*f + 3*b^2*c^2*d*e - 12*a*b*c*d^2*e))/(8*(a^3*c^3*d^6 - b^3*
c^6*d^3 + 3*a*b^2*c^5*d^4 - 3*a^2*b*c^4*d^5))))*(d^3*(a*d - b*c)^3)^(1/2)*(
8*a^2*d^3*e + b^2*c^3*f + 3*b^2*c^2*d*e - 12*a*b*c*d^2*e)*1i)/(4*(a^3*c^3*d
^6 - b^3*c^6*d^3 + 3*a*b^2*c^5*d^4 - 3*a^2*b*c^4*d^5)) - (((a + b*x)^(1/2)*
(b^2*c^2*f + 4*a*b*d^2*e - 5*b^2*c*d*e))/(4*c^2*d) + ((a + b*x)^(3/2)*(b^2*
c^2*f - 4*a*b*d^2*e + 3*b^2*c*d*e))/(4*c^2*(a*d - b*c)))/(d^2*(a + b*x)^2 -
 (2*a*d^2 - 2*b*c*d)*(a + b*x) + a^2*d^2 + b^2*c^2 - 2*a*b*c*d) + (a^(1/2)*
e*atan(((a^(1/2)*e*(((a + b*x)^(1/2)*(b^6*c^6*f^2 + 128*a^4*b^2*d^6*e^2 + 9
*b^6*c^4*d^2*e^2 + 6*b^6*c^5*d*e*f + 256*a^2*b^4*c^2*d^4*e^2 - 72*a*b^5*c^3
*d^3*e^2 - 320*a^3*b^3*c*d^5*e^2 + 16*a^2*b^4*c^3*d^3*e*f - 24*a*b^5*c^4*d^
2*e*f))/(8*(b^2*c^6*d + a^2*c^4*d^3 - 2*a*b*c^5*d^2)) + (a^(1/2)*e*((5*a*b^
5*c^8*d^3*e - a*b^5*c^9*d^2*f - 9*a^2*b^4*c^7*d^4*e + 4*a^3*b^3*c^6*d^5*e +
 a^2*b^4*c^8*d^3*f)/(b^2*c^8*d + a^2*c^6*d^3 - 2*a*b*c^7*d^2) + (a^(1/2)*e*
(a + b*x)^(1/2)*(64*b^5*c^9*d^3 - 256*a*b^4*c^8*d^4 + 320*a^2*b^3*c^7*d^5 -
 128*a^3*b^2*c^6*d^6))/(8*c^3*(b^2*c^6*d + a^2*c^4*d^3 - 2*a*b*c^5*d^2))))/
c^3)*1i)/c^3 + (a^(1/2)*e*(((a + b*x)^(1/2)*(b^6*c^6*f^2 + 128*a^4*b^2*d^6*
e^2 + 9*b^6*c^4*d^2*e^2 + 6*b^6*c^5*d*e*f + 256*a^2*b^4*c^2*d^4*e^2 - 72*a*
b^5*c^3*d^3*e^2 - 320*a^3*b^3*c*d^5*e^2 + 16*a^2*b^4*c^3*d^3*e*f - 24*a*b^5
```

```
*c^4*d^2*e*f))/(8*(b^2*c^6*d + a^2*c^4*d^3 - 2*a*b*c^5*d^2)) - (a^(1/2)*e*(
(5*a*b^5*c^8*d^3*e - a*b^5*c^9*d^2*f - 9*a^2*b^4*c^7*d^4*e + 4*a^3*b^3*c^6*
d^5*e + a^2*b^4*c^8*d^3*f)/(b^2*c^8*d + a^2*c^6*d^3 - 2*a*b*c^7*d^2) - (a^(
1/2)*e*(a + b*x)^(1/2)*(64*b^5*c^9*d^3 - 256*a*b^4*c^8*d^4 + 320*a^2*b^3*c^
7*d^5 - 128*a^3*b^2*c^6*d^6))/(8*c^3*(b^2*c^6*d + a^2*c^4*d^3 - 2*a*b*c^5*d
^2))))/c^3)*1i)/c^3)/(((a*b^6*c^5*e*f^2)/4 - 12*a^2*b^5*c^2*d^3*e^3 - 8*a^4
*b^3*d^5*e^3 + (9*a*b^6*c^3*d^2*e^3)/4 + 18*a^3*b^4*c*d^4*e^3 - 4*a^2*b^5*c
^3*d^2*e^2*f + 2*a^3*b^4*c^2*d^3*e^2*f + (3*a*b^6*c^4*d*e^2*f)/2)/(b^2*c^8*
d + a^2*c^6*d^3 - 2*a*b*c^7*d^2) + (a^(1/2)*e*(((a + b*x)^(1/2)*(b^6*c^6*f^
2 + 128*a^4*b^2*d^6*e^2 + 9*b^6*c^4*d^2*e^2 + 6*b^6*c^5*d*e*f + 256*a^2*b^4
*c^2*d^4*e^2 - 72*a*b^5*c^3*d^3*e^2 - 320*a^3*b^3*c*d^5*e^2 + 16*a^2*b^4*c^
3*d^3*e*f - 24*a*b^5*c^4*d^2*e*f))/(8*(b^2*c^6*d + a^2*c^4*d^3 - 2*a*b*c^5*
d^2)) + (a^(1/2)*e*((5*a*b^5*c^8*d^3*e - a*b^5*c^9*d^2*f - 9*a^2*b^4*c^7*d^
4*e + 4*a^3*b^3*c^6*d^5*e + a^2*b^4*c^8*d^3*f)/(b^2*c^8*d + a^2*c^6*d^3 - 2
*a*b*c^7*d^2) + (a^(1/2)*e*(a + b*x)^(1/2)*(64*b^5*c^9*d^3 - 256*a*b^4*c^8*
d^4 + 320*a^2*b^3*c^7*d^5 - 128*a^3*b^2*c^6*d^6))/(8*c^3*(b^2*c^6*d + a^2*c
^4*d^3 - 2*a*b*c^5*d^2))))/c^3))/c^3 - (a^(1/2)*e*(((a + b*x)^(1/2)*(b^6*c^
6*f^2 + 128*a^4*b^2*d^6*e^2 + 9*b^6*c^4*d^2*e^2 + 6*b^6*c^5*d*e*f + 256*a^2
*b^4*c^2*d^4*e^2 - 72*a*b^5*c^3*d^3*e^2 - 320*a^3*b^3*c*d^5*e^2 + 16*a^2*b^
4*c^3*d^3*e*f - 24*a*b^5*c^4*d^2*e*f))/(8*(b^2*c^6*d + a^2*c^4*d^3 - 2*a*b*
c^5*d^2)) - (a^(1/2)*e*((5*a*b^5*c^8*d^3*e - a*b^5*c^9*d^2*f - 9*a^2*b^4*c^
7*d^4*e + 4*a^3*b^3*c^6*d^5*e + a^2*b^4*c^8*d^3*f)/(b^2*c^8*d + a^2*c^6*d^3
 - 2*a*b*c^7*d^2) - (a^(1/2)*e*(a + b*x)^(1/2)*(64*b^5*c^9*d^3 - 256*a*b^4*
c^8*d^4 + 320*a^2*b^3*c^7*d^5 - 128*a^3*b^2*c^6*d^6))/(8*c^3*(b^2*c^6*d + a
^2*c^4*d^3 - 2*a*b*c^5*d^2))))/c^3))/c^3))*2i)/c^3
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)**(1/2)/x/(d*x+c)**3,x)
```

```
[Out] Timed out
```

3.22 $\int \frac{x^3(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx$

Optimal. Leaf size=111

$$-\frac{\sqrt{1-ax}\,(ax)^{7/2}}{4a^4}-\frac{5\sqrt{1-ax}\,(ax)^{5/2}}{8a^4}-\frac{25\sqrt{1-ax}\,(ax)^{3/2}}{32a^4}-\frac{75\sqrt{1-ax}\,\sqrt{ax}}{64a^4}-\frac{75\sin^{-1}(1-2ax)}{128a^4}$$

```
[Out] 75/128*arcsin(2*a*x-1)/a^4-25/32*(a*x)^(3/2)*(-a*x+1)^(1/2)/a^4-5/8*(a*x)^(
5/2)*(-a*x+1)^(1/2)/a^4-1/4*(a*x)^(7/2)*(-a*x+1)^(1/2)/a^4-75/64*(a*x)^(1/2
)*(-a*x+1)^(1/2)/a^4
```

Rubi [A] time = 0.04, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {16, 80, 50, 53, 619, 216}

$$-\frac{\sqrt{1-ax}\,(ax)^{7/2}}{4a^4}-\frac{5\sqrt{1-ax}\,(ax)^{5/2}}{8a^4}-\frac{25\sqrt{1-ax}\,(ax)^{3/2}}{32a^4}-\frac{75\sqrt{1-ax}\,\sqrt{ax}}{64a^4}-\frac{75\sin^{-1}(1-2ax)}{128a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(1 + a*x))/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-75*Sqrt[a*x]*Sqrt[1 - a*x])/(64*a^4) - (25*(a*x)^(3/2)*Sqrt[1 - a*x])/(32
*a^4) - (5*(a*x)^(5/2)*Sqrt[1 - a*x])/(8*a^4) - ((a*x)^(7/2)*Sqrt[1 - a*x])
/(4*a^4) - (75*ArcSin[1 - 2*a*x])/(128*a^4)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \frac{x^3(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx = \frac{\int \frac{(ax)^{5/2}(1+ax)}{\sqrt{1-ax}}\,dx}{a^3}$$

$$= -\frac{(ax)^{7/2}\sqrt{1-ax}}{4a^4} + \frac{15\int \frac{(ax)^{5/2}}{\sqrt{1-ax}}\,dx}{8a^3}$$

$$= -\frac{5(ax)^{5/2}\sqrt{1-ax}}{8a^4} - \frac{(ax)^{7/2}\sqrt{1-ax}}{4a^4} + \frac{25\int \frac{(ax)^{3/2}}{\sqrt{1-ax}}\,dx}{16a^3}$$

$$= -\frac{25(ax)^{3/2}\sqrt{1-ax}}{32a^4} - \frac{5(ax)^{5/2}\sqrt{1-ax}}{8a^4} - \frac{(ax)^{7/2}\sqrt{1-ax}}{4a^4} + \frac{75\int \frac{\sqrt{ax}}{\sqrt{1-ax}}\,dx}{64a^3}$$

$$= -\frac{75\sqrt{ax}\sqrt{1-ax}}{64a^4} - \frac{25(ax)^{3/2}\sqrt{1-ax}}{32a^4} - \frac{5(ax)^{5/2}\sqrt{1-ax}}{8a^4} - \frac{(ax)^{7/2}\sqrt{1-ax}}{4a^4} + \frac{75\int \frac{1}{\sqrt{ax}\sqrt{1}}}{128a^3}$$

$$= -\frac{75\sqrt{ax}\sqrt{1-ax}}{64a^4} - \frac{25(ax)^{3/2}\sqrt{1-ax}}{32a^4} - \frac{5(ax)^{5/2}\sqrt{1-ax}}{8a^4} - \frac{(ax)^{7/2}\sqrt{1-ax}}{4a^4} + \frac{75\int \frac{1}{\sqrt{ax-a^2}}}{128a^3}$$

$$= -\frac{75\sqrt{ax}\sqrt{1-ax}}{64a^4} - \frac{25(ax)^{3/2}\sqrt{1-ax}}{32a^4} - \frac{5(ax)^{5/2}\sqrt{1-ax}}{8a^4} - \frac{(ax)^{7/2}\sqrt{1-ax}}{4a^4} - \frac{75\,\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{75\sqrt{ax}\sqrt{1-ax}}{64a^4} - \frac{25(ax)^{3/2}\sqrt{1-ax}}{32a^4} - \frac{5(ax)^{5/2}\sqrt{1-ax}}{8a^4} - \frac{(ax)^{7/2}\sqrt{1-ax}}{4a^4} - \frac{75\sin^{-1}(1-}{128a^4}$$

Mathematica [A] time = 0.09, size = 89, normalized size = 0.80

$$\frac{\sqrt{a}\,x\left(16a^4x^4+24a^3x^3+10a^2x^2+25ax-75\right)+75\sqrt{x}\sqrt{1-ax}\,\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{64a^{7/2}\sqrt{-ax(ax-1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(1 + a*x))/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (Sqrt[a]*x*(-75 + 25*a*x + 10*a^2*x^2 + 24*a^3*x^3 + 16*a^4*x^4) + 75*Sqrt[
x]*Sqrt[1 - a*x]*ArcSin[Sqrt[a]*Sqrt[x]])/(64*a^(7/2)*Sqrt[-(a*x*(-1 + a*x)
)])
```

fricas [A] time = 0.88, size = 65, normalized size = 0.59

$$-\frac{\left(16\,a^3x^3+40\,a^2x^2+50\,ax+75\right)\sqrt{ax}\,\sqrt{-ax+1}\,+75\,\arctan\left(\frac{\sqrt{ax}\,\sqrt{-ax+1}}{ax}\right)}{64\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/64*((16*a^3*x^3 + 40*a^2*x^2 + 50*a*x + 75)*sqrt(a*x)*sqrt(-a*x + 1) + 7
5*arctan(sqrt(a*x)*sqrt(-a*x + 1)/(a*x)))/a^4
```

giac [A] time = 1.25, size = 63, normalized size = 0.57

$$-\frac{\left(2\left(4\,ax\left(\frac{2\,x}{a^2}+\frac{5}{a^3}\right)+\frac{25}{a^3}\right)ax+\frac{75}{a^3}\right)\sqrt{ax}\,\sqrt{-ax+1}-\frac{75\,\arcsin\left(\sqrt{ax}\right)}{a^3}}{64\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/64*((2*(4*a*x*(2*x/a^2 + 5/a^3) + 25/a^3)*a*x + 75/a^3)*sqrt(a*x)*sqrt(-
a*x + 1) - 75*arcsin(sqrt(a*x))/a^3)/a
```

maple [C] time = 0.04, size = 132, normalized size = 1.19

$$-\frac{\sqrt{-ax+1}\,\left(32\sqrt{-(ax-1)\,ax}\;a^3x^3\mathrm{csgn}(a)+80\sqrt{-(ax-1)\,ax}\;a^2x^2\mathrm{csgn}(a)+100\sqrt{-(ax-1)\,ax}\;ax\,\mathrm{csgn}(a)}{128\sqrt{ax}\;\sqrt{-(ax-1)\,ax}\;a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x)
```

```
[Out] -1/128*(-a*x+1)^(1/2)*x*(32*csgn(a)*x^3*a^3*(-x*(a*x-1)*a)^(1/2)+80*csgn(a)
*x^2*a^2*(-x*(a*x-1)*a)^(1/2)+100*csgn(a)*(-x*(a*x-1)*a)^(1/2)*x*a+150*csgn
(a)*(-x*(a*x-1)*a)^(1/2)-75*arctan(1/2*csgn(a)*(2*a*x-1)/(-x*(a*x-1)*a)^(1/
2)))*csgn(a)/a^3/(a*x)^(1/2)/(-x*(a*x-1)*a)^(1/2)
```

maxima [A] time = 0.97, size = 105, normalized size = 0.95

$$-\frac{\sqrt{-a^2x^2+ax}\,x^3}{4\,a}-\frac{5\,\sqrt{-a^2x^2+ax}\,x^2}{8\,a^2}-\frac{25\,\sqrt{-a^2x^2+ax}\,x}{32\,a^3}-\frac{75\,\arcsin\left(-\frac{2\,a^2x-a}{a}\right)}{128\,a^4}-\frac{75\,\sqrt{-a^2x^2+ax}}{64\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(-a^2*x^2 + a*x)*x^3/a - 5/8*sqrt(-a^2*x^2 + a*x)*x^2/a^2 - 25/32*
sqrt(-a^2*x^2 + a*x)*x/a^3 - 75/128*arcsin(-(2*a^2*x - a)/a)/a^4 - 75/64*sq
rt(-a^2*x^2 + a*x)/a^4
```

mupad [B] time = 7.78, size = 345, normalized size = 3.11

$$\frac{75\,\mathrm{atan}\left(\frac{\sqrt{a\,x}}{\sqrt{1-a\,x}-1}\right)}{32\,a^4}-\frac{\frac{5\,\sqrt{a\,x}}{4\left(\sqrt{1-a\,x}-1\right)}+\frac{85\,(a\,x)^{3/2}}{12\left(\sqrt{1-a\,x}-1\right)^3}+\frac{33\,(a\,x)^{5/2}}{2\left(\sqrt{1-a\,x}-1\right)^5}-\frac{33\,(a\,x)^{7/2}}{2\left(\sqrt{1-a\,x}-1\right)^7}-\frac{85\,(a\,x)^{9/2}}{12\left(\sqrt{1-a\,x}-1\right)^9}-\frac{5\,(a\,x)^{11/2}}{4\left(\sqrt{1-a\,x}-1\right)^{11}}}{a^4\left(\frac{a\,x}{\left(\sqrt{1-a\,x}-1\right)^2}+1\right)^6}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a*x + 1))/((a*x)^(1/2)*(1 - a*x)^(1/2)),x)
```

```
[Out] (75*atan((a*x)^(1/2)/((1 - a*x)^(1/2) - 1)))/(32*a^4) - ((5*(a*x)^(1/2))/(4
*((1 - a*x)^(1/2) - 1)) + (85*(a*x)^(3/2))/(12*((1 - a*x)^(1/2) - 1)^3) + (
33*(a*x)^(5/2))/(2*((1 - a*x)^(1/2) - 1)^5) - (33*(a*x)^(7/2))/(2*((1 - a*x
)^(1/2) - 1)^7) - (85*(a*x)^(9/2))/(12*((1 - a*x)^(1/2) - 1)^9) - (5*(a*x)^
(11/2))/(4*((1 - a*x)^(1/2) - 1)^11))/(a^4*((a*x)/((1 - a*x)^(1/2) - 1)^2 +
 1)^6) - ((35*(a*x)^(1/2))/(32*((1 - a*x)^(1/2) - 1)) + (805*(a*x)^(3/2))/(
96*((1 - a*x)^(1/2) - 1)^3) + (2681*(a*x)^(5/2))/(96*((1 - a*x)^(1/2) - 1)^
5) + (5053*(a*x)^(7/2))/(96*((1 - a*x)^(1/2) - 1)^7) - (5053*(a*x)^(9/2))/(
96*((1 - a*x)^(1/2) - 1)^9) - (2681*(a*x)^(11/2))/(96*((1 - a*x)^(1/2) - 1)
```

```
^11) - (805*(a*x)^(13/2))/(96*((1 - a*x)^(1/2) - 1)^13) - (35*(a*x)^(15/2))
/(32*((1 - a*x)^(1/2) - 1)^15))/(a^4*((a*x)/((1 - a*x)^(1/2) - 1)^2 + 1)^8)
```

sympy [C] time = 35.80, size = 484, normalized size = 4.36

$$a\left(\begin{cases} -\frac{35i\,\text{acosh}\left(\sqrt{a}\,\sqrt{x}\right)}{64a^5} - \frac{ix^{\frac{9}{2}}}{4\sqrt{a}\,\sqrt{ax-1}} - \frac{ix^{\frac{7}{2}}}{24a^{\frac{3}{2}}\sqrt{ax-1}} - \frac{7ix^{\frac{5}{2}}}{96a^{\frac{5}{2}}\sqrt{ax-1}} - \frac{35ix^{\frac{3}{2}}}{192a^{\frac{7}{2}}\sqrt{ax-1}} + \frac{35i\sqrt{x}}{64a^{\frac{9}{2}}\sqrt{ax-1}} & \text{for } |ax| > 1 \\ \frac{35\,\text{asin}\left(\sqrt{a}\,\sqrt{x}\right)}{64a^5} + \frac{x^{\frac{9}{2}}}{4\sqrt{a}\,\sqrt{-ax+1}} + \frac{x^{\frac{7}{2}}}{24a^{\frac{3}{2}}\sqrt{-ax+1}} + \frac{7x^{\frac{5}{2}}}{96a^{\frac{5}{2}}\sqrt{-ax+1}} + \frac{35x^{\frac{3}{2}}}{192a^{\frac{7}{2}}\sqrt{-ax+1}} - \frac{35\sqrt{x}}{64a^{\frac{9}{2}}\sqrt{-ax+1}} & \text{otherwise} \end{cases}\right) + \begin{cases} -\frac{5i\,\text{aco}\ldots}{} \\ \frac{5\,\text{asin}\left(\ldots\right.}{8a\ldots} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a*x+1)/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

```
[Out] a*Piecewise((-35*I*acosh(sqrt(a)*sqrt(x))/(64*a**5) - I*x**(9/2)/(4*sqrt(a)
*sqrt(a*x - 1)) - I*x**(7/2)/(24*a**(3/2)*sqrt(a*x - 1)) - 7*I*x**(5/2)/(96
*a**(5/2)*sqrt(a*x - 1)) - 35*I*x**(3/2)/(192*a**(7/2)*sqrt(a*x - 1)) + 35*
I*sqrt(x)/(64*a**(9/2)*sqrt(a*x - 1)), Abs(a*x) > 1), (35*asin(sqrt(a)*sqrt
(x))/(64*a**5) + x**(9/2)/(4*sqrt(a)*sqrt(-a*x + 1)) + x**(7/2)/(24*a**(3/2
)*sqrt(-a*x + 1)) + 7*x**(5/2)/(96*a**(5/2)*sqrt(-a*x + 1)) + 35*x**(3/2)/(
192*a**(7/2)*sqrt(-a*x + 1)) - 35*sqrt(x)/(64*a**(9/2)*sqrt(-a*x + 1)), Tru
e)) + Piecewise((-5*I*acosh(sqrt(a)*sqrt(x))/(8*a**4) - I*x**(7/2)/(3*sqrt(
a)*sqrt(a*x - 1)) - I*x**(5/2)/(12*a**(3/2)*sqrt(a*x - 1)) - 5*I*x**(3/2)/(
24*a**(5/2)*sqrt(a*x - 1)) + 5*I*sqrt(x)/(8*a**(7/2)*sqrt(a*x - 1)), Abs(a*
x) > 1), (5*asin(sqrt(a)*sqrt(x))/(8*a**4) + x**(7/2)/(3*sqrt(a)*sqrt(-a*x
+ 1)) + x**(5/2)/(12*a**(3/2)*sqrt(-a*x + 1)) + 5*x**(3/2)/(24*a**(5/2)*sqr
t(-a*x + 1)) - 5*sqrt(x)/(8*a**(7/2)*sqrt(-a*x + 1)), True))
```

3.23 $\int \frac{x^2(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx$

Optimal. Leaf size=87

$$-\frac{\sqrt{1-ax}\,(ax)^{5/2}}{3a^3}-\frac{11\sqrt{1-ax}\,(ax)^{3/2}}{12a^3}-\frac{11\sqrt{1-ax}\,\sqrt{ax}}{8a^3}-\frac{11\sin^{-1}(1-2ax)}{16a^3}$$

```
[Out] 11/16*arcsin(2*a*x-1)/a^3-11/12*(a*x)^(3/2)*(-a*x+1)^(1/2)/a^3-1/3*(a*x)^(5
/2)*(-a*x+1)^(1/2)/a^3-11/8*(a*x)^(1/2)*(-a*x+1)^(1/2)/a^3
```

Rubi [A] time = 0.03, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {16, 80, 50, 53, 619, 216}

$$-\frac{\sqrt{1-ax}\,(ax)^{5/2}}{3a^3}-\frac{11\sqrt{1-ax}\,(ax)^{3/2}}{12a^3}-\frac{11\sqrt{1-ax}\,\sqrt{ax}}{8a^3}-\frac{11\sin^{-1}(1-2ax)}{16a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 + a*x))/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-11*Sqrt[a*x]*Sqrt[1 - a*x])/(8*a^3) - (11*(a*x)^(3/2)*Sqrt[1 - a*x])/(12*
a^3) - ((a*x)^(5/2)*Sqrt[1 - a*x])/(3*a^3) - (11*ArcSin[1 - 2*a*x])/(16*a^3
)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \frac{x^2(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx = \frac{\int \frac{(ax)^{3/2}(1+ax)}{\sqrt{1-ax}}\,dx}{a^2}$$
$$= -\frac{(ax)^{5/2}\sqrt{1-ax}}{3a^3} + \frac{11\int \frac{(ax)^{3/2}}{\sqrt{1-ax}}\,dx}{6a^2}$$
$$= -\frac{11(ax)^{3/2}\sqrt{1-ax}}{12a^3} - \frac{(ax)^{5/2}\sqrt{1-ax}}{3a^3} + \frac{11\int \frac{\sqrt{ax}}{\sqrt{1-ax}}\,dx}{8a^2}$$
$$= -\frac{11\sqrt{ax}\sqrt{1-ax}}{8a^3} - \frac{11(ax)^{3/2}\sqrt{1-ax}}{12a^3} - \frac{(ax)^{5/2}\sqrt{1-ax}}{3a^3} + \frac{11\int \frac{1}{\sqrt{ax}\sqrt{1-ax}}\,dx}{16a^2}$$
$$= -\frac{11\sqrt{ax}\sqrt{1-ax}}{8a^3} - \frac{11(ax)^{3/2}\sqrt{1-ax}}{12a^3} - \frac{(ax)^{5/2}\sqrt{1-ax}}{3a^3} + \frac{11\int \frac{1}{\sqrt{ax-a^2x^2}}\,dx}{16a^2}$$
$$= -\frac{11\sqrt{ax}\sqrt{1-ax}}{8a^3} - \frac{11(ax)^{3/2}\sqrt{1-ax}}{12a^3} - \frac{(ax)^{5/2}\sqrt{1-ax}}{3a^3} - \frac{11\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, a-2a}{16a^4}$$
$$= -\frac{11\sqrt{ax}\sqrt{1-ax}}{8a^3} - \frac{11(ax)^{3/2}\sqrt{1-ax}}{12a^3} - \frac{(ax)^{5/2}\sqrt{1-ax}}{3a^3} - \frac{11\sin^{-1}(1-2ax)}{16a^3}$$

Mathematica [A] time = 0.03, size = 81, normalized size = 0.93

$$\frac{\sqrt{a}\,x\left(8a^3x^3+14a^2x^2+11ax-33\right)+33\sqrt{x}\sqrt{1-ax}\,\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{24a^{5/2}\sqrt{-ax(ax-1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(1 + a*x))/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (Sqrt[a]*x*(-33 + 11*a*x + 14*a^2*x^2 + 8*a^3*x^3) + 33*Sqrt[x]*Sqrt[1 - a*
x]*ArcSin[Sqrt[a]*Sqrt[x]])/(24*a^(5/2)*Sqrt[-(a*x*(-1 + a*x))])
```

fricas [A] time = 1.22, size = 57, normalized size = 0.66

$$-\frac{\left(8\,a^2x^2+22\,ax+33\right)\sqrt{ax}\,\sqrt{-ax+1}\,+33\,\arctan\left(\frac{\sqrt{ax}\,\sqrt{-ax+1}}{ax}\right)}{24\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*((8*a^2*x^2 + 22*a*x + 33)*sqrt(a*x)*sqrt(-a*x + 1) + 33*arctan(sqrt(
a*x)*sqrt(-a*x + 1)/(a*x)))/a^3
```

giac [A] time = 1.34, size = 53, normalized size = 0.61

$$-\frac{\left(2\,ax\left(\frac{4\,x}{a}+\frac{11}{a^2}\right)+\frac{33}{a^2}\right)\sqrt{ax}\,\sqrt{-ax+1}\,-\frac{33\,\arcsin\left(\sqrt{ax}\right)}{a^2}}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*((2*a*x*(4*x/a + 11/a^2) + 33/a^2)*sqrt(a*x)*sqrt(-a*x + 1) - 33*arcs
in(sqrt(a*x))/a^2)/a
```

maple [C] time = 0.02, size = 111, normalized size = 1.28

$$-\frac{\sqrt{-ax+1}\left(16\sqrt{-(ax-1)ax}\,a^2x^2\mathrm{csgn}(a)+44\sqrt{-(ax-1)ax}\,ax\,\mathrm{csgn}(a)-33\arctan\left(\frac{(2ax-1)\mathrm{csgn}(a)}{2\sqrt{-(ax-1)ax}}\right)+66\sqrt{-}\right.}{48\sqrt{ax}\,\sqrt{-(ax-1)ax}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x)
```

```
[Out] -1/48*(-a*x+1)^(1/2)*x*(16*(-(a*x-1)*a*x)^(1/2)*a^2*x^2*csgn(a)+44*(-(a*x-1
)*a*x)^(1/2)*a*x*csgn(a)+66*(-(a*x-1)*a*x)^(1/2)*csgn(a)-33*arctan(1/2*(2*a
*x-1)/(-(a*x-1)*a*x)^(1/2)*csgn(a)))*csgn(a)/a^2/(a*x)^(1/2)/(-(a*x-1)*a*x)
^(1/2)
```

maxima [A] time = 0.99, size = 83, normalized size = 0.95

$$-\frac{\sqrt{-a^2x^2+ax}\,x^2}{3\,a}-\frac{11\,\sqrt{-a^2x^2+ax}\,x}{12\,a^2}-\frac{11\,\arcsin\left(-\frac{2\,a^2x-a}{a}\right)}{16\,a^3}-\frac{11\,\sqrt{-a^2x^2+ax}}{8\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(-a^2*x^2 + a*x)*x^2/a - 11/12*sqrt(-a^2*x^2 + a*x)*x/a^2 - 11/16*
arcsin(-(2*a^2*x - a)/a)/a^3 - 11/8*sqrt(-a^2*x^2 + a*x)/a^3
```

mupad [B] time = 5.92, size = 269, normalized size = 3.09

$$\frac{11\,\mathrm{atan}\left(\frac{\sqrt{a\,x}}{\sqrt{1-a\,x}-1}\right)}{4\,a^3}-\frac{\frac{5\,\sqrt{a\,x}}{4\left(\sqrt{1-a\,x}-1\right)}+\frac{85\,(a\,x)^{3/2}}{12\left(\sqrt{1-a\,x}-1\right)^3}+\frac{33\,(a\,x)^{5/2}}{2\left(\sqrt{1-a\,x}-1\right)^5}-\frac{33\,(a\,x)^{7/2}}{2\left(\sqrt{1-a\,x}-1\right)^7}-\frac{85\,(a\,x)^{9/2}}{12\left(\sqrt{1-a\,x}-1\right)^9}-\frac{5\,(a\,x)^{11/2}}{4\left(\sqrt{1-a\,x}-1\right)^{11}}}{a^3\left(\frac{a\,x}{\left(\sqrt{1-a\,x}-1\right)^2}+1\right)^6}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*x + 1))/((a*x)^(1/2)*(1 - a*x)^(1/2)),x)
```

```
[Out] (11*atan((a*x)^(1/2)/((1 - a*x)^(1/2) - 1)))/(4*a^3) - ((5*(a*x)^(1/2))/(4*
((1 - a*x)^(1/2) - 1)) + (85*(a*x)^(3/2))/(12*((1 - a*x)^(1/2) - 1)^3) + (3
3*(a*x)^(5/2))/(2*((1 - a*x)^(1/2) - 1)^5) - (33*(a*x)^(7/2))/(2*((1 - a*x)
^(1/2) - 1)^7) - (85*(a*x)^(9/2))/(12*((1 - a*x)^(1/2) - 1)^9) - (5*(a*x)^(
11/2))/(4*((1 - a*x)^(1/2) - 1)^11))/(a^3*((a*x)/((1 - a*x)^(1/2) - 1)^2 +
1)^6) - ((3*(a*x)^(1/2))/(2*((1 - a*x)^(1/2) - 1)) + (11*(a*x)^(3/2))/(2*((
1 - a*x)^(1/2) - 1)^3) - (11*(a*x)^(5/2))/(2*((1 - a*x)^(1/2) - 1)^5) - (3*
(a*x)^(7/2))/(2*((1 - a*x)^(1/2) - 1)^7))/(a^3*((a*x)/((1 - a*x)^(1/2) - 1)
^2 + 1)^4)
```

sympy [C] time = 25.60, size = 393, normalized size = 4.52

$$a\left(\begin{cases}-\frac{5i\,\mathrm{acosh}\left(\sqrt{a}\sqrt{x}\right)}{8a^4}-\frac{ix^{\frac{7}{2}}}{3\sqrt{a}\sqrt{ax-1}}-\frac{ix^{\frac{5}{2}}}{12a^{\frac{3}{2}}\sqrt{ax-1}}-\frac{5ix^{\frac{3}{2}}}{24a^{\frac{5}{2}}\sqrt{ax-1}}+\frac{5i\sqrt{x}}{8a^{\frac{7}{2}}\sqrt{ax-1}} & \text{for } |ax|>1\\ \frac{5\,\mathrm{asin}\left(\sqrt{a}\sqrt{x}\right)}{8a^4}+\frac{x^{\frac{7}{2}}}{3\sqrt{a}\sqrt{-ax+1}}+\frac{x^{\frac{5}{2}}}{12a^{\frac{3}{2}}\sqrt{-ax+1}}+\frac{5x^{\frac{3}{2}}}{24a^{\frac{5}{2}}\sqrt{-ax+1}}-\frac{5\sqrt{x}}{8a^{\frac{7}{2}}\sqrt{-ax+1}} & \text{otherwise}\end{cases}\right)+\begin{cases}-\frac{3i\,\mathrm{acosh}\left(\sqrt{a}\sqrt{x}\right)}{4a^3}-\frac{}{2\sqrt{}} \\ \frac{3\,\mathrm{asin}\left(\sqrt{a}\sqrt{x}\right)}{4a^3}+\frac{x}{2\sqrt{a}\sqrt{}}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a*x+1)/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

```
[Out] a*Piecewise((-5*I*acosh(sqrt(a)*sqrt(x))/(8*a**4) - I*x**(7/2)/(3*sqrt(a)*s
qrt(a*x - 1)) - I*x**(5/2)/(12*a**(3/2)*sqrt(a*x - 1)) - 5*I*x**(3/2)/(24*a
**(5/2)*sqrt(a*x - 1)) + 5*I*sqrt(x)/(8*a**(7/2)*sqrt(a*x - 1)), Abs(a*x) >
 1), (5*asin(sqrt(a)*sqrt(x))/(8*a**4) + x**(7/2)/(3*sqrt(a)*sqrt(-a*x + 1)
) + x**(5/2)/(12*a**(3/2)*sqrt(-a*x + 1)) + 5*x**(3/2)/(24*a**(5/2)*sqrt(-a
*x + 1)) - 5*sqrt(x)/(8*a**(7/2)*sqrt(-a*x + 1)), True)) + Piecewise((-3*I*
acosh(sqrt(a)*sqrt(x))/(4*a**3) - I*x**(5/2)/(2*sqrt(a)*sqrt(a*x - 1)) - I*
x**(3/2)/(4*a**(3/2)*sqrt(a*x - 1)) + 3*I*sqrt(x)/(4*a**(5/2)*sqrt(a*x - 1)
), Abs(a*x) > 1), (3*asin(sqrt(a)*sqrt(x))/(4*a**3) + x**(5/2)/(2*sqrt(a)*s
qrt(-a*x + 1)) + x**(3/2)/(4*a**(3/2)*sqrt(-a*x + 1)) - 3*sqrt(x)/(4*a**(5/
2)*sqrt(-a*x + 1)), True))
```

3.24 $\int \frac{x(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx$

Optimal. Leaf size=63

$$-\frac{\sqrt{1-ax}\,(ax)^{3/2}}{2a^2}-\frac{7\sqrt{1-ax}\,\sqrt{ax}}{4a^2}-\frac{7\sin^{-1}(1-2ax)}{8a^2}$$

```
[Out] 7/8*arcsin(2*a*x-1)/a^2-1/2*(a*x)^(3/2)*(-a*x+1)^(1/2)/a^2-7/4*(a*x)^(1/2)*
(-a*x+1)^(1/2)/a^2
```

Rubi [A] time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {16, 80, 50, 53, 619, 216}

$$-\frac{\sqrt{1-ax}\,(ax)^{3/2}}{2a^2}-\frac{7\sqrt{1-ax}\,\sqrt{ax}}{4a^2}-\frac{7\sin^{-1}(1-2ax)}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 + a*x))/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-7*Sqrt[a*x]*Sqrt[1 - a*x])/(4*a^2) - ((a*x)^(3/2)*Sqrt[1 - a*x])/(2*a^2)
- (7*ArcSin[1 - 2*a*x])/(8*a^2)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \frac{x(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx = \frac{\int \frac{\sqrt{ax}(1+ax)}{\sqrt{1-ax}}\,dx}{a}$$

$$= -\frac{(ax)^{3/2}\sqrt{1-ax}}{2a^2} + \frac{7\int \frac{\sqrt{ax}}{\sqrt{1-ax}}\,dx}{4a}$$

$$= -\frac{7\sqrt{ax}\sqrt{1-ax}}{4a^2} - \frac{(ax)^{3/2}\sqrt{1-ax}}{2a^2} + \frac{7\int \frac{1}{\sqrt{ax}\sqrt{1-ax}}\,dx}{8a}$$

$$= -\frac{7\sqrt{ax}\sqrt{1-ax}}{4a^2} - \frac{(ax)^{3/2}\sqrt{1-ax}}{2a^2} + \frac{7\int \frac{1}{\sqrt{ax-a^2x^2}}\,dx}{8a}$$

$$= -\frac{7\sqrt{ax}\sqrt{1-ax}}{4a^2} - \frac{(ax)^{3/2}\sqrt{1-ax}}{2a^2} - \frac{7\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, a-2a^2x\right)}{8a^3}$$

$$= -\frac{7\sqrt{ax}\sqrt{1-ax}}{4a^2} - \frac{(ax)^{3/2}\sqrt{1-ax}}{2a^2} - \frac{7\sin^{-1}(1-2ax)}{8a^2}$$

Mathematica [A] time = 0.03, size = 73, normalized size = 1.16

$$\frac{\sqrt{a}\,x\left(2a^2x^2+5ax-7\right)+7\sqrt{x}\sqrt{1-ax}\,\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}\sqrt{-ax(ax-1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 + a*x))/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (Sqrt[a]*x*(-7 + 5*a*x + 2*a^2*x^2) + 7*Sqrt[x]*Sqrt[1 - a*x]*ArcSin[Sqrt[a
]*Sqrt[x]])/(4*a^(3/2)*Sqrt[-(a*x*(-1 + a*x))])
```

fricas [A] time = 0.64, size = 49, normalized size = 0.78

$$-\frac{(2\,ax+7)\sqrt{ax}\sqrt{-ax+1}+7\,\arctan\left(\frac{\sqrt{ax}\sqrt{-ax+1}}{ax}\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*((2*a*x + 7)*sqrt(a*x)*sqrt(-a*x + 1) + 7*arctan(sqrt(a*x)*sqrt(-a*x +
 1)/(a*x)))/a^2
```

giac [A] time = 1.28, size = 40, normalized size = 0.63

$$-\frac{\sqrt{ax}\sqrt{-ax+1}\left(2\,x+\frac{7}{a}\right)-\frac{7\,\arcsin\left(\sqrt{ax}\right)}{a}}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(sqrt(a*x)*sqrt(-a*x + 1)*(2*x + 7/a) - 7*arcsin(sqrt(a*x))/a)/a
```

maple [C] time = 0.02, size = 90, normalized size = 1.43

$$-\frac{\sqrt{-ax+1}\,\left(4\sqrt{-(ax-1)\,ax}\;ax\,\mathrm{csgn}(a)-7\arctan\left(\frac{(2ax-1)\mathrm{csgn}(a)}{2\sqrt{-(ax-1)ax}}\right)+14\sqrt{-(ax-1)\,ax}\;\mathrm{csgn}(a)\right)x\,\mathrm{csgn}(a)}{8\sqrt{ax}\;\sqrt{-(ax-1)\,ax}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x)
```

```
[Out] -1/8*(-a*x+1)^(1/2)*x/a*(4*(-(a*x-1)*a*x)^(1/2)*a*x*csgn(a)+14*(-(a*x-1)*a*
x)^(1/2)*csgn(a)-7*arctan(1/2*(2*a*x-1)/(-(a*x-1)*a*x)^(1/2)*csgn(a)))*csgn
(a)/(a*x)^(1/2)/(-(a*x-1)*a*x)^(1/2)
```

maxima [A] time = 0.96, size = 61, normalized size = 0.97

$$-\frac{\sqrt{-a^2x^2+ax}\,x}{2\,a}-\frac{7\,\arcsin\left(-\frac{2\,a^2x-a}{a}\right)}{8\,a^2}-\frac{7\,\sqrt{-a^2x^2+ax}}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-a^2*x^2 + a*x)*x/a - 7/8*arcsin(-(2*a^2*x - a)/a)/a^2 - 7/4*sqrt
(-a^2*x^2 + a*x)/a^2
```

mupad [B] time = 4.53, size = 191, normalized size = 3.03

$$\frac{7\,\mathrm{atan}\left(\frac{\sqrt{a\,x}}{\sqrt{1-a\,x}-1}\right)}{2\,a^2}-\frac{\frac{2\,\sqrt{a\,x}}{\sqrt{1-a\,x}-1}-\frac{2\,{(a\,x)}^{3/2}}{{\left(\sqrt{1-a\,x}-1\right)}^3}}{a^2\,{\left(\frac{a\,x}{{\left(\sqrt{1-a\,x}-1\right)}^2}+1\right)}^2}-\frac{\frac{3\,\sqrt{a\,x}}{2\,\left(\sqrt{1-a\,x}-1\right)}+\frac{11\,{(a\,x)}^{3/2}}{2\,{\left(\sqrt{1-a\,x}-1\right)}^3}-\frac{11\,{(a\,x)}^{5/2}}{2\,{\left(\sqrt{1-a\,x}-1\right)}^5}-\frac{3\,{(a\,x)}^{7/2}}{2\,{\left(\sqrt{1-a\,x}-1\right)}^7}}{a^2\,{\left(\frac{a\,x}{{\left(\sqrt{1-a\,x}-1\right)}^2}+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*x + 1))/((a*x)^(1/2)*(1 - a*x)^(1/2)),x)
```

```
[Out] (7*atan((a*x)^(1/2)/((1 - a*x)^(1/2) - 1)))/(2*a^2) - ((2*(a*x)^(1/2))/((1
- a*x)^(1/2) - 1) - (2*(a*x)^(3/2))/((1 - a*x)^(1/2) - 1)^3)/(a^2*((a*x)/((
1 - a*x)^(1/2) - 1)^2 + 1)^2) - ((3*(a*x)^(1/2))/(2*((1 - a*x)^(1/2) - 1))
+ (11*(a*x)^(3/2))/(2*((1 - a*x)^(1/2) - 1)^3) - (11*(a*x)^(5/2))/(2*((1 -
a*x)^(1/2) - 1)^5) - (3*(a*x)^(7/2))/(2*((1 - a*x)^(1/2) - 1)^7))/(a^2*((a*
x)/((1 - a*x)^(1/2) - 1)^2 + 1)^4)
```

sympy [C] time = 20.77, size = 269, normalized size = 4.27

$$a\left(\begin{cases}-\frac{3i\,\mathrm{acosh}\left(\sqrt{a}\,\sqrt{x}\right)}{4a^3}-\frac{ix^{\frac{5}{2}}}{2\sqrt{a}\,\sqrt{ax-1}}-\frac{ix^{\frac{3}{2}}}{4a^{\frac{3}{2}}\sqrt{ax-1}}+\frac{3i\sqrt{x}}{4a^{\frac{5}{2}}\sqrt{ax-1}} & \text{for } |ax|>1\\ \frac{3\,\mathrm{asin}\left(\sqrt{a}\,\sqrt{x}\right)}{4a^3}+\frac{x^{\frac{5}{2}}}{2\sqrt{a}\,\sqrt{-ax+1}}+\frac{x^{\frac{3}{2}}}{4a^{\frac{3}{2}}\sqrt{-ax+1}}-\frac{3\sqrt{x}}{4a^{\frac{5}{2}}\sqrt{-ax+1}} & \text{otherwise}\end{cases}\right)+\begin{cases}-\frac{i\,\mathrm{acosh}\left(\sqrt{a}\,\sqrt{x}\right)}{a^2}-\frac{i\sqrt{x}\,\sqrt{ax-1}}{a^{\frac{3}{2}}}\\ \frac{\mathrm{asin}\left(\sqrt{a}\,\sqrt{x}\right)}{a^2}+\frac{x^{\frac{3}{2}}}{\sqrt{a}\,\sqrt{-ax+1}}-\frac{\sqrt{x}}{a^{\frac{3}{2}}\sqrt{-ax+1}}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*x+1)/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

```
[Out] a*Piecewise((-3*I*acosh(sqrt(a)*sqrt(x))/(4*a**3) - I*x**(5/2)/(2*sqrt(a)*s
qrt(a*x - 1)) - I*x**(3/2)/(4*a**(3/2)*sqrt(a*x - 1)) + 3*I*sqrt(x)/(4*a**(
5/2)*sqrt(a*x - 1)), Abs(a*x) > 1), (3*asin(sqrt(a)*sqrt(x))/(4*a**3) + x**
(5/2)/(2*sqrt(a)*sqrt(-a*x + 1)) + x**(3/2)/(4*a**(3/2)*sqrt(-a*x + 1)) - 3
*sqrt(x)/(4*a**(5/2)*sqrt(-a*x + 1)), True)) + Piecewise((-I*acosh(sqrt(a)*
sqrt(x))/a**2 - I*sqrt(x)*sqrt(a*x - 1)/a**(3/2), Abs(a*x) > 1), (asin(sqrt
(a)*sqrt(x))/a**2 + x**(3/2)/(sqrt(a)*sqrt(-a*x + 1)) - sqrt(x)/(a**(3/2)*s
qrt(-a*x + 1)), True))
```

3.25 $\int \frac{1+ax}{\sqrt{ax}\sqrt{1-ax}}\,dx$

Optimal. Leaf size=37

$$-\frac{\sqrt{ax}\sqrt{1-ax}}{a}-\frac{3\sin^{-1}(1-2ax)}{2a}$$

```
[Out] 3/2*arcsin(2*a*x-1)/a-(a*x)^(1/2)*(-a*x+1)^(1/2)/a
```

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {80, 53, 619, 216}

$$-\frac{\sqrt{ax}\sqrt{1-ax}}{a}-\frac{3\sin^{-1}(1-2ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a*x)/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] -((Sqrt[a*x]*Sqrt[1 - a*x])/a) - (3*ArcSin[1 - 2*a*x])/(2*a)
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+ax}{\sqrt{ax}\sqrt{1-ax}}\,dx &= -\frac{\sqrt{ax}\sqrt{1-ax}}{a}+\frac{3}{2}\int \frac{1}{\sqrt{ax}\sqrt{1-ax}}\,dx \\
&= -\frac{\sqrt{ax}\sqrt{1-ax}}{a}+\frac{3}{2}\int \frac{1}{\sqrt{ax-a^2x^2}}\,dx \\
&= -\frac{\sqrt{ax}\sqrt{1-ax}}{a}-\frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,a-2a^2x\right)}{2a^2} \\
&= -\frac{\sqrt{ax}\sqrt{1-ax}}{a}-\frac{3\sin^{-1}(1-2ax)}{2a}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 61, normalized size = 1.65

$$
\frac{\sqrt{a}\,x(ax-1)+3\sqrt{x}\sqrt{1-ax}\,\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{\sqrt{a}\sqrt{-ax(ax-1)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a*x)/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (Sqrt[a]*x*(-1 + a*x) + 3*Sqrt[x]*Sqrt[1 - a*x]*ArcSin[Sqrt[a]*Sqrt[x]])/(S
qrt[a]*Sqrt[-(a*x*(-1 + a*x))])
```

fricas [A] time = 0.95, size = 43, normalized size = 1.16

$$
-\frac{\sqrt{ax}\sqrt{-ax+1}+3\arctan\left(\frac{\sqrt{ax}\sqrt{-ax+1}}{ax}\right)}{a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(a*x)*sqrt(-a*x + 1) + 3*arctan(sqrt(a*x)*sqrt(-a*x + 1)/(a*x)))/a
```

giac [A] time = 1.24, size = 28, normalized size = 0.76

$$
-\frac{\sqrt{ax}\sqrt{-ax+1}-3\arcsin\left(\sqrt{ax}\right)}{a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -(sqrt(a*x)*sqrt(-a*x + 1) - 3*arcsin(sqrt(a*x)))/a
```

maple [C] time = 0.02, size = 70, normalized size = 1.89

$$
-\frac{\sqrt{-ax+1}\left(-3\arctan\left(\frac{(2ax-1)\,\text{csgn}(a)}{2\sqrt{-(ax-1)ax}}\right)+2\sqrt{-(ax-1)\,ax}\;\text{csgn}(a)\right)x\,\text{csgn}(a)}{2\sqrt{ax}\sqrt{-(ax-1)\,ax}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x)
```

```
[Out] -1/2*(-a*x+1)^(1/2)*x*(2*(-(a*x-1)*a*x)^(1/2)*csgn(a)-3*arctan(1/2*(2*a*x-1
)/(-(a*x-1)*a*x)^(1/2)*csgn(a)))*csgn(a)/(a*x)^(1/2)/(-(a*x-1)*a*x)^(1/2)
```

maxima [A] time = 0.95, size = 41, normalized size = 1.11

$$-\frac{3\arcsin\left(-\frac{2a^2x-a}{a}\right)}{2a}-\frac{\sqrt{-a^2x^2+ax}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/2*arcsin(-(2*a^2*x - a)/a)/a - sqrt(-a^2*x^2 + a*x)/a
```

mupad [B] time = 3.45, size = 118, normalized size = 3.19

$$\frac{2\,\mathrm{atan}\left(\frac{\sqrt{a\,x}}{\sqrt{1-a\,x}-1}\right)}{a}-\frac{4\,\mathrm{atan}\left(\frac{a\left(\sqrt{1-a\,x}-1\right)}{\sqrt{a\,x}\,\sqrt{a^2}}\right)}{\sqrt{a^2}}-\frac{\frac{2\,\sqrt{a\,x}}{\sqrt{1-a\,x}-1}-\frac{2\,(a\,x)^{3/2}}{\left(\sqrt{1-a\,x}-1\right)^3}}{a\left(\frac{a\,x}{\left(\sqrt{1-a\,x}-1\right)^2}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((a*x)^(1/2)*(1 - a*x)^(1/2)),x)
```

```
[Out] (2*atan((a*x)^(1/2)/((1 - a*x)^(1/2) - 1)))/a - (4*atan((a*((1 - a*x)^(1/2)
 - 1))/((a*x)^(1/2)*(a^2)^(1/2))))/(a^2)^(1/2) - ((2*(a*x)^(1/2))/((1 - a*x
)^(1/2) - 1) - (2*(a*x)^(3/2))/((1 - a*x)^(1/2) - 1)^3)/(a*((a*x)/((1 - a*x
)^(1/2) - 1)^2 + 1)^2)
```

sympy [C] time = 11.71, size = 133, normalized size = 3.59

$$a\left(\begin{cases}-\frac{i\,\mathrm{acosh}\left(\sqrt{a}\sqrt{x}\right)}{a^2}-\frac{i\sqrt{x}\sqrt{ax-1}}{a^{\frac{3}{2}}} & \text{for}\ |ax|>1\\ \frac{\mathrm{asin}\left(\sqrt{a}\sqrt{x}\right)}{a^2}+\frac{x^{\frac{3}{2}}}{\sqrt{a}\sqrt{-ax+1}}-\frac{\sqrt{x}}{a^{\frac{3}{2}}\sqrt{-ax+1}} & \text{otherwise}\end{cases}\right)+\begin{cases}-\frac{2i\,\mathrm{acosh}\left(\sqrt{a}\sqrt{x}\right)}{a} & \text{for}\ |ax|>1\\ \frac{2\,\mathrm{asin}\left(\sqrt{a}\sqrt{x}\right)}{a} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

```
[Out] a*Piecewise((-I*acosh(sqrt(a)*sqrt(x))/a**2 - I*sqrt(x)*sqrt(a*x - 1)/a**(3
/2), Abs(a*x) > 1), (asin(sqrt(a)*sqrt(x))/a**2 + x**(3/2)/(sqrt(a)*sqrt(-a
*x + 1)) - sqrt(x)/(a**(3/2)*sqrt(-a*x + 1)), True)) + Piecewise((-2*I*acos
h(sqrt(a)*sqrt(x))/a, Abs(a*x) > 1), (2*asin(sqrt(a)*sqrt(x))/a, True))
```

3.26 $\int \frac{1+ax}{x\sqrt{ax}\sqrt{1-ax}}\,dx$

Optimal. Leaf size=29

$$-\frac{2\sqrt{1-ax}}{\sqrt{ax}}-\sin^{-1}(1-2ax)$$

```
[Out] arcsin(2*a*x-1)-2*(-a*x+1)^(1/2)/(a*x)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {16, 78, 53, 619, 216}

$$-\frac{2\sqrt{1-ax}}{\sqrt{ax}}-\sin^{-1}(1-2ax)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a*x)/(x*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-2*Sqrt[1 - a*x])/Sqrt[a*x] - ArcSin[1 - 2*a*x]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+ax}{x\sqrt{ax}\sqrt{1-ax}}\,dx &= a\int \frac{1+ax}{(ax)^{3/2}\sqrt{1-ax}}\,dx \\
&= -\frac{2\sqrt{1-ax}}{\sqrt{ax}} + a\int \frac{1}{\sqrt{ax}\sqrt{1-ax}}\,dx \\
&= -\frac{2\sqrt{1-ax}}{\sqrt{ax}} + a\int \frac{1}{\sqrt{ax-a^2x^2}}\,dx \\
&= -\frac{2\sqrt{1-ax}}{\sqrt{ax}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, a-2a^2x\right)}{a} \\
&= -\frac{2\sqrt{1-ax}}{\sqrt{ax}} - \sin^{-1}(1-2ax)
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 53, normalized size = 1.83

$$\frac{2\left(ax+\sqrt{a}\sqrt{x}\sqrt{1-ax}\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)-1\right)}{\sqrt{-ax(ax-1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a*x)/(x*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (2*(-1 + a*x + Sqrt[a]*Sqrt[x]*Sqrt[1 - a*x]*ArcSin[Sqrt[a]*Sqrt[x]]))/Sqrt
[-(a*x*(-1 + a*x))]
```

fricas [B] time = 0.98, size = 47, normalized size = 1.62

$$-\frac{2\left(ax\arctan\left(\frac{\sqrt{ax}\sqrt{-ax+1}}{ax}\right)+\sqrt{ax}\sqrt{-ax+1}\right)}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(a*x*arctan(sqrt(a*x)*sqrt(-a*x + 1)/(a*x)) + sqrt(a*x)*sqrt(-a*x + 1))/
(a*x)
```

giac [A] time = 1.23, size = 44, normalized size = 1.52

$$-\frac{\sqrt{-ax+1}-1}{\sqrt{ax}}+\frac{\sqrt{ax}}{\sqrt{-ax+1}-1}+2\arcsin\left(\sqrt{ax}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -(sqrt(-a*x + 1) - 1)/sqrt(a*x) + sqrt(a*x)/(sqrt(-a*x + 1) - 1) + 2*arcsin
(sqrt(a*x))
```

maple [C] time = 0.02, size = 69, normalized size = 2.38

$$\frac{\left(ax\arctan\left(\frac{(2ax-1)\operatorname{csgn}(a)}{2\sqrt{-(ax-1)ax}}\right)-2\sqrt{-(ax-1)ax}\operatorname{csgn}(a)\right)\sqrt{-ax+1}\operatorname{csgn}(a)}{\sqrt{ax}\sqrt{-(ax-1)ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/x/(a*x)^(1/2)/(-a*x+1)^(1/2),x)
```

```
[Out] (arctan(1/2*(2*a*x-1)/(-(a*x-1)*a*x)^(1/2)*csgn(a))*x*a-2*(-(a*x-1)*a*x)^(1
/2)*csgn(a))*(-a*x+1)^(1/2)*csgn(a)/(a*x)^(1/2)/(-(a*x-1)*a*x)^(1/2)
```

maxima [A] time = 0.95, size = 41, normalized size = 1.41

$$-\frac{2\sqrt{-a^2x^2+ax}}{ax}-\arcsin\left(-\frac{2a^2x-a}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(-a^2*x^2 + a*x)/(a*x) - arcsin(-(2*a^2*x - a)/a)
```

mupad [B] time = 2.98, size = 47, normalized size = 1.62

$$-\frac{2\sqrt{1-a\,x}}{\sqrt{a\,x}}-\frac{4\,a\,\mathrm{atan}\left(\frac{a\left(\sqrt{1-a\,x}-1\right)}{\sqrt{a\,x}\,\sqrt{a^2}}\right)}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x*(a*x)^(1/2)*(1 - a*x)^(1/2)),x)
```

```
[Out] - (2*(1 - a*x)^(1/2))/(a*x)^(1/2) - (4*a*atan((a*((1 - a*x)^(1/2) - 1))/((a
*x)^(1/2)*(a^2)^(1/2))))/(a^2)^(1/2)
```

sympy [C] time = 25.62, size = 71, normalized size = 2.45

$$a\left(\begin{cases}-\frac{2i\operatorname{acosh}\left(\sqrt{a}\sqrt{x}\right)}{a} & \text{for } |ax|>1\\ \frac{2\operatorname{asin}\left(\sqrt{a}\sqrt{x}\right)}{a} & \text{otherwise}\end{cases}\right)+\begin{cases}-2\sqrt{-1+\frac{1}{ax}} & \text{for } \frac{1}{|ax|}>1\\ -2i\sqrt{1-\frac{1}{ax}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

```
[Out] a*Piecewise((-2*I*acosh(sqrt(a)*sqrt(x))/a, Abs(a*x) > 1), (2*asin(sqrt(a)*
sqrt(x))/a, True)) + Piecewise((-2*sqrt(-1 + 1/(a*x)), 1/Abs(a*x) > 1), (-2
*I*sqrt(1 - 1/(a*x)), True))
```

3.27 $\int \frac{1+ax}{x^2\sqrt{ax}\sqrt{1-ax}}\,dx$

Optimal. Leaf size=45

$$-\frac{10a\sqrt{1-ax}}{3\sqrt{ax}}-\frac{2a\sqrt{1-ax}}{3(ax)^{3/2}}$$

```
[Out] -2/3*a*(-a*x+1)^(1/2)/(a*x)^(3/2)-10/3*a*(-a*x+1)^(1/2)/(a*x)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = $\{16, 78, 37\}$

$$-\frac{10a\sqrt{1-ax}}{3\sqrt{ax}}-\frac{2a\sqrt{1-ax}}{3(ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a*x)/(x^2*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-2*a*Sqrt[1 - a*x])/(3*(a*x)^(3/2)) - (10*a*Sqrt[1 - a*x])/(3*Sqrt[a*x])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{1+ax}{x^2\sqrt{ax}\sqrt{1-ax}}\,dx &= a^2\int \frac{1+ax}{(ax)^{5/2}\sqrt{1-ax}}\,dx\\
&= -\frac{2a\sqrt{1-ax}}{3(ax)^{3/2}}+\frac{1}{3}\left(5a^2\right)\int \frac{1}{(ax)^{3/2}\sqrt{1-ax}}\,dx\\
&= -\frac{2a\sqrt{1-ax}}{3(ax)^{3/2}}-\frac{10a\sqrt{1-ax}}{3\sqrt{ax}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.64

$$-\frac{2\sqrt{-ax(ax-1)}\,(5ax+1)}{3ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a*x)/(x^2*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-2*Sqrt[-(a*x*(-1 + a*x))]*(1 + 5*a*x))/(3*a*x^2)
```

fricas [A] time = 0.92, size = 27, normalized size = 0.60

$$-\frac{2\,(5\,ax+1)\sqrt{ax}\,\sqrt{-ax+1}}{3\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x^2/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(5*a*x + 1)*sqrt(a*x)*sqrt(-a*x + 1)/(a*x^2)
```

giac [B] time = 1.27, size = 88, normalized size = 1.96

$$-\frac{\frac{a^2\left(\sqrt{-ax+1}-1\right)^3}{(ax)^{\frac{3}{2}}}+\frac{21\,a^2\left(\sqrt{-ax+1}-1\right)}{\sqrt{ax}}-\frac{\left(a^2+\frac{21\,a\left(\sqrt{-ax+1}-1\right)^2}{x}\right)(ax)^{\frac{3}{2}}}{\left(\sqrt{-ax+1}-1\right)^3}}{12\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x^2/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/12*(a^2*(sqrt(-a*x + 1) - 1)^3/(a*x)^(3/2) + 21*a^2*(sqrt(-a*x + 1) - 1)
/sqrt(a*x) - (a^2 + 21*a*(sqrt(-a*x + 1) - 1)^2/x)*(a*x)^(3/2)/(sqrt(-a*x +
 1) - 1)^3)/a
```

maple [A] time = 0.00, size = 25, normalized size = 0.56

$$-\frac{2\,(5ax+1)\,\sqrt{-ax+1}}{3\sqrt{ax}\;x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/x^2/(a*x)^(1/2)/(-a*x+1)^(1/2),x)
```

```
[Out] -2/3*(5*a*x+1)/x/(a*x)^(1/2)*(-a*x+1)^(1/2)
```

maxima [A] time = 0.96, size = 42, normalized size = 0.93

$$-\frac{10\,\sqrt{-a^2x^2+ax}}{3\,x}-\frac{2\,\sqrt{-a^2x^2+ax}}{3\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x^2/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -10/3*sqrt(-a^2*x^2 + a*x)/x - 2/3*sqrt(-a^2*x^2 + a*x)/(a*x^2)
```

mupad [B] time = 2.75, size = 24, normalized size = 0.53

$$-\frac{\sqrt{1-a\,x}\,\left(\frac{10\,a\,x}{3}+\frac{2}{3}\right)}{x\,\sqrt{a\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^2*(a*x)^(1/2)*(1 - a*x)^(1/2)),x)
```

```
[Out] -((1 - a*x)^(1/2)*((10*a*x)/3 + 2/3))/(x*(a*x)^(1/2))
```

sympy [C] time = 15.24, size = 107, normalized size = 2.38

$$a\left(\begin{cases} -2\sqrt{-1+\frac{1}{ax}} & \text{for}\ \frac{1}{|ax|}>1 \\ -2i\sqrt{1-\frac{1}{ax}} & \text{otherwise} \end{cases}\right)+\begin{cases} -\frac{4a\sqrt{-1+\frac{1}{ax}}}{3}-\frac{2\sqrt{-1+\frac{1}{ax}}}{3x} & \text{for}\ \frac{1}{|ax|}>1 \\ -\frac{4ia\sqrt{1-\frac{1}{ax}}}{3}-\frac{2i\sqrt{1-\frac{1}{ax}}}{3x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x**2/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

```
[Out] a*Piecewise((-2*sqrt(-1 + 1/(a*x)), 1/Abs(a*x) > 1), (-2*I*sqrt(1 - 1/(a*x)
), True)) + Piecewise((-4*a*sqrt(-1 + 1/(a*x))/3 - 2*sqrt(-1 + 1/(a*x))/(3*
x), 1/Abs(a*x) > 1), (-4*I*a*sqrt(1 - 1/(a*x))/3 - 2*I*sqrt(1 - 1/(a*x))/(3
*x), True))
```

3.28 $\int \frac{1+ax}{x^3\sqrt{ax}\sqrt{1-ax}}\,dx$

Optimal. Leaf size=73

$$-\frac{12a^2\sqrt{1-ax}}{5\sqrt{ax}}-\frac{6a^2\sqrt{1-ax}}{5(ax)^{3/2}}-\frac{2a^2\sqrt{1-ax}}{5(ax)^{5/2}}$$

```
[Out] -2/5*a^2*(-a*x+1)^(1/2)/(a*x)^(5/2)-6/5*a^2*(-a*x+1)^(1/2)/(a*x)^(3/2)-12/5
*a^2*(-a*x+1)^(1/2)/(a*x)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {16, 78, 45, 37}

$$-\frac{12a^2\sqrt{1-ax}}{5\sqrt{ax}}-\frac{6a^2\sqrt{1-ax}}{5(ax)^{3/2}}-\frac{2a^2\sqrt{1-ax}}{5(ax)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a*x)/(x^3*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-2*a^2*Sqrt[1 - a*x])/(5*(a*x)^(5/2)) - (6*a^2*Sqrt[1 - a*x])/(5*(a*x)^(3/
2)) - (12*a^2*Sqrt[1 - a*x])/(5*Sqrt[a*x])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rubi steps

$$\begin{aligned}\int \frac{1+ax}{x^3\sqrt{ax}\sqrt{1-ax}}\,dx &= a^3\int \frac{1+ax}{(ax)^{7/2}\sqrt{1-ax}}\,dx\\ &= -\frac{2a^2\sqrt{1-ax}}{5(ax)^{5/2}}+\frac{1}{5}\left(9a^3\right)\int \frac{1}{(ax)^{5/2}\sqrt{1-ax}}\,dx\\ &= -\frac{2a^2\sqrt{1-ax}}{5(ax)^{5/2}}-\frac{6a^2\sqrt{1-ax}}{5(ax)^{3/2}}+\frac{1}{5}\left(6a^3\right)\int \frac{1}{(ax)^{3/2}\sqrt{1-ax}}\,dx\\ &= -\frac{2a^2\sqrt{1-ax}}{5(ax)^{5/2}}-\frac{6a^2\sqrt{1-ax}}{5(ax)^{3/2}}-\frac{12a^2\sqrt{1-ax}}{5\sqrt{ax}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 0.51

$$-\frac{2\sqrt{-ax(ax-1)}\left(6a^2x^2+3ax+1\right)}{5ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a*x)/(x^3*Sqrt[a*x]*Sqrt[1 - a*x]),x]
[Out] (-2*Sqrt[-(a*x*(-1 + a*x))]*(1 + 3*a*x + 6*a^2*x^2))/(5*a*x^3)
```

fricas [A] time = 0.62, size = 35, normalized size = 0.48

$$-\frac{2\left(6\,a^2x^2+3\,ax+1\right)\sqrt{ax}\,\sqrt{-ax+1}}{5\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x^3/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
[Out] -2/5*(6*a^2*x^2 + 3*a*x + 1)*sqrt(a*x)*sqrt(-a*x + 1)/(a*x^3)
```

giac [B] time = 1.24, size = 130, normalized size = 1.78

$$-\frac{\frac{a^3\left(\sqrt{-ax+1}-1\right)^5}{(ax)^{\frac{5}{2}}}+\frac{15\,a^3\left(\sqrt{-ax+1}-1\right)^3}{(ax)^{\frac{3}{2}}}+\frac{110\,a^3\left(\sqrt{-ax+1}-1\right)}{\sqrt{ax}}-\frac{\left(a^3+\frac{15\,a^2\left(\sqrt{-ax+1}-1\right)^2}{x}+\frac{110\,a\left(\sqrt{-ax+1}-1\right)^4}{x^2}\right)(ax)^{\frac{5}{2}}}{\left(\sqrt{-ax+1}-1\right)^5}}{80\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x^3/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
[Out] -1/80*(a^3*(sqrt(-a*x + 1) - 1)^5/(a*x)^(5/2) + 15*a^3*(sqrt(-a*x + 1) - 1)
^3/(a*x)^(3/2) + 110*a^3*(sqrt(-a*x + 1) - 1)/sqrt(a*x) - (a^3 + 15*a^2*(sq
rt(-a*x + 1) - 1)^2/x + 110*a*(sqrt(-a*x + 1) - 1)^4/x^2)*(a*x)^(5/2)/(sqrt
(-a*x + 1) - 1)^5)/a
```

maple [A] time = 0.00, size = 33, normalized size = 0.45

$$-\frac{2\left(6a^2x^2+3ax+1\right)\sqrt{-ax+1}}{5\sqrt{ax}\;x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/x^3/(a*x)^(1/2)/(-a*x+1)^(1/2),x)
```

```
[Out] -2/5*(6*a^2*x^2+3*a*x+1)/x^2/(a*x)^(1/2)*(-a*x+1)^(1/2)
```

maxima [A] time = 0.96, size = 62, normalized size = 0.85

$$-\frac{12\sqrt{-a^2x^2+ax}\,a}{5\,x}-\frac{6\sqrt{-a^2x^2+ax}}{5\,x^2}-\frac{2\sqrt{-a^2x^2+ax}}{5\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x^3/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -12/5*sqrt(-a^2*x^2 + a*x)*a/x - 6/5*sqrt(-a^2*x^2 + a*x)/x^2 - 2/5*sqrt(-a
^2*x^2 + a*x)/(a*x^3)
```

mupad [B] time = 2.73, size = 32, normalized size = 0.44

$$-\frac{\sqrt{1-a\,x}\,\left(\frac{12\,a^2\,x^2}{5}+\frac{6\,a\,x}{5}+\frac{2}{5}\right)}{x^2\,\sqrt{a\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^3*(a*x)^(1/2)*(1 - a*x)^(1/2)),x)
```

```
[Out] -((1 - a*x)^(1/2)*((6*a*x)/5 + (12*a^2*x^2)/5 + 2/5))/(x^2*(a*x)^(1/2))
```

sympy [C] time = 17.84, size = 189, normalized size = 2.59

$$a\left(\begin{cases}-\frac{4a\sqrt{-1+\frac{1}{ax}}}{3}-\frac{2\sqrt{-1+\frac{1}{ax}}}{3x} & \text{for}\ \frac{1}{|ax|}>1\\ -\frac{4ia\sqrt{1-\frac{1}{ax}}}{3}-\frac{2i\sqrt{1-\frac{1}{ax}}}{3x} & \text{otherwise}\end{cases}\right)+\begin{cases}-\frac{16a^2\sqrt{-1+\frac{1}{ax}}}{15}-\frac{8a\sqrt{-1+\frac{1}{ax}}}{15x}-\frac{2\sqrt{-1+\frac{1}{ax}}}{5x^2} & \text{for}\ \frac{1}{|ax|}>1\\ -\frac{16ia^2\sqrt{1-\frac{1}{ax}}}{15}-\frac{8ia\sqrt{1-\frac{1}{ax}}}{15x}-\frac{2i\sqrt{1-\frac{1}{ax}}}{5x^2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x**3/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

```
[Out] a*Piecewise((-4*a*sqrt(-1 + 1/(a*x))/3 - 2*sqrt(-1 + 1/(a*x))/(3*x), 1/Abs(
a*x) > 1), (-4*I*a*sqrt(1 - 1/(a*x))/3 - 2*I*sqrt(1 - 1/(a*x))/(3*x), True)
) + Piecewise((-16*a**2*sqrt(-1 + 1/(a*x))/15 - 8*a*sqrt(-1 + 1/(a*x))/(15*
x) - 2*sqrt(-1 + 1/(a*x))/(5*x**2), 1/Abs(a*x) > 1), (-16*I*a**2*sqrt(1 - 1
/(a*x))/15 - 8*I*a*sqrt(1 - 1/(a*x))/(15*x) - 2*I*sqrt(1 - 1/(a*x))/(5*x**2
), True))
```

3.29 $\int \frac{1+ax}{x^4\sqrt{ax}\sqrt{1-ax}}\,dx$

Optimal. Leaf size=97

$$-\frac{208a^3\sqrt{1-ax}}{105\sqrt{ax}}-\frac{104a^3\sqrt{1-ax}}{105(ax)^{3/2}}-\frac{26a^3\sqrt{1-ax}}{35(ax)^{5/2}}-\frac{2a^3\sqrt{1-ax}}{7(ax)^{7/2}}$$

```
[Out] -2/7*a^3*(-a*x+1)^(1/2)/(a*x)^(7/2)-26/35*a^3*(-a*x+1)^(1/2)/(a*x)^(5/2)-10
4/105*a^3*(-a*x+1)^(1/2)/(a*x)^(3/2)-208/105*a^3*(-a*x+1)^(1/2)/(a*x)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {16, 78, 45, 37}

$$-\frac{208a^3\sqrt{1-ax}}{105\sqrt{ax}}-\frac{104a^3\sqrt{1-ax}}{105(ax)^{3/2}}-\frac{26a^3\sqrt{1-ax}}{35(ax)^{5/2}}-\frac{2a^3\sqrt{1-ax}}{7(ax)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a*x)/(x^4*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-2*a^3*Sqrt[1 - a*x])/(7*(a*x)^(7/2)) - (26*a^3*Sqrt[1 - a*x])/(35*(a*x)^(
5/2)) - (104*a^3*Sqrt[1 - a*x])/(105*(a*x)^(3/2)) - (208*a^3*Sqrt[1 - a*x])
/(105*Sqrt[a*x])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+ax}{x^4\sqrt{ax}\sqrt{1-ax}}\,dx &= a^4\int \frac{1+ax}{(ax)^{9/2}\sqrt{1-ax}}\,dx \\
&= -\frac{2a^3\sqrt{1-ax}}{7(ax)^{7/2}}+\frac{1}{7}\left(13a^4\right)\int \frac{1}{(ax)^{7/2}\sqrt{1-ax}}\,dx \\
&= -\frac{2a^3\sqrt{1-ax}}{7(ax)^{7/2}}-\frac{26a^3\sqrt{1-ax}}{35(ax)^{5/2}}+\frac{1}{35}\left(52a^4\right)\int \frac{1}{(ax)^{5/2}\sqrt{1-ax}}\,dx \\
&= -\frac{2a^3\sqrt{1-ax}}{7(ax)^{7/2}}-\frac{26a^3\sqrt{1-ax}}{35(ax)^{5/2}}-\frac{104a^3\sqrt{1-ax}}{105(ax)^{3/2}}+\frac{1}{105}\left(104a^4\right)\int \frac{1}{(ax)^{3/2}\sqrt{1-ax}}\,dx \\
&= -\frac{2a^3\sqrt{1-ax}}{7(ax)^{7/2}}-\frac{26a^3\sqrt{1-ax}}{35(ax)^{5/2}}-\frac{104a^3\sqrt{1-ax}}{105(ax)^{3/2}}-\frac{208a^3\sqrt{1-ax}}{105\sqrt{ax}}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 45, normalized size = 0.46

$$-\frac{2\sqrt{-ax(ax-1)}\left(104a^3x^3+52a^2x^2+39ax+15\right)}{105ax^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a*x)/(x^4*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-2*Sqrt[-(a*x*(-1 + a*x))]*(15 + 39*a*x + 52*a^2*x^2 + 104*a^3*x^3))/(105*
a*x^4)
```

fricas [A] time = 0.71, size = 43, normalized size = 0.44

$$-\frac{2\left(104\,a^3x^3+52\,a^2x^2+39\,ax+15\right)\sqrt{ax}\,\sqrt{-ax+1}}{105\,ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x^4/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/105*(104*a^3*x^3 + 52*a^2*x^2 + 39*a*x + 15)*sqrt(a*x)*sqrt(-a*x + 1)/(a
*x^4)
```

giac [B] time = 1.41, size = 175, normalized size = 1.80

$$-\frac{\frac{15\,a^4\left(\sqrt{-ax+1}-1\right)^7}{(ax)^{\frac{7}{2}}}+\frac{231\,a^4\left(\sqrt{-ax+1}-1\right)^5}{(ax)^{\frac{5}{2}}}+\frac{1435\,a^4\left(\sqrt{-ax+1}-1\right)^3}{(ax)^{\frac{3}{2}}}+\frac{7875\,a^4\left(\sqrt{-ax+1}-1\right)}{\sqrt{ax}}-\frac{\left(15\,a^4+\frac{231\,a^3\left(\sqrt{-ax+1}-1\right)^2}{x}+\frac{1435\,a^2\left(\sqrt{-ax+1}-1\right)}{x^2}\right.}{\left(\sqrt{-ax+1}-1\right)^7}}{6720\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x^4/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6720*(15*a^4*(sqrt(-a*x + 1) - 1)^7/(a*x)^(7/2) + 231*a^4*(sqrt(-a*x + 1
) - 1)^5/(a*x)^(5/2) + 1435*a^4*(sqrt(-a*x + 1) - 1)^3/(a*x)^(3/2) + 7875*a
^4*(sqrt(-a*x + 1) - 1)/sqrt(a*x) - (15*a^4 + 231*a^3*(sqrt(-a*x + 1) - 1)^
2/x + 1435*a^2*(sqrt(-a*x + 1) - 1)^4/x^2 + 7875*a*(sqrt(-a*x + 1) - 1)^6/x
^3)*(a*x)^(7/2)/(sqrt(-a*x + 1) - 1)^7)/a
```

maple [A] time = 0.01, size = 41, normalized size = 0.42

$$-\frac{2\left(104a^3x^3+52a^2x^2+39ax+15\right)\sqrt{-ax+1}}{105\sqrt{ax}\;x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/x^4/(a*x)^(1/2)/(-a*x+1)^(1/2),x)
```

```
[Out] -2/105*(104*a^3*x^3+52*a^2*x^2+39*a*x+15)/x^3/(a*x)^(1/2)*(-a*x+1)^(1/2)
```

maxima [A] time = 0.97, size = 84, normalized size = 0.87

$$-\frac{208\sqrt{-a^2x^2+ax}\,a^2}{105\,x}-\frac{104\sqrt{-a^2x^2+ax}\,a}{105\,x^2}-\frac{26\sqrt{-a^2x^2+ax}}{35\,x^3}-\frac{2\sqrt{-a^2x^2+ax}}{7\,ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x^4/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -208/105*sqrt(-a^2*x^2 + a*x)*a^2/x - 104/105*sqrt(-a^2*x^2 + a*x)*a/x^2 -
26/35*sqrt(-a^2*x^2 + a*x)/x^3 - 2/7*sqrt(-a^2*x^2 + a*x)/(a*x^4)
```

mupad [B] time = 2.77, size = 40, normalized size = 0.41

$$-\frac{\sqrt{1-a\,x}\,\left(\frac{208\,a^3\,x^3}{105}+\frac{104\,a^2\,x^2}{105}+\frac{26\,a\,x}{35}+\frac{2}{7}\right)}{x^3\,\sqrt{a\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^4*(a*x)^(1/2)*(1 - a*x)^(1/2)),x)
```

```
[Out] -((1 - a*x)^(1/2)*((26*a*x)/35 + (104*a^2*x^2)/105 + (208*a^3*x^3)/105 + 2/
7))/(x^3*(a*x)^(1/2))
```

sympy [C] time = 23.15, size = 274, normalized size = 2.82

$$a\left(\begin{cases}-\frac{16a^2\sqrt{-1+\frac{1}{ax}}}{15}-\frac{8a\sqrt{-1+\frac{1}{ax}}}{15x}-\frac{2\sqrt{-1+\frac{1}{ax}}}{5x^2} & \text{for}\ \frac{1}{|ax|}>1\\ -\frac{16ia^2\sqrt{1-\frac{1}{ax}}}{15}-\frac{8ia\sqrt{1-\frac{1}{ax}}}{15x}-\frac{2i\sqrt{1-\frac{1}{ax}}}{5x^2} & \text{otherwise}\end{cases}\right)+\begin{cases}-\frac{32a^3\sqrt{-1+\frac{1}{ax}}}{35}-\frac{16a^2\sqrt{-1+\frac{1}{ax}}}{35x}-\frac{12a\sqrt{-1+\frac{1}{ax}}}{35x^2}-\frac{2\sqrt{-1+\frac{1}{ax}}}{7x^3}\\ -\frac{32ia^3\sqrt{1-\frac{1}{ax}}}{35}-\frac{16ia^2\sqrt{1-\frac{1}{ax}}}{35x}-\frac{12ia\sqrt{1-\frac{1}{ax}}}{35x^2}-\frac{2i\sqrt{1-\frac{1}{ax}}}{7x^3}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x**4/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

```
[Out] a*Piecewise((-16*a**2*sqrt(-1 + 1/(a*x))/15 - 8*a*sqrt(-1 + 1/(a*x))/(15*x)
 - 2*sqrt(-1 + 1/(a*x))/(5*x**2), 1/Abs(a*x) > 1), (-16*I*a**2*sqrt(1 - 1/(
a*x))/15 - 8*I*a*sqrt(1 - 1/(a*x))/(15*x) - 2*I*sqrt(1 - 1/(a*x))/(5*x**2),
 True)) + Piecewise((-32*a**3*sqrt(-1 + 1/(a*x))/35 - 16*a**2*sqrt(-1 + 1/(
a*x))/(35*x) - 12*a*sqrt(-1 + 1/(a*x))/(35*x**2) - 2*sqrt(-1 + 1/(a*x))/(7*
x**3), 1/Abs(a*x) > 1), (-32*I*a**3*sqrt(1 - 1/(a*x))/35 - 16*I*a**2*sqrt(1
 - 1/(a*x))/(35*x) - 12*I*a*sqrt(1 - 1/(a*x))/(35*x**2) - 2*I*sqrt(1 - 1/(a
*x))/(7*x**3), True))
```

3.30 $\int \frac{1+ax}{x^5 \sqrt{ax} \sqrt{1-ax}} \, dx$

Optimal. Leaf size=121

$$-\frac{544a^4\sqrt{1-ax}}{315\sqrt{ax}}-\frac{272a^4\sqrt{1-ax}}{315(ax)^{3/2}}-\frac{68a^4\sqrt{1-ax}}{105(ax)^{5/2}}-\frac{34a^4\sqrt{1-ax}}{63(ax)^{7/2}}-\frac{2a^4\sqrt{1-ax}}{9(ax)^{9/2}}$$

```
[Out] -2/9*a^4*(-a*x+1)^(1/2)/(a*x)^(9/2)-34/63*a^4*(-a*x+1)^(1/2)/(a*x)^(7/2)-68
/105*a^4*(-a*x+1)^(1/2)/(a*x)^(5/2)-272/315*a^4*(-a*x+1)^(1/2)/(a*x)^(3/2)-
544/315*a^4*(-a*x+1)^(1/2)/(a*x)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {16, 78, 45, 37}

$$-\frac{544a^4\sqrt{1-ax}}{315\sqrt{ax}}-\frac{272a^4\sqrt{1-ax}}{315(ax)^{3/2}}-\frac{68a^4\sqrt{1-ax}}{105(ax)^{5/2}}-\frac{34a^4\sqrt{1-ax}}{63(ax)^{7/2}}-\frac{2a^4\sqrt{1-ax}}{9(ax)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a*x)/(x^5*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-2*a^4*Sqrt[1 - a*x])/(9*(a*x)^(9/2)) - (34*a^4*Sqrt[1 - a*x])/(63*(a*x)^(
7/2)) - (68*a^4*Sqrt[1 - a*x])/(105*(a*x)^(5/2)) - (272*a^4*Sqrt[1 - a*x])/
(315*(a*x)^(3/2)) - (544*a^4*Sqrt[1 - a*x])/(315*Sqrt[a*x])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rubi steps

$$\int \frac{1+ax}{x^5\sqrt{ax}\sqrt{1-ax}}\,dx = a^5\int \frac{1+ax}{(ax)^{11/2}\sqrt{1-ax}}\,dx$$
$$= -\frac{2a^4\sqrt{1-ax}}{9(ax)^{9/2}} + \frac{1}{9}\left(17a^5\right)\int \frac{1}{(ax)^{9/2}\sqrt{1-ax}}\,dx$$
$$= -\frac{2a^4\sqrt{1-ax}}{9(ax)^{9/2}} - \frac{34a^4\sqrt{1-ax}}{63(ax)^{7/2}} + \frac{1}{21}\left(34a^5\right)\int \frac{1}{(ax)^{7/2}\sqrt{1-ax}}\,dx$$
$$= -\frac{2a^4\sqrt{1-ax}}{9(ax)^{9/2}} - \frac{34a^4\sqrt{1-ax}}{63(ax)^{7/2}} - \frac{68a^4\sqrt{1-ax}}{105(ax)^{5/2}} + \frac{1}{105}\left(136a^5\right)\int \frac{1}{(ax)^{5/2}\sqrt{1-ax}}\,dx$$
$$= -\frac{2a^4\sqrt{1-ax}}{9(ax)^{9/2}} - \frac{34a^4\sqrt{1-ax}}{63(ax)^{7/2}} - \frac{68a^4\sqrt{1-ax}}{105(ax)^{5/2}} - \frac{272a^4\sqrt{1-ax}}{315(ax)^{3/2}} + \frac{1}{315}\left(272a^5\right)\int \frac{}{(a}$$
$$= -\frac{2a^4\sqrt{1-ax}}{9(ax)^{9/2}} - \frac{34a^4\sqrt{1-ax}}{63(ax)^{7/2}} - \frac{68a^4\sqrt{1-ax}}{105(ax)^{5/2}} - \frac{272a^4\sqrt{1-ax}}{315(ax)^{3/2}} - \frac{544a^4\sqrt{1-ax}}{315\sqrt{ax}}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 0.44

$$-\frac{2\sqrt{-ax(ax-1)}\left(272a^4x^4+136a^3x^3+102a^2x^2+85ax+35\right)}{315ax^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a*x)/(x^5*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-2*Sqrt[-(a*x*(-1 + a*x))]*(35 + 85*a*x + 102*a^2*x^2 + 136*a^3*x^3 + 272*
a^4*x^4))/(315*a*x^5)
```

fricas [A] time = 0.93, size = 51, normalized size = 0.42

$$-\frac{2\left(272\,a^4x^4+136\,a^3x^3+102\,a^2x^2+85\,ax+35\right)\sqrt{ax}\,\sqrt{-ax+1}}{315\,ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x^5/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/315*(272*a^4*x^4 + 136*a^3*x^3 + 102*a^2*x^2 + 85*a*x + 35)*sqrt(a*x)*sq
rt(-a*x + 1)/(a*x^5)
```

giac [B] time = 1.32, size = 217, normalized size = 1.79

$$-\frac{\frac{35\,a^5\left(\sqrt{-ax+1}-1\right)^9}{(ax)^{\frac{9}{2}}}+\frac{585\,a^5\left(\sqrt{-ax+1}-1\right)^7}{(ax)^{\frac{7}{2}}}+\frac{4032\,a^5\left(\sqrt{-ax+1}-1\right)^5}{(ax)^{\frac{5}{2}}}+\frac{17640\,a^5\left(\sqrt{-ax+1}-1\right)^3}{(ax)^{\frac{3}{2}}}+\frac{83790\,a^5\left(\sqrt{-ax+1}-1\right)}{\sqrt{ax}}-\frac{\left(35\,a^5+\frac{585\,a^4}{}\right.}{}}{80640\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x^5/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/80640*(35*a^5*(sqrt(-a*x + 1) - 1)^9/(a*x)^(9/2) + 585*a^5*(sqrt(-a*x +
1) - 1)^7/(a*x)^(7/2) + 4032*a^5*(sqrt(-a*x + 1) - 1)^5/(a*x)^(5/2) + 17640
*a^5*(sqrt(-a*x + 1) - 1)^3/(a*x)^(3/2) + 83790*a^5*(sqrt(-a*x + 1) - 1)/sq
rt(a*x) - (35*a^5 + 585*a^4*(sqrt(-a*x + 1) - 1)^2/x + 4032*a^3*(sqrt(-a*x
+ 1) - 1)^4/x^2 + 17640*a^2*(sqrt(-a*x + 1) - 1)^6/x^3 + 83790*a*(sqrt(-a*x
 + 1) - 1)^8/x^4)*(a*x)^(9/2)/(sqrt(-a*x + 1) - 1)^9)/a
```

maple [A] time = 0.01, size = 49, normalized size = 0.40

$$-\frac{2\left(272a^4x^4+136a^3x^3+102a^2x^2+85ax+35\right)\sqrt{-ax+1}}{315\sqrt{ax}\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/x^5/(a*x)^(1/2)/(-a*x+1)^(1/2),x)
```

```
[Out] -2/315*(272*a^4*x^4+136*a^3*x^3+102*a^2*x^2+85*a*x+35)/x^4/(a*x)^(1/2)*(-a*
x+1)^(1/2)
```

maxima [A] time = 0.97, size = 106, normalized size = 0.88

$$-\frac{544\sqrt{-a^2x^2+ax}\,a^3}{315\,x}-\frac{272\sqrt{-a^2x^2+ax}\,a^2}{315\,x^2}-\frac{68\sqrt{-a^2x^2+ax}\,a}{105\,x^3}-\frac{34\sqrt{-a^2x^2+ax}}{63\,x^4}-\frac{2\sqrt{-a^2x^2+ax}}{9\,ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x^5/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -544/315*sqrt(-a^2*x^2 + a*x)*a^3/x - 272/315*sqrt(-a^2*x^2 + a*x)*a^2/x^2
- 68/105*sqrt(-a^2*x^2 + a*x)*a/x^3 - 34/63*sqrt(-a^2*x^2 + a*x)/x^4 - 2/9*
sqrt(-a^2*x^2 + a*x)/(a*x^5)
```

mupad [B] time = 2.83, size = 48, normalized size = 0.40

$$-\frac{\sqrt{1-a\,x}\left(\frac{544\,a^4\,x^4}{315}+\frac{272\,a^3\,x^3}{315}+\frac{68\,a^2\,x^2}{105}+\frac{34\,a\,x}{63}+\frac{2}{9}\right)}{x^4\,\sqrt{a\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^5*(a*x)^(1/2)*(1 - a*x)^(1/2)),x)
```

```
[Out] -((1 - a*x)^(1/2)*((34*a*x)/63 + (68*a^2*x^2)/105 + (272*a^3*x^3)/315 + (54
4*a^4*x^4)/315 + 2/9))/(x^4*(a*x)^(1/2))
```

sympy [C] time = 33.86, size = 359, normalized size = 2.97

$$a\left(\begin{cases}-\frac{32a^3\sqrt{-1+\frac{1}{ax}}}{35}-\frac{16a^2\sqrt{-1+\frac{1}{ax}}}{35x}-\frac{12a\sqrt{-1+\frac{1}{ax}}}{35x^2}-\frac{2\sqrt{-1+\frac{1}{ax}}}{7x^3} & \text{for}\ \frac{1}{|ax|}>1\\ -\frac{32ia^3\sqrt{1-\frac{1}{ax}}}{35}-\frac{16ia^2\sqrt{1-\frac{1}{ax}}}{35x}-\frac{12ia\sqrt{1-\frac{1}{ax}}}{35x^2}-\frac{2i\sqrt{1-\frac{1}{ax}}}{7x^3} & \text{otherwise}\end{cases}\right)+\begin{cases}-\frac{256a^4\sqrt{-1+\frac{1}{ax}}}{315}-\frac{128a^3\sqrt{-1+\frac{1}{ax}}}{315x}-\frac{32a^2\sqrt{-1+\ldots}}{105x^2}\\ -\frac{256ia^4\sqrt{1-\frac{1}{ax}}}{315}-\frac{128ia^3\sqrt{1-\frac{1}{ax}}}{315x}-\frac{32ia^2\sqrt{1-\frac{1}{ax}}}{105x^2}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x**5/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

```
[Out] a*Piecewise((-32*a**3*sqrt(-1 + 1/(a*x))/35 - 16*a**2*sqrt(-1 + 1/(a*x))/(3
5*x) - 12*a*sqrt(-1 + 1/(a*x))/(35*x**2) - 2*sqrt(-1 + 1/(a*x))/(7*x**3), 1
/Abs(a*x) > 1), (-32*I*a**3*sqrt(1 - 1/(a*x))/35 - 16*I*a**2*sqrt(1 - 1/(a*
x))/(35*x) - 12*I*a*sqrt(1 - 1/(a*x))/(35*x**2) - 2*I*sqrt(1 - 1/(a*x))/(7*
x**3), True)) + Piecewise((-256*a**4*sqrt(-1 + 1/(a*x))/315 - 128*a**3*sqrt
(-1 + 1/(a*x))/(315*x) - 32*a**2*sqrt(-1 + 1/(a*x))/(105*x**2) - 16*a*sqrt(
-1 + 1/(a*x))/(63*x**3) - 2*sqrt(-1 + 1/(a*x))/(9*x**4), 1/Abs(a*x) > 1), (
-256*I*a**4*sqrt(1 - 1/(a*x))/315 - 128*I*a**3*sqrt(1 - 1/(a*x))/(315*x) -
32*I*a**2*sqrt(1 - 1/(a*x))/(105*x**2) - 16*I*a*sqrt(1 - 1/(a*x))/(63*x**3)
 - 2*I*sqrt(1 - 1/(a*x))/(9*x**4), True))
```

3.31 $\int \frac{-1+2ax}{\sqrt{-1+x}\,x^2\sqrt{1+x}}\,dx$

Optimal. Leaf size=39

$$2a\tan^{-1}\left(\sqrt{x-1}\,\sqrt{x+1}\right)-\frac{\sqrt{x-1}\,\sqrt{x+1}}{x}$$

```
[Out] 2*a*arctan((-1+x)^(1/2)*(1+x)^(1/2))-(-1+x)^(1/2)*(1+x)^(1/2)/x
```

Rubi [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {151, 12, 92, 203}

$$2a\tan^{-1}\left(\sqrt{x-1}\,\sqrt{x+1}\right)-\frac{\sqrt{x-1}\,\sqrt{x+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 2*a*x)/(Sqrt[-1 + x]*x^2*Sqrt[1 + x]),x]
```

```
[Out] -((Sqrt[-1 + x]*Sqrt[1 + x])/x) + 2*a*ArcTan[Sqrt[-1 + x]*Sqrt[1 + x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{-1+2ax}{\sqrt{-1+x}\,x^2\sqrt{1+x}}\,dx &= -\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x} + \int \frac{2a}{\sqrt{-1+x}\,x\sqrt{1+x}}\,dx \\ &= -\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x} + (2a)\int \frac{1}{\sqrt{-1+x}\,x\sqrt{1+x}}\,dx \\ &= -\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x} + (2a)\,\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x}\,\sqrt{1+x}\right) \\ &= -\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x} + 2a\tan^{-1}\left(\sqrt{-1+x}\,\sqrt{1+x}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 48, normalized size = 1.23

$$\frac{2a\sqrt{x^2-1}\,x\tan^{-1}\left(\sqrt{x^2-1}\right)-x^2+1}{\sqrt{x-1}\,x\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 2*a*x)/(Sqrt[-1 + x]*x^2*Sqrt[1 + x]),x]
```

```
[Out] (1 - x^2 + 2*a*x*Sqrt[-1 + x^2]*ArcTan[Sqrt[-1 + x^2]])/(Sqrt[-1 + x]*x*Sqr
t[1 + x])
```

fricas [A] time = 0.82, size = 40, normalized size = 1.03

$$\frac{4\,ax\arctan\left(\sqrt{x+1}\,\sqrt{x-1}\,-x\right)-\sqrt{x+1}\,\sqrt{x-1}\,-x}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x-1)/x^2/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] (4*a*x*arctan(sqrt(x + 1)*sqrt(x - 1) - x) - sqrt(x + 1)*sqrt(x - 1) - x)/x
```

giac [A] time = 1.27, size = 43, normalized size = 1.10

$$-4\,a\arctan\left(\frac{1}{2}\left(\sqrt{x+1}\,-\sqrt{x-1}\right)^2\right)-\frac{8}{\left(\sqrt{x+1}\,-\sqrt{x-1}\right)^4+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x-1)/x^2/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] -4*a*arctan(1/2*(sqrt(x + 1) - sqrt(x - 1))^2) - 8/((sqrt(x + 1) - sqrt(x -
 1))^4 + 4)
```

maple [A] time = 0.02, size = 44, normalized size = 1.13

$$\frac{\left(-2ax\arctan\left(\frac{1}{\sqrt{x^2-1}}\right)-\sqrt{x^2-1}\right)\sqrt{x-1}\,\sqrt{x+1}}{\sqrt{x^2-1}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a*x-1)/x^2/(x-1)^(1/2)/(x+1)^(1/2),x)
```

```
[Out] (-2*x*a*arctan(1/(x^2-1)^(1/2))-(x^2-1)^(1/2))*(x-1)^(1/2)*(x+1)^(1/2)/x/(x
^2-1)^(1/2)
```

maxima [A] time = 0.96, size = 21, normalized size = 0.54

$$-2\,a\arcsin\left(\frac{1}{|x|}\right)-\frac{\sqrt{x^2-1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x-1)/x^2/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*a*arcsin(1/abs(x)) - sqrt(x^2 - 1)/x
```

mupad [B] time = 4.08, size = 65, normalized size = 1.67

$$-\frac{\sqrt{x-1}\,\sqrt{x+1}}{x}-a\left(\ln\left(\frac{\left(\sqrt{x-1}-\mathrm{i}\right)^2}{\left(\sqrt{x+1}-1\right)^2}+1\right)-\ln\left(\frac{\sqrt{x-1}-\mathrm{i}}{\sqrt{x+1}-1}\right)\right)2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a*x - 1)/(x^2*(x - 1)^(1/2)*(x + 1)^(1/2)),x)
```

```
[Out] - a*(log(((x - 1)^(1/2) - 1i)^2/((x + 1)^(1/2) - 1)^2 + 1) - log(((x - 1)^(
1/2) - 1i)/((x + 1)^(1/2) - 1)))*2i - ((x - 1)^(1/2)*(x + 1)^(1/2))/x
```

sympy [C] time = 35.80, size = 117, normalized size = 3.00

$$-\frac{aG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2}\\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0\end{matrix}\middle|\frac{1}{x^2}\right)}{2\pi^{\frac{3}{2}}}+\frac{iaG_{6,6}^{2,6}\left(\begin{matrix}0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0\end{matrix}\middle|\frac{e^{2i\pi}}{x^2}\right)}{2\pi^{\frac{3}{2}}}+\frac{G_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{1}{x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{iG_{6,6}^{2,6}\left(\begin{matrix}\frac{1}{2} \cdots\end{matrix}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x-1)/x**2/(-1+x)**(1/2)/(1+x)**(1/2),x)
```

```
[Out] -a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)), x
**(-2))/(2*pi**(3/2)) + I*a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()), ((1/4,
3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/x**2)/(2*pi**(3/2)) + meijerg(((
5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,)), x**(-2))/(4*pi
**(3/2)) + I*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1
, 1, 0)), exp_polar(2*I*pi)/x**2)/(4*pi**(3/2))
```

3.32 $\int \frac{a^2x^2-(1-ax)^2}{\sqrt{-1+x}\,x^2\sqrt{1+x}}\,dx$

Optimal. Leaf size=39

$$2a\tan^{-1}\left(\sqrt{x-1}\sqrt{x+1}\right)-\frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

```
[Out] 2*a*arctan((-1+x)^(1/2)*(1+x)^(1/2))-(-1+x)^(1/2)*(1+x)^(1/2)/x
```

Rubi [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {188, 151, 12, 92, 203}

$$2a\tan^{-1}\left(\sqrt{x-1}\sqrt{x+1}\right)-\frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2*x^2 - (1 - a*x)^2)/(Sqrt[-1 + x]*x^2*Sqrt[1 + x]),x]
```

```
[Out] -((Sqrt[-1 + x]*Sqrt[1 + x])/x) + 2*a*ArcTan[Sqrt[-1 + x]*Sqrt[1 + x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 188

```
Int[(u_)^(m_.)*(v_)^(n_.)*(w_)^(p_.)*(z_)^(q_.), x_Symbol] :> Int[ExpandToS
um[u, x]^m*ExpandToSum[v, x]^n*ExpandToSum[w, x]^p*ExpandToSum[z, x]^q, x]
/; FreeQ[{m, n, p, q}, x] && LinearQ[{u, v, w, z}, x] &&  !LinearMatchQ[{u,
 v, w, z}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{a^2x^2-(1-ax)^2}{\sqrt{-1+x}\,x^2\sqrt{1+x}}\,dx &= \int \frac{-1+2ax}{\sqrt{-1+x}\,x^2\sqrt{1+x}}\,dx\\
&= -\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x}+\int \frac{2a}{\sqrt{-1+x}\,x\sqrt{1+x}}\,dx\\
&= -\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x}+(2a)\int \frac{1}{\sqrt{-1+x}\,x\sqrt{1+x}}\,dx\\
&= -\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x}+(2a)\,\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{-1+x}\,\sqrt{1+x}\right)\\
&= -\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x}+2a\tan^{-1}\left(\sqrt{-1+x}\,\sqrt{1+x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 48, normalized size = 1.23

$$\frac{2a\sqrt{x^2-1}\,x\tan^{-1}\left(\sqrt{x^2-1}\right)-x^2+1}{\sqrt{x-1}\,x\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2*x^2 - (1 - a*x)^2)/(Sqrt[-1 + x]*x^2*Sqrt[1 + x]),x]
```

```
[Out] (1 - x^2 + 2*a*x*Sqrt[-1 + x^2]*ArcTan[Sqrt[-1 + x^2]])/(Sqrt[-1 + x]*x*Sqr
t[1 + x])
```

fricas [A] time = 0.82, size = 40, normalized size = 1.03

$$\frac{4\,ax\arctan\left(\sqrt{x+1}\,\sqrt{x-1}\,-x\right)-\sqrt{x+1}\,\sqrt{x-1}\,-x}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-(-a*x+1)^2)/x^2/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="f
ricas")
```

```
[Out] (4*a*x*arctan(sqrt(x + 1)*sqrt(x - 1) - x) - sqrt(x + 1)*sqrt(x - 1) - x)/x
```

giac [A] time = 1.29, size = 43, normalized size = 1.10

$$-4\,a\arctan\left(\frac{1}{2}\left(\sqrt{x+1}\,-\sqrt{x-1}\right)^2\right)-\frac{8}{\left(\sqrt{x+1}\,-\sqrt{x-1}\right)^4+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-(-a*x+1)^2)/x^2/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="g
iac")
```

```
[Out] -4*a*arctan(1/2*(sqrt(x + 1) - sqrt(x - 1))^2) - 8/((sqrt(x + 1) - sqrt(x -
 1))^4 + 4)
```

maple [A] time = 0.00, size = 44, normalized size = 1.13

$$\frac{\left(-2ax\arctan\left(\frac{1}{\sqrt{x^2-1}}\right)-\sqrt{x^2-1}\right)\sqrt{x-1}\,\sqrt{x+1}}{\sqrt{x^2-1}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2-(-a*x+1)^2)/x^2/(x-1)^(1/2)/(x+1)^(1/2),x)
```

```
[Out] (-2*a*x*arctan(1/(x^2-1)^(1/2))-(x^2-1)^(1/2))*(x-1)^(1/2)*(x+1)^(1/2)/(x^2
-1)^(1/2)/x
```

maxima [A] time = 0.98, size = 21, normalized size = 0.54

$$-2\,a\arcsin\left(\frac{1}{|x|}\right)-\frac{\sqrt{x^2-1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-(-a*x+1)^2)/x^2/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="m
axima")
```

```
[Out] -2*a*arcsin(1/abs(x)) - sqrt(x^2 - 1)/x
```

mupad [B] time = 5.27, size = 444, normalized size = 11.38

$$a\,\ln\left(\frac{\sqrt{x-1}\,-\mathrm{i}}{\sqrt{x+1}\,-1}\right)2\mathrm{i}-a^2\,\mathrm{atan}\left(\frac{1024\,a^6}{1024\,a^5+1024\,a^7+\frac{a^6\left(\sqrt{x-1}-\mathrm{i}\right)1024\mathrm{i}}{\sqrt{x+1}-1}+\frac{a^8\left(\sqrt{x-1}-\mathrm{i}\right)1024\mathrm{i}}{\sqrt{x+1}-1}}+\frac{10}{1024\,a^5+1024\,a^7+\frac{a^6\left(\sqrt{\right.}}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a*x - 1)^2 - a^2*x^2)/(x^2*(x - 1)^(1/2)*(x + 1)^(1/2)),x)
```

```
[Out] a*log(((x - 1)^(1/2) - 1i)/((x + 1)^(1/2) - 1))*2i - a^2*atan((1024*a^6)/(1
024*a^5 + 1024*a^7 + (a^6*((x - 1)^(1/2) - 1i)*1024i)/((x + 1)^(1/2) - 1) +
 (a^8*((x - 1)^(1/2) - 1i)*1024i)/((x + 1)^(1/2) - 1)) + (1024*a^8)/(1024*a
^5 + 1024*a^7 + (a^6*((x - 1)^(1/2) - 1i)*1024i)/((x + 1)^(1/2) - 1) + (a^8
*((x - 1)^(1/2) - 1i)*1024i)/((x + 1)^(1/2) - 1)) - (a^5*((x - 1)^(1/2) - 1
i)*1024i)/(((x + 1)^(1/2) - 1)*(1024*a^5 + 1024*a^7 + (a^6*((x - 1)^(1/2) -
 1i)*1024i)/((x + 1)^(1/2) - 1) + (a^8*((x - 1)^(1/2) - 1i)*1024i)/((x + 1)
^(1/2) - 1))) - (a^7*((x - 1)^(1/2) - 1i)*1024i)/(((x + 1)^(1/2) - 1)*(1024
*a^5 + 1024*a^7 + (a^6*((x - 1)^(1/2) - 1i)*1024i)/((x + 1)^(1/2) - 1) + (a
^8*((x - 1)^(1/2) - 1i)*1024i)/((x + 1)^(1/2) - 1))))*4i - a*log(((x - 1)^(
1/2) - 1i)^2/((x + 1)^(1/2) - 1)^2 + 1)*2i - ((x - 1)^(1/2) - 1i)/(4*((x +
1)^(1/2) - 1)) + a^2*acosh(x) - ((5*((x - 1)^(1/2) - 1i)^2)/(4*((x + 1)^(1/
2) - 1)^2) + 1/4)/(((x - 1)^(1/2) - 1i)^3/((x + 1)^(1/2) - 1)^3 + ((x - 1)^
(1/2) - 1i)/((x + 1)^(1/2) - 1))
```

sympy [C] time = 72.78, size = 117, normalized size = 3.00

$$-\frac{aG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2}\\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0\end{matrix}\middle|\frac{1}{x^2}\right)}{2\pi^{\frac{3}{2}}}+\frac{iaG_{6,6}^{2,6}\left(\begin{matrix}0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0\end{matrix}\middle|\frac{e^{2i\pi}}{x^2}\right)}{2\pi^{\frac{3}{2}}}+\frac{G_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{1}{x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{iG_{6,6}^{2,6}\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\end{matrix}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*x**2-(-a*x+1)**2)/x**2/(-1+x)**(1/2)/(1+x)**(1/2),x)
```

```
[Out] -a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)), x
**(-2))/(2*pi**(3/2)) + I*a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()), ((1/4,
3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/x**2)/(2*pi**(3/2)) + meijerg(((
5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,)), x**(-2))/(4*pi
**(3/2)) + I*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1
, 1, 0)), exp_polar(2*I*pi)/x**2)/(4*pi**(3/2))
```

3.33 $\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+\frac{b(-1+c)x}{a}}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx$

Optimal. Leaf size=145

$$\frac{2\sqrt{a}\,(aBe+A(b-be))\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-c}\,\sqrt{a+bx}}{\sqrt{a}}\right),\frac{1-e}{1-c}\right)}{b^2\sqrt{1-c}\,(1-e)}-\frac{2a^{3/2}BE\left(\sin^{-1}\left(\frac{\sqrt{1-c}\,\sqrt{a+bx}}{\sqrt{a}}\right)|\frac{1-e}{1-c}\right)}{b^2\sqrt{1-c}\,(1-e)}$$

```
[Out] -2*a^(3/2)*B*EllipticE((1-c)^(1/2)*(b*x+a)^(1/2)/a^(1/2),((1-e)/(1-c))^(1/2
))/b^2/(1-e)/(1-c)^(1/2)+2*(a*B*e+A*(-b*e+b))*EllipticF((1-c)^(1/2)*(b*x+a)
^(1/2)/a^(1/2),((1-e)/(1-c))^(1/2))*a^(1/2)/b^2/(1-e)/(1-c)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {158, 113, 119}

$$\frac{2\sqrt{a}\,(aBe+A(b-be))F\left(\sin^{-1}\left(\frac{\sqrt{1-c}\,\sqrt{a+bx}}{\sqrt{a}}\right)|\frac{1-e}{1-c}\right)}{b^2\sqrt{1-c}\,(1-e)}-\frac{2a^{3/2}BE\left(\sin^{-1}\left(\frac{\sqrt{1-c}\,\sqrt{a+bx}}{\sqrt{a}}\right)|\frac{1-e}{1-c}\right)}{b^2\sqrt{1-c}\,(1-e)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(Sqrt[a + b*x]*Sqrt[c + (b*(-1 + c)*x)/a]*Sqrt[e + (b*(-1 + e
)*x)/a]),x]
```

```
[Out] (-2*a^(3/2)*B*EllipticE[ArcSin[(Sqrt[1 - c]*Sqrt[a + b*x])/Sqrt[a]], (1 - e
)/(1 - c)])/(b^2*Sqrt[1 - c]*(1 - e)) + (2*Sqrt[a]*(a*B*e + A*(b - b*e))*El
lipticF[ArcSin[(Sqrt[1 - c]*Sqrt[a + b*x])/Sqrt[a]], (1 - e)/(1 - c)])/(b^2
*Sqrt[1 - c]*(1 - e))
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] && !LtQ[-((b*c - a*d)/d), 0] && !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] && !LtQ[(b*c - a*d)/b, 0])
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] && !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) && !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) && !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rubi steps

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+\frac{b(-1+c)x}{a}}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx = -\frac{(aB)\int \frac{\sqrt{e+\frac{b(-1+e)x}{a}}}{\sqrt{a+bx}\sqrt{c+\frac{b(-1+c)x}{a}}}\,dx}{b(1-e)} + \left(A+\frac{aBe}{b-be}\right)\int \frac{1}{\sqrt{a+bx}\sqrt{c+\frac{b(-1+}{a}}}$$

$$= -\frac{2a^{3/2}BE\left(\sin^{-1}\left(\frac{\sqrt{1-c}\sqrt{a+bx}}{\sqrt{a}}\right)|\frac{1-e}{1-c}\right)}{b^2\sqrt{1-c}(1-e)} + \frac{2\sqrt{a}\left(A+\frac{aBe}{b-be}\right)F\left(\sin^{-1}\left(\frac{\sqrt{1}}{}\right.\right.}{b\sqrt{1-c}}$$

Mathematica [C] time = 1.51, size = 309, normalized size = 2.13

$$-\frac{2\sqrt{\frac{a}{c-1}}(a+bx)^{3/2}\left(\frac{i(e-1)\sqrt{\frac{\frac{a}{a+bx}+c-1}{c-1}}\sqrt{\frac{\frac{a}{a+bx}+e-1}{e-1}}(aBc+A(b-bc))\,\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{a}{c-1}}}{\sqrt{a+bx}}\right),\frac{c-1}{e-1}\right)}{\sqrt{a+bx}} - B\sqrt{\frac{a}{c-1}}\left(\frac{a}{a+bx}+c-1\right)\left(\frac{a}{a+bx}+\right.\right.}{ab^2(e-1)\sqrt{\frac{b(c-1)x}{a}+c}\sqrt{\frac{b(e-1)x}{a}+e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(Sqrt[a + b*x]*Sqrt[c + (b*(-1 + c)*x)/a]*Sqrt[e + (b*(
-1 + e)*x)/a]),x]
```

```
[Out] (-2*Sqrt[a/(-1 + c)]*(a + b*x)^(3/2)*(-(B*Sqrt[a/(-1 + c)]*(-1 + c + a/(a +
 b*x))*(-1 + e + a/(a + b*x))) - (I*a*B*(-1 + e)*Sqrt[(-1 + c + a/(a + b*x)
)/(-1 + c)]*Sqrt[(-1 + e + a/(a + b*x))/(-1 + e)]*EllipticE[I*ArcSinh[Sqrt[
a/(-1 + c)]/Sqrt[a + b*x]], (-1 + c)/(-1 + e)])/Sqrt[a + b*x] + (I*(a*B*c +
 A*(b - b*c))*(-1 + e)*Sqrt[(-1 + c + a/(a + b*x))/(-1 + c)]*Sqrt[(-1 + e +
 a/(a + b*x))/(-1 + e)]*EllipticF[I*ArcSinh[Sqrt[a/(-1 + c)]/Sqrt[a + b*x]]
, (-1 + c)/(-1 + e)])/Sqrt[a + b*x]))/(a*b^2*(-1 + e)*Sqrt[c + (b*(-1 + c)*
x)/a]*Sqrt[e + (b*(-1 + e)*x)/a])
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Ba^2x+Aa^2\right)\sqrt{bx+a}\sqrt{\frac{ac+(bc-b)x}{a}}\sqrt{\frac{ae+(be-b)x}{a}}}{a^3ce-\left(b^3c-b^3-\left(b^3c-b^3\right)e\right)x^3-\left(2\,ab^2c-ab^2-\left(3\,ab^2c-2\,ab^2\right)e\right)x^2-\left(a^2bc-\left(3\,a^2bc-a^2b\right)e\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x+a)^(1/2)/(c+b*(-1+c)*x/a)^(1/2)/(e+b*(-1+e)*x/a)^(1/
2),x, algorithm="fricas")
```

```
[Out] integral((B*a^2*x + A*a^2)*sqrt(b*x + a)*sqrt((a*c + (b*c - b)*x)/a)*sqrt((
a*e + (b*e - b)*x)/a)/(a^3*c*e - (b^3*c - b^3 - (b^3*c - b^3)*e)*x^3 - (2*a
*b^2*c - a*b^2 - (3*a*b^2*c - 2*a*b^2)*e)*x^2 - (a^2*b*c - (3*a^2*b*c - a^2
*b)*e)*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx+A}{\sqrt{bx+a}\sqrt{\frac{b(c-1)x}{a}+c}\sqrt{\frac{b(e-1)x}{a}+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x+a)^(1/2)/(c+b*(-1+c)*x/a)^(1/2)/(e+b*(-1+e)*x/a)^(1/
2),x, algorithm="giac")
```

```
[Out] integrate((B*x + A)/(sqrt(b*x + a)*sqrt(b*(c - 1)*x/a + c)*sqrt(b*(e - 1)*x
/a + e)), x)
```

maple [B] time = 0.05, size = 624, normalized size = 4.30

$$-\frac{2\left(Abc^2\,\mathrm{EllipticF}\left(\sqrt{-\frac{(e-1)(bcx+ac-bx)}{(c-e)a}},\sqrt{-\frac{c-e}{e-1}}\right)-Abce\,\mathrm{EllipticF}\left(\sqrt{-\frac{(e-1)(bcx+ac-bx)}{(c-e)a}},\sqrt{-\frac{c-e}{e-1}}\right)-Bac^2\,\mathrm{Ellipti}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(b*x+a)^(1/2)/(c+b*(c-1)*x/a)^(1/2)/(e+b*(e-1)*x/a)^(1/2),x)
```

```
[Out] -2*(A*EllipticF((-(e-1)*(b*c*x+a*c-b*x)/(c-e)/a)^(1/2),(-(c-e)/(e-1))^(1/2)
)*b*c^2-A*EllipticF((-(e-1)*(b*c*x+a*c-b*x)/(c-e)/a)^(1/2),(-(c-e)/(e-1))^(
1/2))*b*c*e-B*EllipticF((-(e-1)*(b*c*x+a*c-b*x)/(c-e)/a)^(1/2),(-(c-e)/(e-1
))^(1/2))*a*c^2+B*EllipticF((-(e-1)*(b*c*x+a*c-b*x)/(c-e)/a)^(1/2),(-(c-e)/
(e-1))^(1/2))*a*c*e-A*EllipticF((-(e-1)*(b*c*x+a*c-b*x)/(c-e)/a)^(1/2),(-(c
-e)/(e-1))^(1/2))*b*c+A*EllipticF((-(e-1)*(b*c*x+a*c-b*x)/(c-e)/a)^(1/2),(-
(c-e)/(e-1))^(1/2))*b*e+B*EllipticF((-(e-1)*(b*c*x+a*c-b*x)/(c-e)/a)^(1/2),
(-(c-e)/(e-1))^(1/2))*a*c-B*EllipticF((-(e-1)*(b*c*x+a*c-b*x)/(c-e)/a)^(1/2
),(-(c-e)/(e-1))^(1/2))*a*e-B*EllipticE((-(e-1)*(b*c*x+a*c-b*x)/(c-e)/a)^(1
/2),(-(c-e)/(e-1))^(1/2))*a*c+B*EllipticE((-(e-1)*(b*c*x+a*c-b*x)/(c-e)/a)^
(1/2),(-(c-e)/(e-1))^(1/2))*a*e)*((c-1)*(b*e*x+a*e-b*x)/(c-e)/a)^(1/2)*(-(b
*x+a)*(c-1)/a)^(1/2)*(-(e-1)*(b*c*x+a*c-b*x)/(c-e)/a)^(1/2)*a/(b*x+a)^(1/2)
/((b*c*x+a*c-b*x)/a)^(1/2)/((b*e*x+a*e-b*x)/a)^(1/2)/(e-1)/(c-1)^2/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx+A}{\sqrt{bx+a}\sqrt{\frac{b(c-1)x}{a}+c}\sqrt{\frac{b(e-1)x}{a}+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x+a)^(1/2)/(c+b*(-1+c)*x/a)^(1/2)/(e+b*(-1+e)*x/a)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((B*x + A)/(sqrt(b*x + a)*sqrt(b*(c - 1)*x/a + c)*sqrt(b*(e - 1)*x
/a + e)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,x}{\sqrt{c+\frac{b\,x\,(c-1)}{a}}\,\sqrt{e+\frac{b\,x\,(e-1)}{a}}\,\sqrt{a+b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/((c + (b*x*(c - 1))/a)^(1/2)*(e + (b*x*(e - 1))/a)^(1/2)*(a +
 b*x)^(1/2)),x)
```

```
[Out] int((A + B*x)/((c + (b*x*(c - 1))/a)^(1/2)*(e + (b*x*(e - 1))/a)^(1/2)*(a +
 b*x)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x+a)**(1/2)/(c+b*(-1+c)*x/a)**(1/2)/(e+b*(-1+e)*x/a)**
(1/2),x)
```

```
[Out] Timed out
```

3.34 $\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx$

Optimal. Leaf size=221

$$\frac{2\sqrt{a}\,(aBe+A(b-be))\sqrt{\frac{b(c+dx)}{bc-ad}}\ \text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-e}\,\sqrt{a+bx}}{\sqrt{a}}\right),-\frac{ad}{(1-e)(bc-ad)}\right)}{b^2(1-e)^{3/2}\sqrt{c+dx}}-\frac{2aB\sqrt{ad-bc}\,\sqrt{\frac{b(c+dx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}\ \cdots}{\sqrt{a}\ \cdots}\right.\right.}{b^2\sqrt{d}\,(1-e)\sqrt{c+\cdots}}$$

```
[Out] 2*(a*B*e+A*(-b*e+b))*EllipticF((1-e)^(1/2)*(b*x+a)^(1/2)/a^(1/2),(-a*d/(-a*
d+b*c)/(1-e))^(1/2))*a^(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)/b^2/(1-e)^(3/2)/(
d*x+c)^(1/2)-2*a*B*EllipticE(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),(-(-a*d+
b*c)*(1-e)/a/d)^(1/2))*(a*d-b*c)^(1/2)*(b*(d*x+c)/(-a*d+b*c))^(1/2)/b^2/(1-
e)/d^(1/2)/(d*x+c)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$, Rules used = {158, 114, 113, 121, 119}

$$\frac{2\sqrt{a}\,(aBe+A(b-be))\sqrt{\frac{b(c+dx)}{bc-ad}}\,F\left(\sin^{-1}\left(\frac{\sqrt{1-e}\,\sqrt{a+bx}}{\sqrt{a}}\right)|-\frac{ad}{(bc-ad)(1-e)}\right)}{b^2(1-e)^{3/2}\sqrt{c+dx}}-\frac{2aB\sqrt{ad-bc}\,\sqrt{\frac{b(c+dx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{ad-bc}}\right)|\cdots\right.}{b^2\sqrt{d}\,(1-e)\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + (b*(-1 + e)*x)/a]),x]
```

```
[Out] (-2*a*B*Sqrt[-(b*c) + a*d]*Sqrt[(b*(c + d*x))/(b*c - a*d)]*EllipticE[ArcSin
[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], -(((b*c - a*d)*(1 - e))/(a*d)
)])/(b^2*Sqrt[d]*(1 - e)*Sqrt[c + d*x]) + (2*Sqrt[a]*(a*B*e + A*(b - b*e))*
Sqrt[(b*(c + d*x))/(b*c - a*d)]*EllipticF[ArcSin[(Sqrt[1 - e]*Sqrt[a + b*x]
)/Sqrt[a]], -((a*d)/((b*c - a*d)*(1 - e)))])/(b^2*(1 - e)^(3/2)*Sqrt[c + d*
x])
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
```

```
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rubi steps

$$\int \frac{A+Bx}{\sqrt{a+bx}\,\sqrt{c+dx}\,\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx=-\frac{(aB)\int \frac{\sqrt{e+\frac{b(-1+e)x}{a}}}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx}{b(1-e)}+\left(A+\frac{aBe}{b-be}\right)\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}\,\sqrt{e+\frac{b(-\cdots}{}}}\cdots$$

$$=\frac{\left(\left(A+\frac{aBe}{b-be}\right)\sqrt{\frac{b(c+dx)}{bc-ad}}\right)\int \frac{1}{\sqrt{a+bx}\,\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}\,\sqrt{e+\frac{b(-1+e)x}{a}}}\,dx}{\sqrt{c+dx}}-\frac{\left(aB\sqrt{\frac{b(c+d\cdots}{bc-a\cdots}}\right.\cdots}{\cdots}$$

$$=-\frac{2aB\sqrt{-bc+ad}\,\sqrt{\frac{b(c+dx)}{bc-ad}}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{-bc+ad}}\right)\middle|-\frac{(bc-ad)(1-e)}{ad}\right)}{b^2\sqrt{d}\,(1-e)\sqrt{c+dx}}+\frac{2\sqrt{a}\,(\cdots}{\cdots}$$

Mathematica [C] time = 2.25, size = 312, normalized size = 1.41

$$-\frac{2\sqrt{\frac{a}{e-1}}\,(a+bx)^{3/2}\left(\frac{id\sqrt{\frac{\frac{a}{a+bx}+e-1}{e-1}}\,(aBe+A(b-be))\sqrt{\frac{b(c+dx)}{d(a+bx)}}\ \text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{a}{e-1}}}{\sqrt{a+bx}}\right),\frac{(e-1)(bc-ad)}{ad}\right)}{\sqrt{a+bx}}-\frac{bB\sqrt{\frac{a}{e-1}}\,(c+dx)(ae+b(e-1)x)}{(a+bx)^2}-\frac{ia\cdots}{\cdots}\right.}{ab^2d\sqrt{c+dx}\,\sqrt{\frac{b(e-1)x}{a}+e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + (b*(-1 + e)*x)/a]
),x]
```

```
[Out] (-2*Sqrt[a/(-1 + e)]*(a + b*x)^(3/2)*(-((b*B*Sqrt[a/(-1 + e)]*(c + d*x)*(a*
e + b*(-1 + e)*x))/(a + b*x)^2) - (I*a*B*d*Sqrt[(b*(c + d*x))/(d*(a + b*x))
]*Sqrt[(-1 + e + a/(a + b*x))/(-1 + e)]*EllipticE[I*ArcSinh[Sqrt[a/(-1 + e)
]/Sqrt[a + b*x]], ((b*c - a*d)*(-1 + e))/(a*d)])/Sqrt[a + b*x] + (I*d*(a*B*
e + A*(b - b*e))*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(-1 + e + a/(a + b*
x))/(-1 + e)]*EllipticF[I*ArcSinh[Sqrt[a/(-1 + e)]/Sqrt[a + b*x]], ((b*c -
```

```
a*d)*(-1 + e))/(a*d)])/Sqrt[a + b*x]))/(a*b^2*d*Sqrt[c + d*x]*Sqrt[e + (b*(
-1 + e)*x)/a])
```

fricas [F] time = 1.08, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(Bax+Aa)\sqrt{bx+a}\sqrt{dx+c}\sqrt{\frac{ae+(be-b)x}{a}}}{a^2ce+\left(b^2de-b^2d\right)x^3-\left(b^2c+abd-\left(b^2c+2\,abd\right)e\right)x^2-\left(abc-\left(2\,abc+a^2d\right)e\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(e+b*(-1+e)*x/a)^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral((B*a*x + A*a)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt((a*e + (b*e - b)*x)
/a)/(a^2*c*e + (b^2*d*e - b^2*d)*x^3 - (b^2*c + a*b*d - (b^2*c + 2*a*b*d)*e
)*x^2 - (a*b*c - (2*a*b*c + a^2*d)*e)*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx+A}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{\frac{b(e-1)x}{a}+e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(e+b*(-1+e)*x/a)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((B*x + A)/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(b*(e - 1)*x/a + e)),
x)
```

maple [B] time = 0.04, size = 940, normalized size = 4.25

$$2\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{\frac{(bex+ae-bx)d}{ade-bce+bc}}\,\sqrt{-\frac{(bx+a)(e-1)}{a}}\,\sqrt{-\frac{(dx+c)(e-1)b}{ade-bce+bc}}\,\left(Aabd\,e^2\,\mathrm{EllipticF}\left(\sqrt{\frac{(bex+ae-bx)d}{ade-bce+bc}},\sqrt{\frac{ade-bce+bc}{ad}}\right)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(e+b*(e-1)*x/a)^(1/2),x)
```

```
[Out] 2*(b*x+a)^(1/2)*(d*x+c)^(1/2)*((b*e*x+a*e-b*x)/(a*d*e-b*c*e+b*c)*d)^(1/2)*(
-(b*x+a)*(e-1)/a)^(1/2)*(-(d*x+c)*(e-1)/(a*d*e-b*c*e+b*c)*b)^(1/2)*(A*Ellip
ticF(((b*e*x+a*e-b*x)/(a*d*e-b*c*e+b*c)*d)^(1/2),((a*d*e-b*c*e+b*c)/a/d)^(1
/2))*a*b*d*e^2-A*EllipticF(((b*e*x+a*e-b*x)/(a*d*e-b*c*e+b*c)*d)^(1/2),((a*
d*e-b*c*e+b*c)/a/d)^(1/2))*b^2*c*e^2-B*EllipticF(((b*e*x+a*e-b*x)/(a*d*e-b*
c*e+b*c)*d)^(1/2),((a*d*e-b*c*e+b*c)/a/d)^(1/2))*a^2*d*e^2+B*EllipticF(((b*
e*x+a*e-b*x)/(a*d*e-b*c*e+b*c)*d)^(1/2),((a*d*e-b*c*e+b*c)/a/d)^(1/2))*a*b*
c*e^2-A*EllipticF(((b*e*x+a*e-b*x)/(a*d*e-b*c*e+b*c)*d)^(1/2),((a*d*e-b*c*e
+b*c)/a/d)^(1/2))*a*b*d*e+2*A*EllipticF(((b*e*x+a*e-b*x)/(a*d*e-b*c*e+b*c)*
d)^(1/2),((a*d*e-b*c*e+b*c)/a/d)^(1/2))*b^2*c*e+B*EllipticF(((b*e*x+a*e-b*x
)/(a*d*e-b*c*e+b*c)*d)^(1/2),((a*d*e-b*c*e+b*c)/a/d)^(1/2))*a^2*d*e-2*B*Ell
ipticF(((b*e*x+a*e-b*x)/(a*d*e-b*c*e+b*c)*d)^(1/2),((a*d*e-b*c*e+b*c)/a/d)^
(1/2))*a*b*c*e-B*EllipticE(((b*e*x+a*e-b*x)/(a*d*e-b*c*e+b*c)*d)^(1/2),((a*
d*e-b*c*e+b*c)/a/d)^(1/2))*a^2*d*e+B*EllipticE(((b*e*x+a*e-b*x)/(a*d*e-b*c*
e+b*c)*d)^(1/2),((a*d*e-b*c*e+b*c)/a/d)^(1/2))*a*b*c*e-A*EllipticF(((b*e*x+
a*e-b*x)/(a*d*e-b*c*e+b*c)*d)^(1/2),((a*d*e-b*c*e+b*c)/a/d)^(1/2))*b^2*c+B*
EllipticF(((b*e*x+a*e-b*x)/(a*d*e-b*c*e+b*c)*d)^(1/2),((a*d*e-b*c*e+b*c)/a/
d)^(1/2))*a*b*c-B*EllipticE(((b*e*x+a*e-b*x)/(a*d*e-b*c*e+b*c)*d)^(1/2),((a
*d*e-b*c*e+b*c)/a/d)^(1/2))*a*b*c)/((b*e*x+a*e-b*x)/a)^(1/2)/(b*d*x^2+a*d*x
+b*c*x+a*c)/(e-1)^2/b^2/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx + A}{\sqrt{bx + a}\sqrt{dx + c}\sqrt{\frac{b(e-1)x}{a} + e}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(e+b*(-1+e)*x/a)^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*x + A)/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(b*(e - 1)*x/a + e)),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B\,x}{\sqrt{e + \frac{b\,x\,(e-1)}{a}}\,\sqrt{a + b\,x}\,\sqrt{c + d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/((e + (b*x*(e - 1))/a)^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(1/2))
,x)
```

```
[Out] int((A + B*x)/((e + (b*x*(e - 1))/a)^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(1/2))
, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x+a)**(1/2)/(d*x+c)**(1/2)/(e+b*(-1+e)*x/a)**(1/2),x)
```

```
[Out] Timed out
```

3.35 $\int \sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^3\,dx$

Optimal. Leaf size=281

$$\frac{522167393\sqrt{\frac{11}{6}}\,\sqrt{5-2x}\ \text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{4x+1}\right),\frac{1}{3}\right)}{23328\sqrt{2x-5}}+\frac{2}{55}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(5x+7)^4-\frac{427\sqrt{2-3}}{}$$

```
[Out] 522167393/139968*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2
)*(5-2*x)^(1/2)/(-5+2*x)^(1/2)-6489123157/699840*EllipticE(2/11*(2-3*x)^(1/
2)*11^(1/2),1/2*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1/2)-1182926269
/1603800*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)-12243139/356400*(7+5*x)
*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)-17561/8910*(7+5*x)^2*(2-3*x)^(1
/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)-427/2970*(7+5*x)^3*(2-3*x)^(1/2)*(-5+2*x)^
(1/2)*(1+4*x)^(1/2)+2/55*(7+5*x)^4*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/
2)
```

Rubi [A] time = 0.39, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {161, 1600, 1615, 158, 114, 113, 121, 119}

$$\frac{2}{55}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(5x+7)^4-\frac{427\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(5x+7)^3}{2970}-\frac{17561\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(}{8910}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^3,x]
```

```
[Out] (-1182926269*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/1603800 - (1224313
9*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x))/356400 - (17561*Sqr
t[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2)/8910 - (427*Sqrt[2 - 3
*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^3)/2970 + (2*Sqrt[2 - 3*x]*Sqrt[
-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^4)/55 - (6489123157*Sqrt[11]*Sqrt[-5 + 2*
x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(699840*Sqrt[5 - 2*
x]) + (522167393*Sqrt[11/6]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[
1 + 4*x]], 1/3])/(23328*Sqrt[-5 + 2*x])
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
```

```
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 161

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[(2*(a + b*x)^(m + 1)*Sqrt[
c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*(2*m + 5)), x] + Dist[1/(b*(2*m +
5)), Int[((a + b*x)^m*Simp[3*b*c*e*g - a*(d*e*g + c*f*g + c*e*h) + 2*(b*(d*
e*g + c*f*g + c*e*h) - a*(d*f*g + d*e*h + c*f*h))*x - (3*a*d*f*h - b*(d*f*g
 + d*e*h + c*f*h))*x^2, x])/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] &&  !LtQ[m,
-1]
```

Rule 1600

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[(2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/
(d*f*h*(2*m + 3)), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^3\,dx = \frac{2}{55}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^4 + \frac{1}{55}\int \frac{(7+5x)^3\left(-3-11\right.}{\sqrt{2-3x}\,\sqrt{-5+}}$$

$$= -\frac{427\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^3}{2970} + \frac{2}{55}\sqrt{2-3x}\,\sqrt{-5+2x}$$

$$= -\frac{17561\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^2}{8910} - \frac{427\sqrt{2-3x}\,\sqrt{-5+}}{29}$$

$$= -\frac{12243139\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)}{356400} - \frac{17561\sqrt{2-3x}\,\sqrt{}}{}$$

$$= -\frac{1182926269\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{1603800} - \frac{12243139\sqrt{2-3x}\,\sqrt{-5}}{356}$$

$$= -\frac{1182926269\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{1603800} - \frac{12243139\sqrt{2-3x}\,\sqrt{-5}}{356}$$

$$= -\frac{1182926269\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{1603800} - \frac{12243139\sqrt{2-3x}\,\sqrt{-5}}{356}$$

$$= -\frac{1182926269\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{1603800} - \frac{12243139\sqrt{2-3x}\,\sqrt{-5}}{356}$$

Mathematica [A] time = 0.44, size = 135, normalized size = 0.48

$$\frac{57438413230\sqrt{66}\,\sqrt{5-2x}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{4x+1}\right),\frac{1}{3}\right) + 24\sqrt{2-3x}\,\sqrt{4x+1}\,\left(29160000x^5 + 67338000\right.}{1539}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^3,x]
```

```
[Out] (24*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(3325071575 - 797747975*x - 670058262*x^2 -
 167736600*x^3 + 67338000*x^4 + 29160000*x^5) - 71380354727*Sqrt[66]*Sqrt[5
 - 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3] + 57438413230*Sqrt
[66]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(15396
480*Sqrt[-5 + 2*x])
```

fricas [F] time = 1.05, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(125\,x^3 + 525\,x^2 + 735\,x + 343\right)\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^3*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((125*x^3 + 525*x^2 + 735*x + 343)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt
(-3*x + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (5\,x+7)^3\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^3*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((5*x + 7)^3*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

maple [A] time = 0.05, size = 160, normalized size = 0.57

$$\frac{\sqrt{-3x+2}\,\sqrt{2x-5}\,\sqrt{4x+1}\,\left(4199040000x^7+7947072000x^6-28894190400x^5-88040305728x^4-706465\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7+5*x)^3*(-3*x+2)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2),x)
```

```
[Out] 1/7698240*(-3*x+2)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(4199040000*x^7+794707
2000*x^6+86157619845*11^(1/2)*(-3*x+2)^(1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*El
lipticF(2/11*(22-33*x)^(1/2),1/2*I*2^(1/2))-71380354727*11^(1/2)*(-3*x+2)^(
1/2)*(5-2*x)^(1/2)*(4*x+1)^(1/2)*EllipticE(2/11*(22-33*x)^(1/2),1/2*I*2^(1/
2))-28894190400*x^5-88040305728*x^4-70646534280*x^3+542756583588*x^2-180358
343100*x-79801717800)/(24*x^3-70*x^2+21*x+10)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (5\,x+7)^3\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^3*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((5*x + 7)^3*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,(5\,x+7)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^3,x)
```

```
[Out] int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**3*(2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2),x)
```

```
[Out] Timed out
```

3.36 $\int \sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^2\,dx$

Optimal. Leaf size=243

$$\frac{5592499\sqrt{\frac{11}{6}}\,\sqrt{5-2x}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{4x+1}\right),\frac{1}{3}\right)}{3888\sqrt{2x-5}}+\frac{2}{45}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(5x+7)^3-\frac{61}{270}\sqrt{2-3x}$$

```
[Out] 5592499/23328*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(
5-2*x)^(1/2)/(-5+2*x)^(1/2)-17746949/29160*EllipticE(2/11*(2-3*x)^(1/2)*11^
(1/2),1/2*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1/2)-5256763/97200*(2
-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)-8141/2700*(7+5*x)*(2-3*x)^(1/2)*(-
5+2*x)^(1/2)*(1+4*x)^(1/2)-61/270*(7+5*x)^2*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1
+4*x)^(1/2)+2/45*(7+5*x)^3*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {161, 1600, 1615, 158, 114, 113, 121, 119}

$$\frac{2}{45}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(5x+7)^3-\frac{61}{270}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(5x+7)^2-\frac{8141\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(5x}{2700}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2,x]
```

```
[Out] (-5256763*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/97200 - (8141*Sqrt[2
 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x))/2700 - (61*Sqrt[2 - 3*x]*Sqr
t[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2)/270 + (2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x
]*Sqrt[1 + 4*x]*(7 + 5*x)^3)/45 - (17746949*Sqrt[11]*Sqrt[-5 + 2*x]*Ellipti
cE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(29160*Sqrt[5 - 2*x]) + (5592
499*Sqrt[11/6]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/
3])/(3888*Sqrt[-5 + 2*x])
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
```

```
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 161

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[(2*(a + b*x)^(m + 1)*Sqrt[
c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*(2*m + 5)), x] + Dist[1/(b*(2*m +
5)), Int[((a + b*x)^m*Simp[3*b*c*e*g - a*(d*e*g + c*f*g + c*e*h) + 2*(b*(d*
e*g + c*f*g + c*e*h) - a*(d*f*g + d*e*h + c*f*h))*x - (3*a*d*f*h - b*(d*f*g
 + d*e*h + c*f*h))*x^2, x])/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] &&  !LtQ[m,
-1]
```

Rule 1600

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[(2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/
(d*f*h*(2*m + 3)), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2\,dx = \frac{2}{45}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3 + \frac{1}{45}\int \frac{(7+5x)^2\left(-3-11\right.}{\sqrt{2-3x}\sqrt{-5+}}$$

$$= -\frac{61}{270}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2 + \frac{2}{45}\sqrt{2-3x}\sqrt{-5+2x}$$

$$= -\frac{8141\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}{2700} - \frac{61}{270}\sqrt{2-3x}\sqrt{-5+2}$$

$$= -\frac{5256763\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{97200} - \frac{8141\sqrt{2-3x}\sqrt{-5+2x}\sqrt{}}{2700}$$

$$= -\frac{5256763\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{97200} - \frac{8141\sqrt{2-3x}\sqrt{-5+2x}\sqrt{}}{2700}$$

$$= -\frac{5256763\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{97200} - \frac{8141\sqrt{2-3x}\sqrt{-5+2x}\sqrt{}}{2700}$$

$$= -\frac{5256763\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{97200} - \frac{8141\sqrt{2-3x}\sqrt{-5+2x}\sqrt{}}{2700}$$

Mathematica [A] time = 0.28, size = 130, normalized size = 0.53

$$\frac{27962495\sqrt{66}\sqrt{5-2x}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right) + 6\sqrt{2-3x}\sqrt{4x+1}\left(216000x^4+147600x^3-1649\right.}{116640\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2,x]
```

```
[Out] (6*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(6902575 - 2933650*x - 1649952*x^2 + 147600*
x^3 + 216000*x^4) - 35493898*Sqrt[66]*Sqrt[5 - 2*x]*EllipticE[ArcSin[Sqrt[3
/11]*Sqrt[1 + 4*x]], 1/3] + 27962495*Sqrt[66]*Sqrt[5 - 2*x]*EllipticF[ArcSi
n[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(116640*Sqrt[-5 + 2*x])
```

fricas [F] time = 1.16, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(25\,x^2+70\,x+49\right)\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^2*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((25*x^2 + 70*x + 49)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (5\,x+7)^2\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^2*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm
="giac")
```

[Out] integrate((5*x + 7)^2*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)

maple [A] time = 0.01, size = 155, normalized size = 0.64

$$\sqrt{-3x+2}\ \sqrt{2x-5}\ \sqrt{4x+1}\ \left(15552000x^6+4147200x^5-125816544x^4-163495440x^3+604794324x^2-17\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((7+5*x)^2*(-3*x+2)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2),x)

[Out] 1/116640*(-3*x+2)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(15552000*x^6+83887485*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))-70987796*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticE(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))+4147200*x^5-125816544*x^4-163495440*x^3+604794324*x^2-171873450*x-82830900)/(24*x^3-70*x^2+21*x+10)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (5\,x+7)^2\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((7+5*x)^2*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm="maxima")

[Out] integrate((5*x + 7)^2*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,(5\,x+7)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^2,x)

[Out] int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^2, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((7+5*x)**2*(2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2),x)

[Out] Timed out

3.37 $\int \sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)\,dx$

Optimal. Leaf size=193

$$\frac{72479\sqrt{\frac{11}{6}}\,\sqrt{5-2x}\ \text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{4x+1}\right),\frac{1}{3}\right)}{756\sqrt{2x-5}}+\frac{5}{28}\sqrt{2-3x}\,(2x-5)^{3/2}(4x+1)^{3/2}+\frac{136}{105}\sqrt{2-3x}\,\sqrt{2x-5}\,($$

```
[Out] 5/28*(-5+2*x)^(3/2)*(1+4*x)^(3/2)*(2-3*x)^(1/2)+72479/4536*EllipticF(1/11*3
3^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*x)^(1/2)/(-5+2*x)^(1/2)+13
6/105*(1+4*x)^(3/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)-954811/22680*EllipticE(2/1
1*(2-3*x)^(1/2)*11^(1/2),1/2*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1/
2)-20911/3780*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {154, 158, 114, 113, 121, 119}

$$\frac{5}{28}\sqrt{2-3x}\,(2x-5)^{3/2}(4x+1)^{3/2}+\frac{136}{105}\sqrt{2-3x}\,\sqrt{2x-5}\,(4x+1)^{3/2}-\frac{20911\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{3780}+\frac{72479\sqrt{\frac{11}{6}}\,\sqrt{5}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x),x]
```

```
[Out] (-20911*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/3780 + (136*Sqrt[2 - 3*
x]*Sqrt[-5 + 2*x]*(1 + 4*x)^(3/2))/105 + (5*Sqrt[2 - 3*x]*(-5 + 2*x)^(3/2)*
(1 + 4*x)^(3/2))/28 - (954811*Sqrt[11]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[(2*S
qrt[2 - 3*x])/Sqrt[11]], -1/2])/(22680*Sqrt[5 - 2*x]) + (72479*Sqrt[11/6]*S
qrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(756*Sqrt[-5
 + 2*x])
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] && !LtQ[-((b*c - a*d)/d), 0] && !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] && !LtQ[(b*c - a*d)/b, 0])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) && !LtQ[-((b*c - a*d)/d), 0]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] && !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) && !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) && !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
```

```
-(f/d)] || PosQ[-(f/b)]))
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rubi steps

$$\int \sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)\,dx = \frac{5}{28}\sqrt{2-3x}\,(-5+2x)^{3/2}(1+4x)^{3/2} + \frac{1}{28}\int \frac{\left(\frac{1249}{2}-1088x\right)\sqrt{-5+}}{\sqrt{2-3x}}$$

$$= \frac{136}{105}\sqrt{2-3x}\,\sqrt{-5+2x}\,(1+4x)^{3/2} + \frac{5}{28}\sqrt{2-3x}\,(-5+2x)^{3/2}(1+$$

$$= -\frac{20911\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{3780} + \frac{136}{105}\sqrt{2-3x}\,\sqrt{-5+2x}\,(1+$$

$$= -\frac{20911\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{3780} + \frac{136}{105}\sqrt{2-3x}\,\sqrt{-5+2x}\,(1+$$

$$= -\frac{20911\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{3780} + \frac{136}{105}\sqrt{2-3x}\,\sqrt{-5+2x}\,(1+$$

$$= -\frac{20911\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{3780} + \frac{136}{105}\sqrt{2-3x}\,\sqrt{-5+2x}\,(1+$$

Mathematica [A] time = 0.25, size = 125, normalized size = 0.65

$$\frac{724790\sqrt{66}\,\sqrt{5-2x}\ \text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{4x+1}\right),\frac{1}{3}\right) + 24\sqrt{2-3x}\,\sqrt{4x+1}\,\left(5400x^3-6066x^2-37975x\right.}{45360\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x),x]
```

```
[Out] (24*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(48475 - 37975*x - 6066*x^2 + 5400*x^3) - 9
54811*Sqrt[66]*Sqrt[5 - 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/
3] + 724790*Sqrt[66]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x
]], 1/3])/(45360*Sqrt[-5 + 2*x])
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left((5\,x+7)\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (5\,x+7)\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

maple [A] time = 0.01, size = 150, normalized size = 0.78

$$\frac{\sqrt{-3x+2}\,\sqrt{2x-5}\,\sqrt{4x+1}\,\left(777600x^5-1197504x^4-5234040x^3+9404484x^2-1997100x-954811\sqrt{11}\,\sqrt{-3}\right.}{544320x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7+5*x)*(-3*x+2)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2),x)
```

```
[Out] 1/22680*(-3*x+2)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(1087185*11^(1/2)*(-3*x+
2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(-33*x+22)^(1/2),1/2*I
*2^(1/2))-954811*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*Ellip
ticE(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))+777600*x^5-1197504*x^4-5234040*x^
3+9404484*x^2-1997100*x-1163400)/(24*x^3-70*x^2+21*x+10)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (5\,x+7)\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\left(5\,x+7\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7),x)
[Out] int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7), x)
```

sympy [**F(-1**)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)*(2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2),x)
[Out] Timed out
```

3.38 $\int \sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,dx$

Optimal. Leaf size=162

$$\frac{121\sqrt{\frac{11}{6}}\,\sqrt{5-2x}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{4x+1}\right),\frac{1}{3}\right)}{18\sqrt{2x-5}}+\frac{1}{10}\sqrt{2-3x}\,\sqrt{2x-5}\,(4x+1)^{3/2}-\frac{22}{45}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x-$$

```
[Out] 121/108*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*x)
^(1/2)/(-5+2*x)^(1/2)+1/10*(1+4*x)^(3/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)-847/2
70*EllipticE(2/11*(2-3*x)^(1/2)*11^(1/2),1/2*I*2^(1/2))*11^(1/2)*(-5+2*x)^(
1/2)/(5-2*x)^(1/2)-22/45*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {101, 154, 158, 114, 113, 121, 119}

$$\frac{1}{10}\sqrt{2-3x}\,\sqrt{2x-5}\,(4x+1)^{3/2}-\frac{22}{45}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}+\frac{121\sqrt{\frac{11}{6}}\,\sqrt{5-2x}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{4x+1}\right)|\frac{1}{3}\right)}{18\sqrt{2x-5}}-\frac{847}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x],x]
```

```
[Out] (-22*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/45 + (Sqrt[2 - 3*x]*Sqrt[-
5 + 2*x]*(1 + 4*x)^(3/2))/10 - (847*Sqrt[11]*Sqrt[-5 + 2*x]*EllipticE[ArcSi
n[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(270*Sqrt[5 - 2*x]) + (121*Sqrt[11/6]
*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(18*Sqrt[-
5 + 2*x])
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[((a + b*x)^m*(c + d*x)^n*(e + f*x)^(p + 1))/(f
*(m + n + p + 1)), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rubi steps

$$\int \sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,dx = \frac{1}{10}\sqrt{2-3x}\,\sqrt{-5+2x}\,(1+4x)^{3/2} - \frac{1}{10}\int \frac{\left(\frac{99}{2}-44x\right)\sqrt{1+4x}}{\sqrt{2-3x}\,\sqrt{-5+2x}}\,dx$$

$$= -\frac{22}{45}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} + \frac{1}{10}\sqrt{2-3x}\,\sqrt{-5+2x}\,(1+4x)^{3/2} + \frac{1}{90}\cdots$$

$$= -\frac{22}{45}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} + \frac{1}{10}\sqrt{2-3x}\,\sqrt{-5+2x}\,(1+4x)^{3/2} + \frac{84\cdots}{90\cdots}$$

$$= -\frac{22}{45}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} + \frac{1}{10}\sqrt{2-3x}\,\sqrt{-5+2x}\,(1+4x)^{3/2} + \frac{\left(12\cdots\right.}{\cdots}$$

$$= -\frac{22}{45}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} + \frac{1}{10}\sqrt{2-3x}\,\sqrt{-5+2x}\,(1+4x)^{3/2} - \frac{84\cdots}{\cdots}$$

Mathematica [A] time = 0.18, size = 120, normalized size = 0.74

$$\frac{605\sqrt{66}\sqrt{5-2x}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)+6\sqrt{2-3x}\sqrt{4x+1}\left(72x^2-250x+175\right)-847\sqrt{66}\sqrt{5-}}{540\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x],x]
```

```
[Out] (6*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(175 - 250*x + 72*x^2) - 847*Sqrt[66]*Sqrt[5
 - 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3] + 605*Sqrt[66]*Sqr
t[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(540*Sqrt[-5 +
 2*x])
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

maple [A] time = 0.01, size = 145, normalized size = 0.90

$$\frac{\sqrt{-3x+2}\sqrt{2x-5}\sqrt{4x+1}\left(5184x^4-20160x^3+19236x^2-2250x-1694\sqrt{11}\sqrt{-3x+2}\sqrt{-2x+5}\sqrt{4x+1}\ \text{I}\right.}{12960x^3-37800x^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x+2)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2),x)
```

```
[Out] 1/540*(-3*x+2)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(1815*11^(1/2)*(-3*x+2)^(1
/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(-33*x+22)^(1/2),1/2*I*2^(1
/2))-1694*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticE(2/
11*(-33*x+22)^(1/2),1/2*I*2^(1/2))+5184*x^4-20160*x^3+19236*x^2-2250*x-2100
)/(24*x^3-70*x^2+21*x+10)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2),x)
[Out] int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2),x)
[Out] Integral(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1), x)
```

3.39 $\int \frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{7+5x}\,dx$

Optimal. Leaf size=182

$$-\frac{1253\sqrt{\frac{2}{33}}\sqrt{5-2x}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{375\sqrt{2x-5}}+\frac{2}{15}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}-\frac{427\sqrt{11}\sqrt{2x-5}\,E\left(\sin \cdots\right.}{225\sqrt{5-\cdots}}$$

```
[Out] -1253/12375*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-
2*x)^(1/2)/(-5+2*x)^(1/2)-2691/1375*EllipticPi(2/11*(2-3*x)^(1/2)*11^(1/2),
55/124,1/2*I*2^(1/2))*(5-2*x)^(1/2)*11^(1/2)/(-5+2*x)^(1/2)-427/225*Ellipti
cE(2/11*(2-3*x)^(1/2)*11^(1/2),1/2*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*
x)^(1/2)+2/15*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {161, 1607, 168, 538, 537, 158, 114, 113, 121, 119}

$$\frac{2}{15}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}-\frac{1253\sqrt{\frac{2}{33}}\sqrt{5-2x}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{375\sqrt{2x-5}}-\frac{427\sqrt{11}\sqrt{2x-5}\,E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\cdots\right.\right.}{225\sqrt{5-2x}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x),x]
```

```
[Out] (2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/15 - (427*Sqrt[11]*Sqrt[-5 +
 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(225*Sqrt[5 - 2*
x]) - (1253*Sqrt[2/33]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4
*x]], 1/3])/(375*Sqrt[-5 + 2*x]) - (2691*Sqrt[5 - 2*x]*EllipticPi[55/124, A
rcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(125*Sqrt[11]*Sqrt[-5 + 2*x])
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
```

```
-(f/d)] || PosQ[-(f/b)]))
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 161

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[(2*(a + b*x)^(m + 1)*Sqrt[
c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*(2*m + 5)), x] + Dist[1/(b*(2*m +
5)), Int[((a + b*x)^m*Simp[3*b*c*e*g - a*(d*e*g + c*f*g + c*e*h) + 2*(b*(d*
e*g + c*f*g + c*e*h) - a*(d*f*g + d*e*h + c*f*h))*x - (3*a*d*f*h - b*(d*f*g
 + d*e*h + c*f*h))*x^2, x])/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] &&  !LtQ[m,
-1]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 1607

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
```

```
q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{7+5x}\,dx = \frac{2}{15}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}+\frac{1}{15}\int \frac{-3-1190x+854x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}$$

$$=\frac{2}{15}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}+\frac{1}{15}\int \frac{-\frac{11928}{25}+\frac{854x}{5}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx+\frac{27}{1}$$

$$=\frac{2}{15}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}-\frac{1253}{375}\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx+$$

$$=\frac{2}{15}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}-\frac{\left(1253\sqrt{\frac{2}{11}}\sqrt{5-2x}\right)\int \frac{1}{\sqrt{2-3x}\sqrt{\frac{10}{11}-\frac{4x}{11}}\sqrt{1+4x}}}{375\sqrt{-5+2x}}$$

$$=\frac{2}{15}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}-\frac{427\sqrt{11}\sqrt{-5+2x}\,E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{225\sqrt{5-2x}}$$

Mathematica [A] time = 0.82, size = 139, normalized size = 0.76

$$\frac{\sqrt{2x-5}\left(-3759\sqrt{11}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)+1650\sqrt{2-3x}\sqrt{5-2x}\sqrt{4x+1}-23485\sqrt{11}\,E\left(\sin^{-1}\left(\frac{2\sqrt{2}}{\sqrt{}}\right.\right.\right.}{12375\sqrt{5-2x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x),x]
```

```
[Out] (Sqrt[-5 + 2*x]*(1650*Sqrt[2 - 3*x]*Sqrt[5 - 2*x]*Sqrt[1 + 4*x] - 23485*Sqr
t[11]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] - 3759*Sqrt[11]*E
llipticF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 24219*Sqrt[11]*Ellipti
cPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/(12375*Sqrt[5 - 2*x
])
```

fricas [F] time = 1.09, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}}{5\,x+7},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x),x, algorithm="
fricas")
```

```
[Out] integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}}{5\,x+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7), x)
```

maple [A] time = 0.02, size = 183, normalized size = 1.01

$$-\frac{\sqrt{-3x+2}\,\sqrt{2x-5}\,\sqrt{4x+1}\,\left(-39600x^3+115500x^2-34650x+23485\sqrt{11}\,\sqrt{-3x+2}\,\sqrt{-2x+5}\,\sqrt{4x+1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x+2)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)/(7+5*x),x)
```

```
[Out] -1/12375*(-3*x+2)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(3759*11^(1/2)*(-3*x+2)
^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(-33*x+22)^(1/2),1/2*I*2
^(1/2))+23485*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*Elliptic
E(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))-24219*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+
5)^(1/2)*(4*x+1)^(1/2)*EllipticPi(2/11*(-33*x+22)^(1/2),55/124,1/2*I*2^(1/2
))-39600*x^3+115500*x^2-34650*x-16500)/(24*x^3-70*x^2+21*x+10)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}}{5\,x+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}{5\,x+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{5x+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)/(5*x + 7), x)
```

3.40 $\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^2}\,dx$

Optimal. Leaf size=189

$$\frac{152\sqrt{\frac{2}{33}}\sqrt{5-2x}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{125\sqrt{2x-5}}-\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{5(5x+7)}+\frac{6\sqrt{11}\sqrt{2x-5}\,E\left(\sin^{-1}\left(\frac{2\sqrt{2-}}{\sqrt{1}}\right.\right.}{25\sqrt{5-2x}}$$

```
[Out] 152/4125*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*x
)^(1/2)/(-5+2*x)^(1/2)+26859/85250*EllipticPi(2/11*(2-3*x)^(1/2)*11^(1/2),5
5/124,1/2*I*2^(1/2))*(5-2*x)^(1/2)*11^(1/2)/(-5+2*x)^(1/2)+6/25*EllipticE(2
/11*(2-3*x)^(1/2)*11^(1/2),1/2*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(
1/2)-1/5*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)
```

Rubi [A] time = 0.21, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {160, 1607, 168, 538, 537, 158, 114, 113, 121, 119}

$$-\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{5(5x+7)}+\frac{152\sqrt{\frac{2}{33}}\sqrt{5-2x}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{125\sqrt{2x-5}}+\frac{6\sqrt{11}\sqrt{2x-5}\,E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\right.}{25\sqrt{5-2x}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^2,x]
```

```
[Out] -(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(5*(7 + 5*x)) + (6*Sqrt[11]*S
qrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(25*Sqrt
[5 - 2*x]) + (152*Sqrt[2/33]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt
[1 + 4*x]], 1/3])/(125*Sqrt[-5 + 2*x]) + (26859*Sqrt[5 - 2*x]*EllipticPi[55
/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(7750*Sqrt[11]*Sqrt[-5 + 2
*x])
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] && !LtQ[-((b*c - a*d)/d), 0] && !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] && !LtQ[(b*c - a*d)/b, 0])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) && !LtQ[-((b*c - a*d)/d), 0]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] && !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) && !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) && !(SimplerQ[e + f*x,
```

```
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 160

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[((a + b*x)^(m + 1)*Sqrt[c
+ d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*(m + 1)), x] - Dist[1/(2*b*(m + 1)),
 Int[((a + b*x)^(m + 1)*Simp[d*e*g + c*f*g + c*e*h + 2*(d*f*g + d*e*h + c*f
*h)*x + 3*d*f*h*x^2, x])/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 1607

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
```

```
q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^2}\,dx = -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{5(7+5x)} + \frac{1}{10}\int \frac{-21+140x-72x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\,(7+5x)}$$

$$= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{5(7+5x)} + \frac{1}{10}\int \frac{\frac{1204}{25}-\frac{72x}{5}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx - \frac{895}{25}$$

$$= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{5(7+5x)} - \frac{18}{25}\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}}\,dx + \frac{152}{125}\int \frac{}{\sqrt{2-}}$$

$$= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{5(7+5x)} + \frac{\left(152\sqrt{\frac{2}{11}}\sqrt{5-2x}\right)\int \frac{1}{\sqrt{2-3x}\sqrt{\frac{10}{11}-\frac{4x}{11}}\sqrt{1+4x}}\,d}{125\sqrt{-5+2x}}$$

$$= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{5(7+5x)} + \frac{6\sqrt{11}\sqrt{-5+2x}\,E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{25\sqrt{5-2x}} +$$

Mathematica [A] time = 0.82, size = 130, normalized size = 0.69

$$\frac{\sqrt{2x-5}\left(\frac{3\sqrt{11}\left(9424\,\text{EllipticF}\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)+20460E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)-26859\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)\right)}{\sqrt{5-2x}}-\frac{51150\sqrt{2-3x}\sqrt{4x+1}}{5x+7}\right)}{255750}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^2,x]
```

```
[Out] (Sqrt[-5 + 2*x]*((-51150*Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(7 + 5*x) + (3*Sqrt[1
1]*(20460*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 9424*Ellipt
icF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] - 26859*EllipticPi[55/124, Ar
cSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/Sqrt[5 - 2*x]))/255750
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{25\,x^2+70\,x+49},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^2,x, algorithm
="fricas")
```

```
[Out] integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(25*x^2 + 70*x + 49), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{(5\,x+7)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^2,x, algorithm
="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^2, x)
```

maple [B] time = 0.03, size = 320, normalized size = 1.69

$$\frac{\sqrt{-3x+2}\,\sqrt{2x-5}\,\sqrt{4x+1}\,\left(-409200x^3+1193500x^2+102300\sqrt{11}\,\sqrt{-3x+2}\,\sqrt{-2x+5}\,\sqrt{4x+1}\,x\,\mathrm{Ellipti}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x+2)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)/(7+5*x)^2,x)
```

```
[Out] 1/85250*(-3*x+2)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(47120*11^(1/2)*(-3*x+2)
^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(-33*x+22)^(1/2),1/2*I*2
^(1/2))*x+102300*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*Ellip
ticE(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))*x-134295*11^(1/2)*(-3*x+2)^(1/2)*
(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticPi(2/11*(-33*x+22)^(1/2),55/124,1/2*I*
2^(1/2))*x+65968*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*Ellip
ticF(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))+143220*11^(1/2)*(-3*x+2)^(1/2)*(-
2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticE(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))-1
88013*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticPi(2/11*
(-33*x+22)^(1/2),55/124,1/2*I*2^(1/2))-409200*x^3+1193500*x^2-358050*x-1705
00)/(24*x^3-70*x^2+21*x+10)/(7+5*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}}{(5\,x+7)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^2,x, algorithm
="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}{(5\,x+7)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^2,x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**2,x)
```

```
[Out] Timed out
```

3.41 $\int \frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{(7+5x)^3}\,dx$

Optimal. Leaf size=227

$$\frac{397\sqrt{\frac{3}{22}}\,\sqrt{5-2x}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{4x+1}\right),\frac{1}{3}\right)}{89125\sqrt{2x-5}}+\frac{8953\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{556140(5x+7)}-\frac{\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+}}{10(5x+7)^2}$$

```
[Out] 397/1960750*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-
2*x)^(1/2)/(-5+2*x)^(1/2)-14832503/3160729000*EllipticPi(2/11*(2-3*x)^(1/2)
*11^(1/2),55/124,1/2*I*2^(1/2))*(5-2*x)^(1/2)*11^(1/2)/(-5+2*x)^(1/2)-8953/
1390350*EllipticE(2/11*(2-3*x)^(1/2)*11^(1/2),1/2*I*2^(1/2))*11^(1/2)*(-5+2
*x)^(1/2)/(5-2*x)^(1/2)-1/10*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+
5*x)^2+8953/556140*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)
```

Rubi [A] time = 0.31, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {160, 1604, 1607, 168, 538, 537, 158, 114, 113, 121, 119}

$$\frac{8953\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{556140(5x+7)}-\frac{\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{10(5x+7)^2}+\frac{397\sqrt{\frac{3}{22}}\,\sqrt{5-2x}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{4x+1}\right)|\frac{1}{3}\right)}{89125\sqrt{2x-5}}-\frac{895}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^3,x]
```

```
[Out] -(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(10*(7 + 5*x)^2) + (8953*Sqrt
[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(556140*(7 + 5*x)) - (8953*Sqrt[11]
*Sqrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(13903
50*Sqrt[5 - 2*x]) + (397*Sqrt[3/22]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/1
1]*Sqrt[1 + 4*x]], 1/3])/(89125*Sqrt[-5 + 2*x]) - (14832503*Sqrt[5 - 2*x]*E
llipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(287339000*Sqr
t[11]*Sqrt[-5 + 2*x])
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
```

```
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 160

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[((a + b*x)^(m + 1)*Sqrt[c
+ d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*(m + 1)), x] - Dist[1/(2*b*(m + 1)),
 Int[((a + b*x)^(m + 1)*Simp[d*e*g + c*f*g + c*e*h + 2*(d*f*g + d*e*h + c*f
*h)*x + 3*d*f*h*x^2, x])/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 1604

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt
[e + f*x]*Sqrt[g + h*x])/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), x]
```

```
- Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m
 + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m +
 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
+ c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2
*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^
2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
+ c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 1607

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
 q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^3}\,dx = -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{10(7+5x)^2} + \frac{1}{20}\int \frac{-21+140x-72x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\,(7+5x)^2}$$

$$= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{10(7+5x)^2} + \frac{8953\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{556140(7+5x)} + \frac{\int \frac{-106}{\sqrt{2-3}}}{}$$

$$= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{10(7+5x)^2} + \frac{8953\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{556140(7+5x)} + \frac{\int \frac{-2}{\sqrt{2-3}}}{}$$

$$= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{10(7+5x)^2} + \frac{8953\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{556140(7+5x)} + \frac{1191\int}{}$$

$$= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{10(7+5x)^2} + \frac{8953\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{556140(7+5x)} + \frac{(1191\sqrt{}}{}$$

$$= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{10(7+5x)^2} + \frac{8953\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{556140(7+5x)} - \frac{8953\sqrt{}}{}$$

Mathematica [A] time = 0.75, size = 134, normalized size = 0.59

$$\frac{\sqrt{2x-5}\left(\frac{\sqrt{11}\left(5759676\,\text{EllipticF}\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)-61059460E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)+44497509\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)\right)}{\sqrt{5-2x}} + \frac{17050\sqrt{2-3x}}{}\right.}{9482187000}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^3,x]
```

```
[Out] (Sqrt[-5 + 2*x]*((17050*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7057 + 44765*x))/(7 +
5*x)^2 + (Sqrt[11]*(-61059460*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]],
 -1/2] + 5759676*EllipticF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 4449
```

```
7509*EllipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/Sqrt[5
- 2*x]))/9482187000
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}}{125\,x^3+525\,x^2+735\,x+343},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^3,x, algorithm
="fricas")
```

```
[Out] integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(125*x^3 + 525*x^2 + 73
5*x + 343), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}}{(5\,x+7)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^3,x, algorithm
="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^3, x)
```

maple [B] time = 0.03, size = 461, normalized size = 2.03

$$\sqrt{-3x+2}\,\sqrt{2x-5}\,\sqrt{4x+1}\,\left(18317838000x^4-50539303100x^3-1526486500\sqrt{11}\,\sqrt{-3x+2}\,\sqrt{-2x+5}\,\sqrt{4x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x+2)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)/(5*x+7)^3,x)
```

```
[Out] 1/9482187000*(-3*x+2)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(143991900*11^(1/2)
*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(-33*x+22)^(1/2
),1/2*I*2^(1/2))*x^2-1526486500*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x
+1)^(1/2)*EllipticE(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))*x^2+1112437725*11^
(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticPi(2/11*(-33*x+22
)^(1/2),55/124,1/2*I*2^(1/2))*x^2+403177320*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5
)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))*x-4274
162200*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticE(2/11*
(-33*x+22)^(1/2),1/2*I*2^(1/2))*x+3114825630*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+
5)^(1/2)*(4*x+1)^(1/2)*EllipticPi(2/11*(-33*x+22)^(1/2),55/124,1/2*I*2^(1/2
))*x+282224124*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*Ellipti
cF(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))-2991913540*11^(1/2)*(-3*x+2)^(1/2)*
(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticE(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))
+2180377941*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticPi
(2/11*(-33*x+22)^(1/2),55/124,1/2*I*2^(1/2))+18317838000*x^4-50539303100*x^
3+7605578750*x^2+10159191350*x+1203218500)/(24*x^3-70*x^2+21*x+10)/(5*x+7)^
2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}}{(5\,x+7)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^3,x, algorithm
="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2-3x}\,\sqrt{4x+1}\,\sqrt{2x-5}}{(5x+7)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^3,x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**3,x)
```

```
[Out] Timed out
```

3.42 $\int \frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{(7+5x)^4}\,dx$

Optimal. Leaf size=263

$$\frac{24957247\sqrt{5-2x}\ \text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{4956597750\sqrt{66}\sqrt{2x-5}}+\frac{16830401\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{30929169960(5x+7)}+\frac{8953\sqrt{2-3x}\ \cdots}{166842\cdots}$$

```
[Out] 15664616449/175780782606000*EllipticPi(2/11*(2-3*x)^(1/2)*11^(1/2),55/124,1
/2*I*2^(1/2))*(5-2*x)^(1/2)*11^(1/2)/(-5+2*x)^(1/2)+24957247/327135451500*E
llipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*x)^(1/2)/(-
5+2*x)^(1/2)-16830401/77322924900*EllipticE(2/11*(2-3*x)^(1/2)*11^(1/2),1/2
*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1/2)-1/15*(2-3*x)^(1/2)*(-5+2*
x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^3+8953/1668420*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*
(1+4*x)^(1/2)/(7+5*x)^2+16830401/30929169960*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(
1+4*x)^(1/2)/(7+5*x)
```

Rubi [A] time = 0.40, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {160, 1604, 1607, 168, 538, 537, 158, 114, 113, 121, 119}

$$\frac{16830401\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{30929169960(5x+7)}+\frac{8953\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{1668420(5x+7)^2}-\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{15(5x+7)^3}+\frac{24957247\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^4,x]
```

```
[Out] -(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(15*(7 + 5*x)^3) + (8953*Sqrt
[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(1668420*(7 + 5*x)^2) + (16830401*S
qrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(30929169960*(7 + 5*x)) - (16830
401*Sqrt[11]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -
1/2])/(77322924900*Sqrt[5 - 2*x]) + (24957247*Sqrt[5 - 2*x]*EllipticF[ArcSi
n[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(4956597750*Sqrt[66]*Sqrt[-5 + 2*x]) + (
15664616449*Sqrt[5 - 2*x]*EllipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[
11]], -1/2])/(15980071146000*Sqrt[11]*Sqrt[-5 + 2*x])
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
```

```
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 160

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[((a + b*x)^(m + 1)*Sqrt[c
+ d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*(m + 1)), x] - Dist[1/(2*b*(m + 1)),
 Int[((a + b*x)^(m + 1)*Simp[d*e*g + c*f*g + c*e*h + 2*(d*f*g + d*e*h + c*f
*h)*x + 3*d*f*h*x^2, x])/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 1604

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
```

```
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt
[e + f*x]*Sqrt[g + h*x])/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), x]
- Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m
 + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m +
 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
+ c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2
*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^
2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
+ c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 1607

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
 q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^4}\,dx = -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{15(7+5x)^3} + \frac{1}{30}\int \frac{-21+140x-72x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\,(7+5$$

$$= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{15(7+5x)^3} + \frac{8953\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1668420(7+5x)^2} + \frac{\int}{\sqrt{}}$$

$$= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{15(7+5x)^3} + \frac{8953\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1668420(7+5x)^2} + \frac{168}{}$$

$$= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{15(7+5x)^3} + \frac{8953\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1668420(7+5x)^2} + \frac{168}{}$$

$$= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{15(7+5x)^3} + \frac{8953\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1668420(7+5x)^2} + \frac{168}{}$$

$$= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{15(7+5x)^3} + \frac{8953\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1668420(7+5x)^2} + \frac{168}{}$$

$$= -\frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{15(7+5x)^3} + \frac{8953\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1668420(7+5x)^2} + \frac{168}{}$$

Mathematica [A] time = 0.79, size = 139, normalized size = 0.53

$$\frac{\sqrt{2x-5}\left(\frac{\sqrt{11}\left(120693246492\,\text{EllipticF}\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)-114783334820E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)-46993849347\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)\right.}{\sqrt{5-2x}}\right.}{527342347818000}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^4,x]
```

```
[Out] (Sqrt[-5 + 2*x]*((17050*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(-75460017 + 2007981640
*x + 420760025*x^2))/(7 + 5*x)^3 + (Sqrt[11]*(-114783334820*EllipticE[ArcSi
n[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 120693246492*EllipticF[ArcSin[(2*Sqr
t[2 - 3*x])/Sqrt[11]], -1/2] - 46993849347*EllipticPi[55/124, ArcSin[(2*Sqr
t[2 - 3*x])/Sqrt[11]], -1/2]))/Sqrt[5 - 2*x]))/527342347818000
```

fricas [F] time = 1.10, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}}{625\,x^4+3500\,x^3+7350\,x^2+6860\,x+2401}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^4,x, algorithm
="fricas")
```

```
[Out] integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(625*x^4 + 3500*x^3 + 7
350*x^2 + 6860*x + 2401), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}}{(5\,x+7)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^4,x, algorithm
="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^4, x)
```

maple [B] time = 0.03, size = 602, normalized size = 2.29

$$\frac{\sqrt{-3x+2}\,\sqrt{2x-5}\,\sqrt{4x+1}\,\left(172175002230000x^5+319488997250500x^4-14347916852500\sqrt{11}\,\sqrt{-3x+2}\,\sqrt{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x+2)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)/(5*x+7)^4,x)
```

```
[Out] 1/527342347818000*(-3*x+2)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(1508665581150
0*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(-33*
x+22)^(1/2),1/2*I*2^(1/2))*x^3-14347916852500*11^(1/2)*(-3*x+2)^(1/2)*(-2*x
+5)^(1/2)*(4*x+1)^(1/2)*EllipticE(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))*x^3-
5874231168375*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*Elliptic
Pi(2/11*(-33*x+22)^(1/2),55/124,1/2*I*2^(1/2))*x^3+63363954408300*11^(1/2)*
(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(-33*x+22)^(1/2)
,1/2*I*2^(1/2))*x^2-60261250780500*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(
4*x+1)^(1/2)*EllipticE(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))*x^2-24671770907
175*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticPi(2/11*(-
33*x+22)^(1/2),55/124,1/2*I*2^(1/2))*x^2+88709536171620*11^(1/2)*(-3*x+2)^(
1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(-33*x+22)^(1/2),1/2*I*2^(
1/2))*x-84365751092700*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)
*EllipticE(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))*x-34540479270045*11^(1/2)*(
-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticPi(2/11*(-33*x+22)^(1/2)
,55/124,1/2*I*2^(1/2))*x+41397783546756*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1
/2)*(4*x+1)^(1/2)*EllipticF(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))-3937068384
3260*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticE(2/11*(-
33*x+22)^(1/2),1/2*I*2^(1/2))-16118890326021*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+
```

```
5)^(1/2)*(4*x+1)^(1/2)*EllipticPi(2/11*(-33*x+22)^(1/2),55/124,1/2*I*2^(1/2
))+172175002230000*x^5+319488997250500*x^4-2276751199345150*x^3+88075894075
4000*x^2+315342410533150*x-12865932898500)/(24*x^3-70*x^2+21*x+10)/(5*x+7)^
3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}}{(5\,x+7)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^4,x, algorithm
="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}{(5\,x+7)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^4,x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**4,x)
```

```
[Out] Timed out
```

3.43 $\int \frac{\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+hx}}{a+bx}\,dx$

Optimal. Leaf size=570

$$\frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\left(3a^2dfh^2-3abh^2(cf+de)-\left(b^2(dg(fg-eh)-ch(2eh+fg))\right)\right)\text{EllipticF}\left(\sin^{-1}\left(\right.\right.}{3b^3d\sqrt{f}\,h\sqrt{e+fx}\,\sqrt{g+hx}}$$

```
[Out] 2/3*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/b-2/3*(3*a*d*f*h-b*(c*f*h+d*e
*h+d*f*g))*EllipticE(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/
(-c*h+d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(h*x+g)^(1/
2)/b^2/d/h/f^(1/2)/(f*x+e)^(1/2)/(d*(h*x+g)/(-c*h+d*g))^(1/2)+2/3*(3*a^2*d*
f*h^2-3*a*b*(c*f+d*e)*h^2-b^2*(d*g*(-e*h+f*g)-c*h*(2*e*h+f*g)))*EllipticF(f
^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*(c*
f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)/b^3/
d/h/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)-2*(-a*f+b*e)*(-a*h+b*g)*EllipticPi(
f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),-b*(-c*f+d*e)/(-a*d+b*c)/f,((-c*f+d*e
)*h/f/(-c*h+d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h
*x+g)/(-c*h+d*g))^(1/2)/b^3/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

Rubi [A] time = 1.29, antiderivative size = 570, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {161, 1607, 169, 538, 537, 158, 114, 113, 121, 120}

$$\frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\left(3a^2dfh^2-3abh^2(cf+de)+b^2(-(dg(fg-eh)-ch(2eh+fg)))\right)F\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+d}}{\sqrt{cf-de}}\right.\right.}{3b^3d\sqrt{f}\,h\sqrt{e+fx}\,\sqrt{g+hx}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(a + b*x),x]
```

```
[Out] (2*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(3*b) - (2*Sqrt[-(d*e) + c*f]
*(3*a*d*f*h - b*(d*f*g + d*e*h + c*f*h))*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sq
rt[g + h*x]*EllipticE[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], (
(d*e - c*f)*h)/(f*(d*g - c*h))])/(3*b^2*d*Sqrt[f]*h*Sqrt[e + f*x]*Sqrt[(d*(
g + h*x))/(d*g - c*h)]) + (2*Sqrt[-(d*e) + c*f]*(3*a^2*d*f*h^2 - 3*a*b*(d*e
 + c*f)*h^2 - b^2*(d*g*(f*g - e*h) - c*h*(f*g + 2*e*h)))*Sqrt[(d*(e + f*x))
/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqr
t[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(3*b^3*d
*Sqrt[f]*h*Sqrt[e + f*x]*Sqrt[g + h*x]) - (2*(b*e - a*f)*Sqrt[-(d*e) + c*f]
*(b*g - a*h)*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h)
]*EllipticPi[-((b*(d*e - c*f))/((b*c - a*d)*f)), ArcSin[(Sqrt[f]*Sqrt[c + d
*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(b^3*Sqrt[f]*Sq
rt[e + f*x]*Sqrt[g + h*x])
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d),
 0] && GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a +
b*x, e + f*x] && (PosQ[-((b*c - a*d)/d)] || NegQ[-((b*e - a*f)/f)])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 161

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[(2*(a + b*x)^(m + 1)*Sqrt[
c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*(2*m + 5)), x] + Dist[1/(b*(2*m +
5)), Int[((a + b*x)^m*Simp[3*b*c*e*g - a*(d*e*g + c*f*g + c*e*h) + 2*(b*(d*
e*g + c*f*g + c*e*h) - a*(d*f*g + d*e*h + c*f*h))*x - (3*a*d*f*h - b*(d*f*g
 + d*e*h + c*f*h))*x^2, x])/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] &&  !LtQ[m,
-1]
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

Rule 1607

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
 q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+hx}}{a+bx}\,dx = \frac{2\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+hx}}{3b} + \frac{\int \frac{3bceg-a(deg+cfg+ceh)+2(b(deg+cfg+ceh)-a(dfg+deh+c}{(a+bx)\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+}}}{3b}$$

$$= \frac{2\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+hx}}{3b} + \frac{\int \frac{2deg+2cfg-\frac{3adfg}{b}+2ceh-\frac{3adeh}{b}-\frac{3acfh}{b}+\frac{3a^2dfh}{b^2}+\left(dfg+deh\right.}{\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+hx}}}{3b}$$

$$= \frac{2\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+hx}}{3b} - \frac{(2(bc-ad)(be-af)(bg-ah))\,\text{Subst}\left(\int \frac{}{(bc-ad-}\right.}{b^3}$$

$$= \frac{2\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+hx}}{3b} - \frac{\left(2(bc-ad)(be-af)(bg-ah)\sqrt{\frac{d(e+fx)}{de-cf}}\right)\text{Subst}}{b^3\sqrt{}}$$

$$= \frac{2\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+hx}}{3b} - \frac{2\sqrt{-de+cf}\,(3adfh-b(dfg+deh+cfh))\sqrt{\frac{d}{}}}{3b^2d\sqrt{f}\,h\sqrt{e+}}$$

$$= \frac{2\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+hx}}{3b} - \frac{2\sqrt{-de+cf}\,(3adfh-b(dfg+deh+cfh))\sqrt{\frac{d}{}}}{3b^2d\sqrt{f}\,h\sqrt{e+}}$$

Mathematica [C] time = 14.19, size = 1820, normalized size = 3.19

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(a + b*x),x]
```

[Out] (2*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(3*b) + ((c + d*x)^(3/2)*(2*b^2*d*f^2*g*Sqrt[-c + (d*g)/h]*h + 2*b^2*d*e*f*Sqrt[-c + (d*g)/h]*h^2 + 2*b^2*c*f^2*Sqrt[-c + (d*g)/h]*h^2 - 6*a*b*d*f^2*Sqrt[-c + (d*g)/h]*h^2 + (2*b^2*d^3*e*f*g^2*Sqrt[-c + (d*g)/h])/(c + d*x)^2 - (2*b^2*c*d^2*f^2*g^2*Sqrt[-c + (d*g)/h])/(c + d*x)^2 + (2*b^2*d^3*e^2*g*Sqrt[-c + (d*g)/h]*h)/(c + d*x)^2 - (2*b^2*c*d^2*e*f*g*Sqrt[-c + (d*g)/h]*h)/(c + d*x)^2 - (6*a*b*d^3*e*f*g*Sqrt[-c + (d*g)/h]*h)/(c + d*x)^2 + (6*a*b*c*d^2*f^2*g*Sqrt[-c + (d*g)/h]*h)/(c + d*x)^2 - (2*b^2*c*d^2*e^2*Sqrt[-c + (d*g)/h]*h^2)/(c + d*x)^2 + (6*a*b*c*d^2*e*f*Sqrt[-c + (d*g)/h]*h^2)/(c + d*x)^2 + (2*b^2*c^3*f^2*Sqrt[-c + (d*g)/h]*h^2)/(c + d*x)^2 - (6*a*b*c^2*d*f^2*Sqrt[-c + (d*g)/h]*h^2)/(c + d*x)^2 + (2*b^2*d^2*f^2*g^2*Sqrt[-c + (d*g)/h])/(c + d*x) + (4*b^2*d^2*e*f*g*Sqrt[-c + (d*g)/h]*h)/(c + d*x) - (2*b^2*c*d*f^2*g*Sqrt[-c + (d*g)/h]*h)/(c + d*x) - (6*a*b*d^2*f^2*g*Sqrt[-c + (d*g)/h]*h)/(c + d*x) + (2*b^2*d^2*e^2*Sqrt[-c + (d*g)/h]*h^2)/(c + d*x) - (2*b^2*c*d*e*f*Sqrt[-c + (d*g)/h]*h^2)/(c + d*x) - (6*a*b*d^2*e*f*Sqrt[-c + (d*g)/h]*h^2)/(c + d*x) - (4*b^2*c^2*f^2*Sqrt[-c + (d*g)/h]*h^2)/(c + d*x) + (12*a*b*c*d*f^2*Sqrt[-c + (d*g)/h]*h^2)/(c + d*x) - ((2*I)*b*f*(-(d*g) + c*h)*(-3*a*d*f*h + b*(d*f*g + d*e*h + c*f*h))*Sqrt[1 - c/(c + d*x) + (d*e)/(f*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (d*g)/(h*(c + d*x))]*EllipticE[I*ArcSinh[Sqrt[-c + (d*g)/h]/Sqrt[c + d*x]], (d*e*h - c*f*h)/(d*f*g - c*f*h)])/Sqrt[c + d*x] - ((2*I)*b*d*f*(-(d*g) + c*h)*(-(b*f*g) - 2*b*e*h + 3*a*f*h)*Sqrt[1 - c/(c + d*x) + (d*e)/(f*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (d*g)/(h*(c + d*x))]*EllipticF[I*ArcSinh[Sqrt[-c + (d*g)/h]/Sqrt[c + d*x]], (d*e*h - c*f*h)/(d*f*g - c*f*h)])/Sqrt[c + d*x] + ((6*I)*b^2*d^2*e*f*g*h*Sqrt[1 - c/(c + d*x) + (d*e)/(f*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (d*g)/(h*(c + d*x))]*EllipticPi[-((b*c*h - a*d*h)/(b*d*g - b*c*h)), I*ArcSinh[Sqrt[-c + (d*g)/h]/Sqrt[c + d*x]], (d*e*h - c*f*h)/(d*f*g - c*f*h)])/Sqrt[c + d*x] - ((6*I)*a*b*d^2*f^2*g*h*Sqrt[1 - c/(c + d*x) + (d*e)/(f*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (d*g)/(h*(c + d*x))]*EllipticPi[-((b*c*h - a*d*h)/(b*d*g - b*c*h)), I*ArcSinh[Sqrt[-c + (d*g)/h]/Sqrt[c + d*x]], (d*e*h - c*f*h)/(d*f*g - c*f*h)])/Sqrt[c + d*x] - ((6*I)*a*b*d^2*e*f*h^2*Sqrt[1 - c/(c + d*x) + (d*e)/(f*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (d*g)/(h*(c + d*x))]*EllipticPi[-((b*c*h - a*d*h)/(b*d*g - b*c*h)), I*ArcSinh[Sqrt[-c + (d*g)/h]/Sqrt[c + d*x]], (d*e*h - c*f*h)/(d*f*g - c*f*h)])/Sqrt[c + d*x] + ((6*I)*a^2*d^2*f^2*h^2*Sqrt[1 - c/(c + d*x) + (d*e)/(f*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (d*g)/(h*(c + d*x))]*EllipticPi[-((b*c*h - a*d*h)/(b*d*g - b*c*h)), I*ArcSinh[Sqrt[-c + (d*g)/h]/Sqrt[c + d*x]], (d*e*h - c*f*h)/(d*f*g - c*f*h)])/Sqrt[c + d*x]))/(3*b^3*d^2*f*Sqrt[-c + (d*g)/h]*h*Sqrt[e + ((c + d*x)*(f - (c*f)/(c + d*x)))/d]*Sqrt[g + ((c + d*x)*(h - (c*h)/(c + d*x)))/d])

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(b*x+a),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(b*x+a),x, algorithm="giac")

[Out] integrate(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)/(b*x + a), x)

maple [B] time = 0.07, size = 3678, normalized size = 6.45

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(b*x+a),x)
```

```
[Out] 2/3*(x*b^2*c*d^2*e*f*h^2-3*((d*x+c)/(c*f-d*e)*f)^(1/2)*(-(h*x+g)*d/(c*h-d*g
))^(1/2)*(-(f*x+e)/(c*f-d*e)*d)^(1/2)*EllipticPi(((d*x+c)/(c*f-d*e)*f)^(1/2
),-(c*f-d*e)*b/f/(a*d-b*c),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*b^2*c*d^2*e*f*g
*h+((d*x+c)/(c*f-d*e)*f)^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x+e)/(c*f-
d*e)*d)^(1/2)*EllipticE(((d*x+c)/(c*f-d*e)*f)^(1/2),((c*f-d*e)*h/f/(c*h-d*g
))^(1/2))*b^2*c*d^2*e*f*g*h+3*((d*x+c)/(c*f-d*e)*f)^(1/2)*(-(h*x+g)*d/(c*h-
d*g))^(1/2)*(-(f*x+e)/(c*f-d*e)*d)^(1/2)*EllipticPi(((d*x+c)/(c*f-d*e)*f)^(
1/2),-(c*f-d*e)*b/f/(a*d-b*c),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*a*b*c*d^2*e*
f*h^2-3*((d*x+c)/(c*f-d*e)*f)^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x+e)/
(c*f-d*e)*d)^(1/2)*EllipticE(((d*x+c)/(c*f-d*e)*f)^(1/2),((c*f-d*e)*h/f/(c*
h-d*g))^(1/2))*a*b*c*d^2*e*f*h^2-3*((d*x+c)/(c*f-d*e)*f)^(1/2)*(-(h*x+g)*d/
(c*h-d*g))^(1/2)*(-(f*x+e)/(c*f-d*e)*d)^(1/2)*EllipticE(((d*x+c)/(c*f-d*e)*
f)^(1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*a*b*c*d^2*f^2*g*h+3*((d*x+c)/(c*f
-d*e)*f)^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x+e)/(c*f-d*e)*d)^(1/2)*El
lipticE(((d*x+c)/(c*f-d*e)*f)^(1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*a*b*d^
3*e*f*g*h+3*((d*x+c)/(c*f-d*e)*f)^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x
+e)/(c*f-d*e)*d)^(1/2)*EllipticPi(((d*x+c)/(c*f-d*e)*f)^(1/2),-(c*f-d*e)*b/
f/(a*d-b*c),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*a*b*c*d^2*f^2*g*h-3*((d*x+c)/(
c*f-d*e)*f)^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x+e)/(c*f-d*e)*d)^(1/2)
*EllipticPi(((d*x+c)/(c*f-d*e)*f)^(1/2),-(c*f-d*e)*b/f/(a*d-b*c),((c*f-d*e)
*h/f/(c*h-d*g))^(1/2))*a*b*d^3*e*f*g*h+x^2*b^2*c*d^2*f^2*h^2+x^2*b^2*d^3*e*
f*h^2+x^2*b^2*d^3*f^2*g*h-((d*x+c)/(c*f-d*e)*f)^(1/2)*(-(h*x+g)*d/(c*h-d*g)
)^(1/2)*(-(f*x+e)/(c*f-d*e)*d)^(1/2)*EllipticE(((d*x+c)/(c*f-d*e)*f)^(1/2),
((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*b^2*c^3*f^2*h^2+x*b^2*c*d^2*f^2*g*h+x*b^2*
d^3*e*f*g*h+x^3*b^2*d^3*f^2*h^2-3*((d*x+c)/(c*f-d*e)*f)^(1/2)*(-(h*x+g)*d/(
c*h-d*g))^(1/2)*(-(f*x+e)/(c*f-d*e)*d)^(1/2)*EllipticF(((d*x+c)/(c*f-d*e)*f
)^(1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*a*b*c^2*d*f^2*h^2+2*((d*x+c)/(c*f-
d*e)*f)^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x+e)/(c*f-d*e)*d)^(1/2)*Ell
ipticF(((d*x+c)/(c*f-d*e)*f)^(1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*b^2*c^2
*d*e*f*h^2+((d*x+c)/(c*f-d*e)*f)^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x+
e)/(c*f-d*e)*d)^(1/2)*EllipticF(((d*x+c)/(c*f-d*e)*f)^(1/2),((c*f-d*e)*h/f/
(c*h-d*g))^(1/2))*b^2*c^2*d*f^2*g*h+3*((d*x+c)/(c*f-d*e)*f)^(1/2)*(-(h*x+g)
*d/(c*h-d*g))^(1/2)*(-(f*x+e)/(c*f-d*e)*d)^(1/2)*EllipticE(((d*x+c)/(c*f-d*
e)*f)^(1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*a*b*c^2*d*f^2*h^2+3*((d*x+c)/(
c*f-d*e)*f)^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x+e)/(c*f-d*e)*d)^(1/2)
*EllipticPi(((d*x+c)/(c*f-d*e)*f)^(1/2),-(c*f-d*e)*b/f/(a*d-b*c),((c*f-d*e)
*h/f/(c*h-d*g))^(1/2))*a^2*d^3*e*f*h^2-3*((d*x+c)/(c*f-d*e)*f)^(1/2)*(-(h*x
+g)*d/(c*h-d*g))^(1/2)*(-(f*x+e)/(c*f-d*e)*d)^(1/2)*EllipticPi(((d*x+c)/(c*
f-d*e)*f)^(1/2),-(c*f-d*e)*b/f/(a*d-b*c),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*a
*b*d^3*e^2*h^2+3*((d*x+c)/(c*f-d*e)*f)^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(
-(f*x+e)/(c*f-d*e)*d)^(1/2)*EllipticPi(((d*x+c)/(c*f-d*e)*f)^(1/2),-(c*f-d*
e)*b/f/(a*d-b*c),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*b^2*d^3*e^2*g*h-((d*x+c)/
(c*f-d*e)*f)^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x+e)/(c*f-d*e)*d)^(1/2
)*EllipticF(((d*x+c)/(c*f-d*e)*f)^(1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*b^
2*c*d^2*f^2*g^2-((d*x+c)/(c*f-d*e)*f)^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-
(f*x+e)/(c*f-d*e)*d)^(1/2)*EllipticF(((d*x+c)/(c*f-d*e)*f)^(1/2),((c*f-d*e)
*h/f/(c*h-d*g))^(1/2))*b^2*d^3*e^2*g*h+((d*x+c)/(c*f-d*e)*f)^(1/2)*(-(h*x+g
)*d/(c*h-d*g))^(1/2)*(-(f*x+e)/(c*f-d*e)*d)^(1/2)*EllipticF(((d*x+c)/(c*f-d
*e)*f)^(1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*b^2*d^3*e*f*g^2+((d*x+c)/(c*f
-d*e)*f)^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x+e)/(c*f-d*e)*d)^(1/2)*El
lipticE(((d*x+c)/(c*f-d*e)*f)^(1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*b^2*c*
d^2*e^2*h^2+((d*x+c)/(c*f-d*e)*f)^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x
```

```
+e)/(c*f-d*e)*d)^(1/2)*EllipticE(((d*x+c)/(c*f-d*e)*f)^(1/2),((c*f-d*e)*h/f
/(c*h-d*g))^(1/2))*b^2*c*d^2*f^2*g^2-((d*x+c)/(c*f-d*e)*f)^(1/2)*(-(h*x+g)*
d/(c*h-d*g))^(1/2)*(-(f*x+e)/(c*f-d*e)*d)^(1/2)*EllipticE(((d*x+c)/(c*f-d*e
)*f)^(1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*b^2*d^3*e^2*g*h-((d*x+c)/(c*f-d
*e)*f)^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x+e)/(c*f-d*e)*d)^(1/2)*Elli
pticE(((d*x+c)/(c*f-d*e)*f)^(1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*b^2*d^3*
e*f*g^2-3*((d*x+c)/(c*f-d*e)*f)^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x+e
)/(c*f-d*e)*d)^(1/2)*EllipticPi(((d*x+c)/(c*f-d*e)*f)^(1/2),-(c*f-d*e)*b/f/
(a*d-b*c),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*a^2*c*d^2*f^2*h^2+3*((d*x+c)/(c*
f-d*e)*f)^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x+e)/(c*f-d*e)*d)^(1/2)*E
llipticF(((d*x+c)/(c*f-d*e)*f)^(1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*a^2*c
*d^2*f^2*h^2-3*((d*x+c)/(c*f-d*e)*f)^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(
f*x+e)/(c*f-d*e)*d)^(1/2)*EllipticF(((d*x+c)/(c*f-d*e)*f)^(1/2),((c*f-d*e)*
h/f/(c*h-d*g))^(1/2))*a^2*d^3*e*f*h^2+3*((d*x+c)/(c*f-d*e)*f)^(1/2)*(-(h*x+
g)*d/(c*h-d*g))^(1/2)*(-(f*x+e)/(c*f-d*e)*d)^(1/2)*EllipticF(((d*x+c)/(c*f-
d*e)*f)^(1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*a*b*d^3*e^2*h^2-2*((d*x+c)/(
c*f-d*e)*f)^(1/2)*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x+e)/(c*f-d*e)*d)^(1/2)
*EllipticF(((d*x+c)/(c*f-d*e)*f)^(1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*b^2
*c*d^2*e^2*h^2+b^2*c*d^2*e*f*g*h)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)
/b^3/h/f/d^2/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g
*x+c*e*g)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(b*x+a),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)/(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2))/(a + b*x),x)
```

```
[Out] int(((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2))/(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+hx}}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*(f*x+e)**(1/2)*(h*x+g)**(1/2)/(b*x+a),x)
```

```
[Out] Integral(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)/(a + b*x), x)
```

3.44 $\int \frac{\sqrt{2-3x}\,\sqrt{1+4x}\,(7+5x)^3}{\sqrt{-5+2x}}\,dx$

Optimal. Leaf size=243

$$-\frac{2161804579\sqrt{\frac{11}{6}}\sqrt{5-2x}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{54432\sqrt{2x-5}}+\frac{1}{9}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\,(5x+7)^3+\frac{1679}{756}\sqrt{2}$$

```
[Out] -2161804579/326592*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1
/2)*(5-2*x)^(1/2)/(-5+2*x)^(1/2)+2629157597/163296*EllipticE(2/11*(2-3*x)^(
1/2)*11^(1/2),1/2*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1/2)+46134551
/38880*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)+26291/540*(7+5*x)*(2-3*x)
^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)+1679/756*(7+5*x)^2*(2-3*x)^(1/2)*(-5+2*
x)^(1/2)*(1+4*x)^(1/2)+1/9*(7+5*x)^3*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(
1/2)
```

Rubi [A] time = 0.30, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {162, 1600, 1615, 158, 114, 113, 121, 119}

$$\frac{1}{9}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\,(5x+7)^3+\frac{1679}{756}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\,(5x+7)^2+\frac{26291}{540}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\,(5$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7 + 5*x)^3)/Sqrt[-5 + 2*x],x]
```

```
[Out] (46134551*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/38880 + (26291*Sqrt[2
 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x))/540 + (1679*Sqrt[2 - 3*x]*S
qrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2)/756 + (Sqrt[2 - 3*x]*Sqrt[-5 + 2*x
]*Sqrt[1 + 4*x]*(7 + 5*x)^3)/9 + (2629157597*Sqrt[11]*Sqrt[-5 + 2*x]*Ellipt
icE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(163296*Sqrt[5 - 2*x]) - (21
61804579*Sqrt[11/6]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]
], 1/3])/(54432*Sqrt[-5 + 2*x])
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
```

```
0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 162

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*
(x_)])/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(2*(a + b*x)^m*Sqrt[c +
d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(d*(2*m + 3)), x] - Dist[1/(d*(2*m + 3)),
 Int[((a + b*x)^(m - 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[2
*b*c*e*g*m + a*(c*(f*g + e*h) - 2*d*e*g*(m + 1)) - (b*(2*d*e*g - c*(f*g + e
*h)*(2*m + 1)) - a*(2*c*f*h - d*(2*m + 1)*(f*g + e*h)))*x - (2*a*d*f*h*m +
b*(d*(f*g + e*h) - 2*c*f*h*(m + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, m}, x] && IntegerQ[2*m] && GtQ[m, 0]
```

Rule 1600

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[(2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/
(d*f*h*(2*m + 3)), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}(7+5x)^3}{\sqrt{-5+2x}}\,dx = \frac{1}{9}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3 - \frac{1}{18}\int \frac{(7+5x)^2\left(-699-565x+335\right.}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4}}$$

$$= \frac{1679}{756}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2 + \frac{1}{9}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7$$

$$= \frac{26291}{540}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) + \frac{1679}{756}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4}$$

$$= \frac{46134551\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{38880} + \frac{26291}{540}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7$$

$$= \frac{46134551\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{38880} + \frac{26291}{540}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7$$

$$= \frac{46134551\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{38880} + \frac{26291}{540}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7$$

$$= \frac{46134551\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{38880} + \frac{26291}{540}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7$$

Mathematica [A] time = 0.39, size = 130, normalized size = 0.53

$$\frac{-2161804579\sqrt{66}\sqrt{5-2x}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)+6\sqrt{2-3x}\sqrt{4x+1}\left(1512000x^4+8614800x^3+\right.}{326592\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7 + 5*x)^3)/Sqrt[-5 + 2*x],x]
```

```
[Out] (6*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(-455686385 + 51484034*x + 21329208*x^2 + 86
14800*x^3 + 1512000*x^4) + 2629157597*Sqrt[66]*Sqrt[5 - 2*x]*EllipticE[ArcS
in[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3] - 2161804579*Sqrt[66]*Sqrt[5 - 2*x]*Elli
pticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(326592*Sqrt[-5 + 2*x])
```

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(125\,x^3+525\,x^2+735\,x+343\right)\sqrt{4\,x+1}\sqrt{-3\,x+2}}{\sqrt{2\,x-5}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^3*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((125*x^3 + 525*x^2 + 735*x + 343)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqr
t(2*x - 5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5\,x+7)^3\sqrt{4\,x+1}\sqrt{-3\,x+2}}{\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((7+5*x)^3*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm ="giac")

[Out] integrate((5*x + 7)^3*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)

maple [A] time = 0.03, size = 155, normalized size = 0.64

$$-\frac{\sqrt{-3x+2}\,\sqrt{4x+1}\,\sqrt{2x-5}\,\left(-108864000x^6-574905600x^5-1259114976x^4-2963596608x^3+34609891\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x+7)^3*(-3*x+2)^(1/2)*(4*x+1)^(1/2)/(2*x-5)^(1/2),x)

[Out] -1/326592*(-3*x+2)^(1/2)*(4*x+1)^(1/2)*(2*x-5)^(1/2)*(-108864000*x^6+6485413737*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))-5258315194*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticE(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))-574905600*x^5-1259114976*x^4-2963596608*x^3+34609891236*x^2-13052783142*x-5468236620)/(24*x^3-70*x^2+21*x+10)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x+7)^3\sqrt{4x+1}\,\sqrt{-3x+2}}{\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((7+5*x)^3*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm ="maxima")

[Out] integrate((5*x + 7)^3*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2-3x}\,\sqrt{4x+1}\,(5x+7)^3}{\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(5*x + 7)^3)/(2*x - 5)^(1/2),x)

[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(5*x + 7)^3)/(2*x - 5)^(1/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((7+5*x)**3*(2-3*x)**(1/2)*(1+4*x)**(1/2)/(-5+2*x)**(1/2),x)

[Out] Timed out

3.45 $\int \frac{\sqrt{2-3x}\,\sqrt{1+4x}\,(7+5x)^2}{\sqrt{-5+2x}}\,dx$

Optimal. Leaf size=205

$$-\frac{1679161\sqrt{\frac{11}{6}}\sqrt{5-2x}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{756\sqrt{2x-5}}+\frac{1}{7}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\,(5x+7)^2+\frac{173}{60}\sqrt{2-3x}$$

```
[Out] -1679161/4536*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(
5-2*x)^(1/2)/(-5+2*x)^(1/2)+8198333/9072*EllipticE(2/11*(2-3*x)^(1/2)*11^(1
/2),1/2*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1/2)+73207/1080*(2-3*x)
^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)+173/60*(7+5*x)*(2-3*x)^(1/2)*(-5+2*x)^(
1/2)*(1+4*x)^(1/2)+1/7*(7+5*x)^2*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {162, 1600, 1615, 158, 114, 113, 121, 119}

$$\frac{1}{7}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\,(5x+7)^2+\frac{173}{60}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\,(5x+7)+\frac{73207\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{1080}-\frac{16}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2)/Sqrt[-5 + 2*x],x]
```

```
[Out] (73207*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/1080 + (173*Sqrt[2 - 3*x
]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x))/60 + (Sqrt[2 - 3*x]*Sqrt[-5 + 2*x
]*Sqrt[1 + 4*x]*(7 + 5*x)^2)/7 + (8198333*Sqrt[11]*Sqrt[-5 + 2*x]*EllipticE
[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(9072*Sqrt[5 - 2*x]) - (1679161
*Sqrt[11/6]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])
/(756*Sqrt[-5 + 2*x])
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
```

```
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 162

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*
(x_)])/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(2*(a + b*x)^m*Sqrt[c +
d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(d*(2*m + 3)), x] - Dist[1/(d*(2*m + 3)),
 Int[((a + b*x)^(m - 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[2
*b*c*e*g*m + a*(c*(f*g + e*h) - 2*d*e*g*(m + 1)) - (b*(2*d*e*g - c*(f*g + e
*h)*(2*m + 1)) - a*(2*c*f*h - d*(2*m + 1)*(f*g + e*h)))*x - (2*a*d*f*h*m +
b*(d*(f*g + e*h) - 2*c*f*h*(m + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, m}, x] && IntegerQ[2*m] && GtQ[m, 0]
```

Rule 1600

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[(2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/
(d*f*h*(2*m + 3)), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\,\sqrt{1+4x}\,(7+5x)^2}{\sqrt{-5+2x}}\,dx = \frac{1}{7}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^2 - \frac{1}{14}\int \frac{(7+5x)\left(-543-175x+242\right.}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4}}$$

$$= \frac{173}{60}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x) + \frac{1}{7}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+$$

$$= \frac{73207\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{1080} + \frac{173}{60}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)$$

$$= \frac{73207\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{1080} + \frac{173}{60}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)$$

$$= \frac{73207\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{1080} + \frac{173}{60}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)$$

$$= \frac{73207\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{1080} + \frac{173}{60}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)$$

Mathematica [A] time = 0.32, size = 125, normalized size = 0.61

$$\frac{-6716644\sqrt{66}\,\sqrt{5-2x}\ \text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{4x+1}\right),\frac{1}{3}\right) + 12\sqrt{2-3x}\,\sqrt{4x+1}\,\left(10800x^3+46836x^2+10259\right.}{18144\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2)/Sqrt[-5 + 2*x],x]
```

```
[Out] (12*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(-717955 + 102592*x + 46836*x^2 + 10800*x^3
) + 8198333*Sqrt[66]*Sqrt[5 - 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x
]], 1/3] - 6716644*Sqrt[66]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[
1 + 4*x]], 1/3])/(18144*Sqrt[-5 + 2*x])
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(25\,x^2+70\,x+49\right)\sqrt{4\,x+1}\,\sqrt{-3\,x+2}}{\sqrt{2\,x-5}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^2*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((25*x^2 + 70*x + 49)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5\,x+7)^2\sqrt{4\,x+1}\,\sqrt{-3\,x+2}}{\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^2*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((5*x + 7)^2*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

maple [A] time = 0.02, size = 150, normalized size = 0.73

$$-\frac{\sqrt{-3x+2}\ \sqrt{4x+1}\ \sqrt{2x-5}\ \left(-777600x^5-3048192x^4-5851944x^3+55332552x^2-20307546x-8198333\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x+7)^2*(-3*x+2)^(1/2)*(4*x+1)^(1/2)/(2*x-5)^(1/2),x)
```

```
[Out] -1/9072*(-3*x+2)^(1/2)*(4*x+1)^(1/2)*(2*x-5)^(1/2)*(10074966*11^(1/2)*(-3*x
+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(-33*x+22)^(1/2),1/2*
I*2^(1/2))-8198333*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*Ell
ipticE(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))-777600*x^5-3048192*x^4-5851944*
x^3+55332552*x^2-20307546*x-8615460)/(24*x^3-70*x^2+21*x+10)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5\,x+7)^2\sqrt{4\,x+1}\ \sqrt{-3\,x+2}}{\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^2*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((5*x + 7)^2*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2-3\,x}\ \sqrt{4\,x+1}\ (5\,x+7)^2}{\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(5*x + 7)^2)/(2*x - 5)^(1/2),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(5*x + 7)^2)/(2*x - 5)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**2*(2-3*x)**(1/2)*(1+4*x)**(1/2)/(-5+2*x)**(1/2),x)
```

```
[Out] Timed out
```

3.46 $\int \frac{\sqrt{2-3x}\,\sqrt{1+4x}\,(7+5x)}{\sqrt{-5+2x}}\,dx$

Optimal. Leaf size=162

$$-\frac{4543\sqrt{\frac{11}{6}}\,\sqrt{5-2x}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{4x+1}\right),\frac{1}{3}\right)}{36\sqrt{2x-5}}+\frac{1}{4}\sqrt{2-3x}\,\sqrt{2x-5}\,(4x+1)^{3/2}+\frac{95}{18}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x}$$

```
[Out] -4543/216*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*
x)^(1/2)/(-5+2*x)^(1/2)+1/4*(1+4*x)^(3/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)+1397
/27*EllipticE(2/11*(2-3*x)^(1/2)*11^(1/2),1/2*I*2^(1/2))*11^(1/2)*(-5+2*x)^
(1/2)/(5-2*x)^(1/2)+95/18*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {154, 158, 114, 113, 121, 119}

$$\frac{1}{4}\sqrt{2-3x}\,\sqrt{2x-5}\,(4x+1)^{3/2}+\frac{95}{18}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}-\frac{4543\sqrt{\frac{11}{6}}\,\sqrt{5-2x}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{4x+1}\right)|\frac{1}{3}\right)}{36\sqrt{2x-5}}+\frac{139}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7 + 5*x))/Sqrt[-5 + 2*x],x]
```

```
[Out] (95*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/18 + (Sqrt[2 - 3*x]*Sqrt[-5
 + 2*x]*(1 + 4*x)^(3/2))/4 + (1397*Sqrt[11]*Sqrt[-5 + 2*x]*EllipticE[ArcSin
[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(27*Sqrt[5 - 2*x]) - (4543*Sqrt[11/6]*
Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(36*Sqrt[-5
 + 2*x])
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] && !LtQ[-((b*c - a*d)/d), 0] && !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] && !LtQ[(b*c - a*d)/b, 0])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) && !LtQ[-((b*c - a*d)/d), 0]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] && !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) && !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) && !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
```

```
-(f/d)] || PosQ[-(f/b)]))
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\,\sqrt{1+4x}\,(7+5x)}{\sqrt{-5+2x}}\,dx = \frac{1}{4}\sqrt{2-3x}\,\sqrt{-5+2x}\,(1+4x)^{3/2} + \frac{1}{20}\int \frac{\left(\frac{1065}{2}-950x\right)\sqrt{1+4x}}{\sqrt{2-3x}\,\sqrt{-5+2x}}\,dx$$

$$= \frac{95}{18}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} + \frac{1}{4}\sqrt{2-3x}\,\sqrt{-5+2x}\,(1+4x)^{3/2} - \frac{1}{180}\int$$

$$= \frac{95}{18}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} + \frac{1}{4}\sqrt{2-3x}\,\sqrt{-5+2x}\,(1+4x)^{3/2} - \frac{1397}{9}\int$$

$$= \frac{95}{18}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} + \frac{1}{4}\sqrt{2-3x}\,\sqrt{-5+2x}\,(1+4x)^{3/2} - \frac{\left(4543\sqrt{\right.}}{}$$

$$= \frac{95}{18}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} + \frac{1}{4}\sqrt{2-3x}\,\sqrt{-5+2x}\,(1+4x)^{3/2} + \frac{1397\sqrt{}}{}$$

Mathematica [A] time = 0.21, size = 120, normalized size = 0.74

$$\frac{-4543\sqrt{66}\,\sqrt{5-2x}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{4x+1}\right),\frac{1}{3}\right) + 6\sqrt{2-3x}\,\sqrt{4x+1}\,\left(72x^2+218x-995\right) + 5588\sqrt{6}}{216\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7 + 5*x))/Sqrt[-5 + 2*x],x]
```

[Out] (6*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(-995 + 218*x + 72*x^2) + 5588*Sqrt[66]*Sqrt[5 - 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3] - 4543*Sqrt[66]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(216*Sqrt[-5 + 2*x])

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(5\,x+7)\sqrt{4\,x+1}\,\sqrt{-3\,x+2}}{\sqrt{2\,x-5}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((7+5*x)*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm="fricas")

[Out] integral((5*x + 7)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5\,x+7)\sqrt{4\,x+1}\,\sqrt{-3\,x+2}}{\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((7+5*x)*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm="giac")

[Out] integrate((5*x + 7)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)

maple [A] time = 0.01, size = 145, normalized size = 0.90

$$-\frac{\sqrt{-3x+2}\,\sqrt{4x+1}\,\sqrt{2x-5}\,\left(-5184x^4-13536x^3+79044x^2-27234x-11176\sqrt{11}\,\sqrt{-3x+2}\,\sqrt{-2x+5}\,\sqrt{4x}\right.}{216\left(24x^3-7\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x+7)*(-3*x+2)^(1/2)*(4*x+1)^(1/2)/(2*x-5)^(1/2),x)

[Out] -1/216*(-3*x+2)^(1/2)*(4*x+1)^(1/2)*(2*x-5)^(1/2)*(13629*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))-11176*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticE(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))-5184*x^4-13536*x^3+79044*x^2-27234*x-11940)/(24*x^3-70*x^2+21*x+10)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5\,x+7)\sqrt{4\,x+1}\,\sqrt{-3\,x+2}}{\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((7+5*x)*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm="maxima")

[Out] integrate((5*x + 7)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\left(5\,x+7\right)}{\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(5*x + 7))/(2*x - 5)^(1/2),x)
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(5*x + 7))/(2*x - 5)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)*(2-3*x)**(1/2)*(1+4*x)**(1/2)/(-5+2*x)**(1/2),x)
[Out] Timed out
```

3.47 $\int \frac{\sqrt{2-3x}\,\sqrt{1+4x}}{\sqrt{-5+2x}}\,dx$

Optimal. Leaf size=131

$$-\frac{11\sqrt{\frac{22}{3}}\,\sqrt{5-2x}\ \text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{4x+1}\right),\frac{1}{3}\right)}{3\sqrt{2x-5}}+\frac{1}{3}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}+\frac{55\sqrt{11}\,\sqrt{2x-5}\,E\left(\sin^{-1}\left(\frac{2\cdot}{}\right.\right.}{18\sqrt{5-2x}}$$

```
[Out] -11/9*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*x)^(
1/2)/(-5+2*x)^(1/2)+55/18*EllipticE(2/11*(2-3*x)^(1/2)*11^(1/2),1/2*I*2^(1/
2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1/2)+1/3*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*
(1+4*x)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {101, 158, 114, 113, 121, 119}

$$\frac{1}{3}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}-\frac{11\sqrt{\frac{22}{3}}\,\sqrt{5-2x}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{4x+1}\right)|\frac{1}{3}\right)}{3\sqrt{2x-5}}+\frac{55\sqrt{11}\,\sqrt{2x-5}\,E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\right.}{18\sqrt{5-2x}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x],x]
```

```
[Out] (Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/3 + (55*Sqrt[11]*Sqrt[-5 + 2*x
]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(18*Sqrt[5 - 2*x]) -
 (11*Sqrt[22/3]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1
/3])/(3*Sqrt[-5 + 2*x])
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[((a + b*x)^m*(c + d*x)^n*(e + f*x)^(p + 1))/(f
*(m + n + p + 1)), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}}\,dx = \frac{1}{3}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} - \frac{1}{3}\int \frac{-\frac{33}{2}+55x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$$

$$= \frac{1}{3}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} - \frac{55}{6}\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}}\,dx - \frac{121}{3}\int \frac{1}{\sqrt{2-3x}\sqrt{-5}\cdots}$$

$$= \frac{1}{3}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} - \frac{\left(11\sqrt{22}\sqrt{5-2x}\right)\int \frac{1}{\sqrt{2-3x}\sqrt{\frac{10}{11}-\frac{4x}{11}}\sqrt{1+4x}}\,dx}{3\sqrt{-5+2x}} - \frac{\left(55\sqrt{\cdots}\right.}{\cdots}$$

$$= \frac{1}{3}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} + \frac{55\sqrt{11}\sqrt{-5+2x}\,E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{18\sqrt{5-2x}} - \frac{11\sqrt{\frac{22}{3}}\cdots}{\cdots}$$

Mathematica [A] time = 0.20, size = 115, normalized size = 0.88

$$\frac{-44\sqrt{66}\sqrt{5-2x}\text{ EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)+12\sqrt{2-3x}\sqrt{4x+1}(2x-5)+55\sqrt{66}\sqrt{5-2x}\,E\left(\sin\cdots\right.}{36\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x],x]
```

```
[Out] (12*Sqrt[2 - 3*x]*(-5 + 2*x)*Sqrt[1 + 4*x] + 55*Sqrt[66]*Sqrt[5 - 2*x]*Elli
pticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3] - 44*Sqrt[66]*Sqrt[5 - 2*x]*El
lipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(36*Sqrt[-5 + 2*x])
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{4\,x+1}\sqrt{-3\,x+2}}{\sqrt{2\,x-5}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{4\,x+1}\,\sqrt{-3\,x+2}}{\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

maple [A] time = 0.01, size = 140, normalized size = 1.07

$$\frac{\sqrt{-3x+2}\,\sqrt{4x+1}\,\sqrt{2x-5}\,\left(144x^3-420x^2+126x+55\sqrt{11}\,\sqrt{-3x+2}\,\sqrt{-2x+5}\,\sqrt{4x+1}\,\operatorname{EllipticE}\left(\frac{2\sqrt{-33x+}}{11}\right.\right.}{432x^3-1260x^2+378x+180}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x+2)^(1/2)*(4*x+1)^(1/2)/(2*x-5)^(1/2),x)
[Out] 1/18*(-3*x+2)^(1/2)*(4*x+1)^(1/2)*(2*x-5)^(1/2)*(55*11^(1/2)*(-3*x+2)^(1/2)
*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticE(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2)
)-66*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(-
33*x+22)^(1/2),1/2*I*2^(1/2))+144*x^3-420*x^2+126*x+60)/(24*x^3-70*x^2+21*x
+10)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{4\,x+1}\,\sqrt{-3\,x+2}}{\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}}{\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/(2*x - 5)^(1/2),x)
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/(2*x - 5)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2-3x}\,\sqrt{4x+1}}{\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)/(-5+2*x)**(1/2),x)
[Out] Integral(sqrt(2 - 3*x)*sqrt(4*x + 1)/sqrt(2*x - 5), x)
```

3.48 $\int \frac{\sqrt{2-3x}\,\sqrt{1+4x}}{\sqrt{-5+2x}\,(7+5x)}\,dx$

Optimal. Leaf size=151

$$-\frac{41\sqrt{\frac{2}{33}}\,\sqrt{5-2x}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{4x+1}\right),\frac{1}{3}\right)}{25\sqrt{2x-5}}+\frac{2\sqrt{11}\,\sqrt{2x-5}\,E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{5\sqrt{5-2x}}+\frac{69\sqrt{5-2x}\,\Pi}{2}$$

```
[Out] -41/825*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*x)
^(1/2)/(-5+2*x)^(1/2)+69/275*EllipticPi(2/11*(2-3*x)^(1/2)*11^(1/2),55/124,
1/2*I*2^(1/2))*(5-2*x)^(1/2)*11^(1/2)/(-5+2*x)^(1/2)+2/5*EllipticE(2/11*(2-
3*x)^(1/2)*11^(1/2),1/2*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {163, 168, 538, 537, 158, 114, 113, 121, 119}

$$-\frac{41\sqrt{\frac{2}{33}}\,\sqrt{5-2x}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{4x+1}\right)|\frac{1}{3}\right)}{25\sqrt{2x-5}}+\frac{2\sqrt{11}\,\sqrt{2x-5}\,E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{5\sqrt{5-2x}}+\frac{69\sqrt{5-2x}\,\Pi\left(\frac{55}{124};\sin\right.}{25\sqrt{11}\,\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)),x]
```

```
[Out] (2*Sqrt[11]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1
/2])/(5*Sqrt[5 - 2*x]) - (41*Sqrt[2/33]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt
[3/11]*Sqrt[1 + 4*x]], 1/3])/(25*Sqrt[-5 + 2*x]) + (69*Sqrt[5 - 2*x]*Ellipt
icPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(25*Sqrt[11]*Sqrt[-
5 + 2*x])
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
```

```
-(f/d)] || PosQ[-(f/b)]))
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 163

```
Int[(Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)])/(((a_.) + (b_.)*(x_
))*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[((b*e - a*f)*(b*g - a*h))/b
^2, Int[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + D
ist[1/b^2, Int[Simp[b*f*g + b*e*h - a*f*h + b*f*h*x, x]/(Sqrt[c + d*x]*Sqrt
[e + f*x]*Sqrt[g + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)}\,dx = \frac{1}{25}\int \frac{109-60x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx - \frac{713}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}$$

$$= -\left(\frac{6}{5}\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}}\,dx\right) - \frac{41}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx + \frac{1426}{25}\text{Sub}$$

$$= -\frac{\left(41\sqrt{\frac{2}{11}}\sqrt{5-2x}\right)\int \frac{1}{\sqrt{2-3x}\sqrt{\frac{10}{11}-\frac{4x}{11}}\sqrt{1+4x}}\,dx}{25\sqrt{-5+2x}} + \frac{\left(1426\sqrt{\frac{3}{11}}\sqrt{5-2x}\right)\text{Subst}\left(\int \frac{}{(31-}\right.}{25\sqrt{-}}$$

$$= \frac{2\sqrt{11}\sqrt{-5+2x}\,E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{5\sqrt{5-2x}} - \frac{41\sqrt{\frac{2}{33}}\sqrt{5-2x}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right.\right.}{25\sqrt{-5+2x}}$$

Mathematica [A] time = 0.42, size = 95, normalized size = 0.63

$$\frac{\sqrt{5-2x}\left(41\,\text{EllipticF}\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)-110E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)+69\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)\right)}{25\sqrt{22x-55}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)),x]
```

```
[Out] (Sqrt[5 - 2*x]*(-110*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] +
41*EllipticF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 69*EllipticPi[55/1
24, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/(25*Sqrt[-55 + 22*x])
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{10x^2-11x-35},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)/(-5+2*x)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(10*x^2 - 11*x - 35), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{4x+1}\sqrt{-3x+2}}{(5x+7)\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)/(-5+2*x)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)*sqrt(2*x - 5)), x)
```

maple [A] time = 0.02, size = 76, normalized size = 0.50

$$\frac{\left(-110\,\text{EllipticE}\left(\frac{2\sqrt{-33x+22}}{11},\frac{i\sqrt{2}}{2}\right)+41\,\text{EllipticF}\left(\frac{2\sqrt{-33x+22}}{11},\frac{i\sqrt{2}}{2}\right)+69\,\text{EllipticPi}\left(\frac{2\sqrt{-33x+22}}{11},\frac{55}{124},\frac{i\sqrt{2}}{2}\right)\right)\sqrt{-2x}}{275\sqrt{2x-5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x+2)^(1/2)*(4*x+1)^(1/2)/(5*x+7)/(2*x-5)^(1/2),x)
```

```
[Out] 1/275*(41*EllipticF(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))-110*EllipticE(2/11
*(-33*x+22)^(1/2),1/2*I*2^(1/2))+69*EllipticPi(2/11*(-33*x+22)^(1/2),55/124
,1/2*I*2^(1/2)))*(-2*x+5)^(1/2)*11^(1/2)/(2*x-5)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{4x+1}\sqrt{-3x+2}}{(5x+7)\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)/(-5+2*x)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)*sqrt(2*x - 5)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2-3x}\,\sqrt{4x+1}}{\sqrt{2x-5}\,(5x+7)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)/(-5+2*x)**(1/2),x)
```

```
[Out] Timed out
```

3.49 $\int \frac{\sqrt{2-3x}\,\sqrt{1+4x}}{\sqrt{-5+2x}\,(7+5x)^2}\,dx$

Optimal. Leaf size=189

$$-\frac{2\sqrt{\frac{6}{11}}\,\sqrt{5-2x}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{4x+1}\right),\frac{1}{3}\right)}{25\sqrt{2x-5}}+\frac{\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{39(5x+7)}-\frac{2\sqrt{11}\,\sqrt{2x-5}\,E\left(\sin^{-1}\left(\frac{2\sqrt{}}{}\right.\right.}{195\sqrt{5-2x}}$$

```
[Out] -2/275*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*x)^
(1/2)/(-5+2*x)^(1/2)-6101/221650*EllipticPi(2/11*(2-3*x)^(1/2)*11^(1/2),55/
124,1/2*I*2^(1/2))*(5-2*x)^(1/2)*11^(1/2)/(-5+2*x)^(1/2)-2/195*EllipticE(2/
11*(2-3*x)^(1/2)*11^(1/2),1/2*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1
/2)+1/39*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)
```

Rubi [A] time = 0.21, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {164, 1607, 168, 538, 537, 158, 114, 113, 121, 119}

$$\frac{\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{39(5x+7)}-\frac{2\sqrt{\frac{6}{11}}\,\sqrt{5-2x}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{4x+1}\right)|\frac{1}{3}\right)}{25\sqrt{2x-5}}-\frac{2\sqrt{11}\,\sqrt{2x-5}\,E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right.}{195\sqrt{5-2x}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^2),x]
```

```
[Out] (Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(39*(7 + 5*x)) - (2*Sqrt[11]*S
qrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(195*Sqr
t[5 - 2*x]) - (2*Sqrt[6/11]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[
1 + 4*x]], 1/3])/(25*Sqrt[-5 + 2*x]) - (6101*Sqrt[5 - 2*x]*EllipticPi[55/12
4, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(20150*Sqrt[11]*Sqrt[-5 + 2*x
])
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] && !LtQ[-((b*c - a*d)/d), 0] && !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] && !LtQ[(b*c - a*d)/b, 0])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) && !LtQ[-((b*c - a*d)/d), 0]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] && !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) && !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) && !(SimplerQ[e + f*x,
```

```
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 164

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*
(x_)])/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[((a + b*x)^(m + 1)*Sqrt[
c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((m + 1)*(b*c - a*d)), x] - Dist[1/(2
*(m + 1)*(b*c - a*d)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*
Sqrt[g + h*x]))*Simp[c*(f*g + e*h) + d*e*g*(2*m + 3) + 2*(c*f*h + d*(m + 2)
*(f*g + e*h))*x + d*f*h*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 1607

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
```

```
q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^2}\,dx = \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{39(7+5x)} - \frac{1}{78}\int \frac{-29+120x-24x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx$$

$$= \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{39(7+5x)} - \frac{1}{78}\int \frac{\frac{768}{25}-\frac{24x}{5}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx + \frac{6101\int \frac{}{\sqrt{2}}}{}$$

$$= \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{39(7+5x)} + \frac{2}{65}\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}}\,dx - \frac{6}{25}\int \frac{1}{\sqrt{2-3x}\sqrt{-5}}$$

$$= \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{39(7+5x)} - \frac{\left(6\sqrt{\frac{2}{11}}\sqrt{5-2x}\right)\int \frac{1}{\sqrt{2-3x}\sqrt{\frac{10}{11}-\frac{4x}{11}}\sqrt{1+4x}}\,dx}{25\sqrt{-5+2x}} - \frac{(6101}{}$$

$$= \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{39(7+5x)} - \frac{2\sqrt{11}\sqrt{-5+2x}\,E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{195\sqrt{5-2x}} - \frac{2\sqrt{\frac{6}{11}}}{}$$

Mathematica [A] time = 0.66, size = 130, normalized size = 0.69

$$\frac{3\sqrt{55-22x}\left(14508\,\text{EllipticF}\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)+6820E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)-18303\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{2\sqrt{2}}{\sqrt{}}\right.\right.\right.}{1994850\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^2),x]
```

```
[Out] ((51150*Sqrt[2 - 3*x]*(-5 + 2*x)*Sqrt[1 + 4*x])/(7 + 5*x) + 3*Sqrt[55 - 22*
x]*(6820*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 14508*Ellipt
icF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] - 18303*EllipticPi[55/124, Ar
cSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/(1994850*Sqrt[-5 + 2*x])
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{50x^3+15x^2-252x-245}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^2/(-5+2*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(50*x^3 + 15*x^2 - 252*
x - 245), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{4x+1}\sqrt{-3x+2}}{(5x+7)^2\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^2/(-5+2*x)^(1/2),x, algorithm ="giac")

[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^2*sqrt(2*x - 5)), x)

maple [B] time = 0.02, size = 320, normalized size = 1.69

$$-\frac{\sqrt{-3x+2}\,\sqrt{4x+1}\,\sqrt{2x-5}\,\left(-409200x^3+1193500x^2+34100\sqrt{11}\,\sqrt{-3x+2}\,\sqrt{-2x+5}\,\sqrt{4x+1}\,x\,\mathrm{EllipticE}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-3*x+2)^(1/2)*(4*x+1)^(1/2)/(5*x+7)^2/(2*x-5)^(1/2),x)

[Out] -1/664950*(-3*x+2)^(1/2)*(4*x+1)^(1/2)*(2*x-5)^(1/2)*(72540*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))*x+34100*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticE(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))*x-91515*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticPi(2/11*(-33*x+22)^(1/2),55/124,1/2*I*2^(1/2))*x+101556*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))+47740*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticE(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))-128121*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticPi(2/11*(-33*x+22)^(1/2),55/124,1/2*I*2^(1/2))-409200*x^3+1193500*x^2-358050*x-170500)/(24*x^3-70*x^2+21*x+10)/(5*x+7)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{4\,x+1}\,\sqrt{-3\,x+2}}{(5\,x+7)^2\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^2/(-5+2*x)^(1/2),x, algorithm ="maxima")

[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^2*sqrt(2*x - 5)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}}{\sqrt{2\,x-5}\,(5\,x+7)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^2),x)

[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**2/(-5+2*x)**(1/2),x)

[Out] Timed out

3.50 $\int \frac{\sqrt{2-3x}\,\sqrt{1+4x}}{\sqrt{-5+2x}\,(7+5x)^3}\,dx$

Optimal. Leaf size=225

$$-\frac{6101\sqrt{5-2x}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{231725\sqrt{66}\sqrt{2x-5}}-\frac{361\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{481988(5x+7)}+\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+}}{78(5x+7)^2}$$

```
[Out] -6655867/8217895400*EllipticPi(2/11*(2-3*x)^(1/2)*11^(1/2),55/124,1/2*I*2^(
1/2))*(5-2*x)^(1/2)*11^(1/2)/(-5+2*x)^(1/2)-6101/15293850*EllipticF(1/11*33
^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*x)^(1/2)/(-5+2*x)^(1/2)+361
/1204970*EllipticE(2/11*(2-3*x)^(1/2)*11^(1/2),1/2*I*2^(1/2))*11^(1/2)*(-5+
2*x)^(1/2)/(5-2*x)^(1/2)+1/78*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7
+5*x)^2-361/481988*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)
```

Rubi [A] time = 0.30, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {164, 1604, 1607, 168, 538, 537, 158, 114, 113, 121, 119}

$$-\frac{361\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{481988(5x+7)}+\frac{\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{78(5x+7)^2}-\frac{6101\sqrt{5-2x}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{231725\sqrt{66}\sqrt{2x-5}}+\frac{361\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^3),x]
```

```
[Out] (Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(78*(7 + 5*x)^2) - (361*Sqrt[2
 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(481988*(7 + 5*x)) + (361*Sqrt[11]*Sq
rt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(1204970*
Sqrt[5 - 2*x]) - (6101*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4
*x]], 1/3])/(231725*Sqrt[66]*Sqrt[-5 + 2*x]) - (6655867*Sqrt[5 - 2*x]*Ellip
ticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(747081400*Sqrt[11
]*Sqrt[-5 + 2*x])
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] && !LtQ[-((b*c - a*d)/d), 0] && !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] && !LtQ[(b*c - a*d)/b, 0])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) && !LtQ[-((b*c - a*d)/d), 0]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] && !(SimplerQ[c + d*x, a + b*
```

```
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 164

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*
(x_)])/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[((a + b*x)^(m + 1)*Sqrt[
c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((m + 1)*(b*c - a*d)), x] - Dist[1/(2
*(m + 1)*(b*c - a*d)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*
Sqrt[g + h*x]))*Simp[c*(f*g + e*h) + d*e*g*(2*m + 3) + 2*(c*f*h + d*(m + 2)
*(f*g + e*h))*x + d*f*h*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 1604

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt
```

```
[e + f*x]*Sqrt[g + h*x])/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), x]
- Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m
 + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m +
 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
+ c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2
*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^
2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
+ c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 1607

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
 q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}\,(7+5x)^3}\,dx = \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{78(7+5x)^2} - \frac{1}{156}\int \frac{-37+100x+24x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\,(7+5x)^2}\,dx$$

$$= \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{78(7+5x)^2} - \frac{361\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{481988(7+5x)} - \frac{\int \frac{-272145+48528}{\sqrt{2-3x}\sqrt{-5+2x}}}{86755}$$

$$= \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{78(7+5x)^2} - \frac{361\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{481988(7+5x)} - \frac{\int \frac{\frac{1880568}{25}+\frac{7797}{5}}{\sqrt{2-3x}\sqrt{-5+2x}}}{8675784}$$

$$= \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{78(7+5x)^2} - \frac{361\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{481988(7+5x)} - \frac{1083\int \frac{\sqrt{-5+2}}{\sqrt{2-3x}\sqrt{1}}}{1204970}$$

$$= \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{78(7+5x)^2} - \frac{361\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{481988(7+5x)} - \frac{\left(6101\sqrt{5-2x}\right)}{231725}$$

$$= \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{78(7+5x)^2} - \frac{361\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{481988(7+5x)} + \frac{361\sqrt{11}\sqrt{-5+}}{12}$$

Mathematica [A] time = 0.58, size = 135, normalized size = 0.60

$$\frac{-3\sqrt{55-22x}\left(-9834812\,\text{EllipticF}\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)+2462020E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)+6655867\Pi\left(\frac{55}{12}\right.\right.}{24653686200\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^3),x]
```

```
[Out] ((-17050*Sqrt[2 - 3*x]*(-5 + 2*x)*Sqrt[1 + 4*x]*(-10957 + 5415*x))/(7 + 5*x
)^2 - 3*Sqrt[55 - 22*x]*(2462020*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11
]], -1/2] - 9834812*EllipticF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 6
```

```
655867*EllipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/(2465
3686200*Sqrt[-5 + 2*x])
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}}{250\,x^4+425\,x^3-1155\,x^2-2989\,x-1715}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^3/(-5+2*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(250*x^4 + 425*x^3 - 11
55*x^2 - 2989*x - 1715), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{4\,x+1}\,\sqrt{-3\,x+2}}{(5\,x+7)^3\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^3/(-5+2*x)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^3*sqrt(2*x - 5)), x)
```

maple [B] time = 0.02, size = 461, normalized size = 2.05

$$-\frac{\sqrt{-3x+2}\,\sqrt{4x+1}\,\sqrt{2x-5}\,\left(2215818000x^4-10946406900x^3-184651500\sqrt{11}\,\sqrt{-3x+2}\,\sqrt{-2x+5}\,\sqrt{4x+1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x+2)^(1/2)*(4*x+1)^(1/2)/(5*x+7)^3/(2*x-5)^(1/2),x)
```

```
[Out] -1/24653686200*(-3*x+2)^(1/2)*(4*x+1)^(1/2)*(2*x-5)^(1/2)*(737610900*11^(1/
2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(-33*x+22)^(1
/2),1/2*I*2^(1/2))*x^2-184651500*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*
x+1)^(1/2)*EllipticE(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))*x^2-499190025*11^
(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticPi(2/11*(-33*x+22
)^(1/2),55/124,1/2*I*2^(1/2))*x^2+2065310520*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+
5)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))*x-517
024200*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticE(2/11*
(-33*x+22)^(1/2),1/2*I*2^(1/2))*x-1397732070*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+
5)^(1/2)*(4*x+1)^(1/2)*EllipticPi(2/11*(-33*x+22)^(1/2),55/124,1/2*I*2^(1/2
))*x+1445717364*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*Ellipt
icF(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))-361916940*11^(1/2)*(-3*x+2)^(1/2)*
(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticE(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))
-978412449*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticPi(
2/11*(-33*x+22)^(1/2),55/124,1/2*I*2^(1/2))+2215818000*x^4-10946406900*x^3+
15016020250*x^2-2999896350*x-1868168500)/(24*x^3-70*x^2+21*x+10)/(5*x+7)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{4\,x+1}\,\sqrt{-3\,x+2}}{(5\,x+7)^3\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^3/(-5+2*x)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^3*sqrt(2*x - 5)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}}{\sqrt{2\,x-5}\,{\left(5\,x+7\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^3),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**3/(-5+2*x)**(1/2),x)
```

```
[Out] Timed out
```

3.51 $\int \frac{\sqrt{2-3x}\,(7+5x)^3}{\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx$

Optimal. Leaf size=205

$$-\frac{25260049\sqrt{\frac{11}{6}}\,\sqrt{5-2x}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{4x+1}\right),\frac{1}{3}\right)}{6048\sqrt{2x-5}}+\frac{5}{28}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(5x+7)^2+\frac{121}{24}\sqrt{2-3}$$

```
[Out] -25260049/36288*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)
*(5-2*x)^(1/2)/(-5+2*x)^(1/2)+15629623/9072*EllipticE(2/11*(2-3*x)^(1/2)*11
^(1/2),1/2*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1/2)+110743/864*(2-3
*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)+121/24*(7+5*x)*(2-3*x)^(1/2)*(-5+2*x
)^(1/2)*(1+4*x)^(1/2)+5/28*(7+5*x)^2*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(
1/2)
```

Rubi [A] time = 0.21, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {174, 1600, 1615, 158, 114, 113, 121, 119}

$$\frac{5}{28}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(5x+7)^2+\frac{121}{24}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(5x+7)+\frac{110743}{864}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}-$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*(7 + 5*x)^3)/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (110743*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/864 + (121*Sqrt[2 - 3*x
]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x))/24 + (5*Sqrt[2 - 3*x]*Sqrt[-5 + 2
*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2)/28 + (15629623*Sqrt[11]*Sqrt[-5 + 2*x]*Ellip
ticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(9072*Sqrt[5 - 2*x]) - (252
60049*Sqrt[11/6]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]],
1/3])/(6048*Sqrt[-5 + 2*x])
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
```

```
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 174

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.
)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(2*b*(a + b*x)^(m - 1)
*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(f*h*(2*m + 1)), x] - Dist[1/(f
*h*(2*m + 1)), Int[((a + b*x)^(m - 2)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]))*Simp[a*b*(d*e*g + c*(f*g + e*h)) + 2*b^2*c*e*g*(m - 1) - a^2*c*f*h*
(2*m + 1) + (b^2*(2*m - 1)*(d*e*g + c*(f*g + e*h)) - a^2*d*f*h*(2*m + 1) +
2*a*b*(d*f*g + d*e*h - 2*c*f*h*m))*x - b*(a*d*f*h*(4*m - 1) + b*(c*f*h - 2*
d*(f*g + e*h)*m))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x]
&& IntegerQ[2*m] && GtQ[m, 1]
```

Rule 1600

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[(2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/
(d*f*h*(2*m + 3)), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}(7+5x)^3}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{5}{28}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2 - \frac{1}{56}\int \frac{(7+5x)\left(-7223+2667x+16940x\right.}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}$$

$$= \frac{121}{24}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) + \frac{5}{28}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2 +$$

$$= \frac{110743}{864}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} + \frac{121}{24}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) + \frac{5}{28}$$

$$= \frac{110743}{864}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} + \frac{121}{24}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) + \frac{5}{28}$$

$$= \frac{110743}{864}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} + \frac{121}{24}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) + \frac{5}{28}$$

$$= \frac{110743}{864}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} + \frac{121}{24}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) + \frac{5}{28}$$

Mathematica [A] time = 0.37, size = 125, normalized size = 0.61

$$\frac{-25260049\sqrt{66}\sqrt{5-2x}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right) + 30\sqrt{2-3x}\sqrt{4x+1}\left(10800x^3+64224x^2+1885\right.}{36288\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*(7 + 5*x)^3)/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (30*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(-1041565 + 188566*x + 64224*x^2 + 10800*x^
3) + 31259246*Sqrt[66]*Sqrt[5 - 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4
*x]], 1/3] - 25260049*Sqrt[66]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sq
rt[1 + 4*x]], 1/3])/(36288*Sqrt[-5 + 2*x])
```

fricas [F] time = 1.05, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(125x^3+525x^2+735x+343\right)\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{8x^2-18x-5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^3*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((125*x^3 + 525*x^2 + 735*x + 343)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt
(-3*x + 2)/(8*x^2 - 18*x - 5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x+7)^3\sqrt{-3x+2}}{\sqrt{4x+1}\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^3*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((5*x + 7)^3*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

maple [A] time = 0.03, size = 150, normalized size = 0.73

$$-\frac{\sqrt{-3x+2}\,\sqrt{2x-5}\,\sqrt{4x+1}\,\left(-3888000x^5-21500640x^4-57602160x^3+407101740x^2-144920790x-62\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x+7)^3*(-3*x+2)^(1/2)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] -1/36288*(-3*x+2)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(75780147*11^(1/2)*(-3*
x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(-33*x+22)^(1/2),1/2
*I*2^(1/2))-62518492*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*E
llipticE(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))-3888000*x^5-21500640*x^4-5760
2160*x^3+407101740*x^2-144920790*x-62493900)/(24*x^3-70*x^2+21*x+10)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5\,x+7)^3\sqrt{-3\,x+2}}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^3*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((5*x + 7)^3*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2-3\,x}\,(5\,x+7)^3}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(5*x + 7)^3)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(5*x + 7)^3)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**3*(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Timed out
```

3.52 $\int \frac{\sqrt{2-3x}\,(7+5x)^2}{\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx$

Optimal. Leaf size=167

$$-\frac{17533\sqrt{\frac{11}{6}}\,\sqrt{5-2x}\,\mathrm{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{4x+1}\right),\frac{1}{3}\right)}{72\sqrt{2x-5}}+\frac{1}{4}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(5x+7)+\frac{68}{9}\sqrt{2-3x}\,\sqrt{2x}$$

```
[Out] -17533/432*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2
*x)^(1/2)/(-5+2*x)^(1/2)+44569/432*EllipticE(2/11*(2-3*x)^(1/2)*11^(1/2),1/
2*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1/2)+68/9*(2-3*x)^(1/2)*(-5+2
*x)^(1/2)*(1+4*x)^(1/2)+1/4*(7+5*x)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1
/2)
```

Rubi [A] time = 0.15, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {174, 1615, 158, 114, 113, 121, 119}

$$\frac{1}{4}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(5x+7)+\frac{68}{9}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}-\frac{17533\sqrt{\frac{11}{6}}\,\sqrt{5-2x}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{4x+1}\right)|\right.}{72\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*(7 + 5*x)^2)/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (68*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/9 + (Sqrt[2 - 3*x]*Sqrt[-5
+ 2*x]*Sqrt[1 + 4*x]*(7 + 5*x))/4 + (44569*Sqrt[11]*Sqrt[-5 + 2*x]*Elliptic
E[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(432*Sqrt[5 - 2*x]) - (17533*S
qrt[11/6]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(
72*Sqrt[-5 + 2*x])
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
```

```
-(f/d)] || PosQ[-(f/b)]))
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 174

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.
)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(2*b*(a + b*x)^(m - 1)
*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(f*h*(2*m + 1)), x] - Dist[1/(f
*h*(2*m + 1)), Int[((a + b*x)^(m - 2)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]))*Simp[a*b*(d*e*g + c*(f*g + e*h)) + 2*b^2*c*e*g*(m - 1) - a^2*c*f*h*
(2*m + 1) + (b^2*(2*m - 1)*(d*e*g + c*(f*g + e*h)) - a^2*d*f*h*(2*m + 1) +
2*a*b*(d*f*g + d*e*h - 2*c*f*h*m))*x - b*(a*d*f*h*(4*m - 1) + b*(c*f*h - 2*
d*(f*g + e*h)*m))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x]
&& IntegerQ[2*m] && GtQ[m, 1]
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\,(7+5x)^2}{\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx = \frac{1}{4}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x) - \frac{1}{40}\int \frac{-5155+3605x+10880x^2}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx$$

$$= \frac{68}{9}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} + \frac{1}{4}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x) - \frac{\int \frac{-899460}{\sqrt{2-3x}\,\sqrt{-}}}{4}$$

$$= \frac{68}{9}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} + \frac{1}{4}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x) - \frac{44569}{144}\int$$

$$= \frac{68}{9}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} + \frac{1}{4}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x) - \frac{\left(17533\sqrt{\frac{1}{2}}\right.}{}$$

$$= \frac{68}{9}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x} + \frac{1}{4}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x) + \frac{44569\sqrt{11}}{}$$

Mathematica [A] time = 0.30, size = 120, normalized size = 0.72

$$\frac{-35066\sqrt{66}\,\sqrt{5-2x}\ \text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{4x+1}\right),\frac{1}{3}\right) + 120\sqrt{2-3x}\,\sqrt{4x+1}\,\left(18x^2+89x-335\right) + 44569\sqrt{6}}{864\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*(7 + 5*x)^2)/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (120*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(-335 + 89*x + 18*x^2) + 44569*Sqrt[66]*Sq
rt[5 - 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3] - 35066*Sqrt[6
6]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(864*Sqr
t[-5 + 2*x])
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(25\,x^2+70\,x+49\right)\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}}{8\,x^2-18\,x-5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^2*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((25*x^2 + 70*x + 49)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(8
*x^2 - 18*x - 5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5\,x+7)^2\sqrt{-3\,x+2}}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^2*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((5*x + 7)^2*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

maple [A] time = 0.02, size = 145, normalized size = 0.87

$$-\frac{\sqrt{-3x+2}\,\sqrt{2x-5}\,\sqrt{4x+1}\,\left(-12960x^4-58680x^3+270060x^2-89820x-44569\sqrt{11}\,\sqrt{-3x+2}\,\sqrt{-2x+5}\right.}{432\left(24x^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x+7)^2*(-3*x+2)^(1/2)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] -1/432*(-3*x+2)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(52599*11^(1/2)*(-3*x+2)^
(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(-33*x+22)^(1/2),1/2*I*2^
(1/2))-44569*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticE
(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))-12960*x^4-58680*x^3+270060*x^2-89820*
x-40200)/(24*x^3-70*x^2+21*x+10)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5\,x+7)^2\sqrt{-3\,x+2}}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^2*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((5*x + 7)^2*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2-3\,x}\,(5\,x+7)^2}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(5*x + 7)^2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(5*x + 7)^2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2-3x}\,(5x+7)^2}{\sqrt{2x-5}\,\sqrt{4x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**2*(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*(5*x + 7)**2/(sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

3.53 $\int \frac{\sqrt{2-3x}\,(7+5x)}{\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx$

Optimal. Leaf size=131

$$-\frac{179\sqrt{\frac{11}{6}}\sqrt{5-2x}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{12\sqrt{2x-5}}+\frac{5}{12}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}+\frac{241\sqrt{11}\sqrt{2x-5}\,E\left(\sin^{-1}\right.}{36\sqrt{5-2}}$$

```
[Out] -179/72*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*x)
^(1/2)/(-5+2*x)^(1/2)+241/36*EllipticE(2/11*(2-3*x)^(1/2)*11^(1/2),1/2*I*2^
(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1/2)+5/12*(2-3*x)^(1/2)*(-5+2*x)^(1
/2)*(1+4*x)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {154, 158, 114, 113, 121, 119}

$$\frac{5}{12}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}-\frac{179\sqrt{\frac{11}{6}}\sqrt{5-2x}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{12\sqrt{2x-5}}+\frac{241\sqrt{11}\sqrt{2x-5}\,E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)\right.}{36\sqrt{5-2x}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*(7 + 5*x))/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (5*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/12 + (241*Sqrt[11]*Sqrt[-5 +
 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(36*Sqrt[5 - 2*x
]) - (179*Sqrt[11/6]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x
]], 1/3])/(12*Sqrt[-5 + 2*x])
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{2-3x}\,(7+5x)}{\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx &= \frac{5}{12}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}+\frac{1}{12}\int \frac{\frac{441}{2}-482x}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx\\ &= \frac{5}{12}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}-\frac{241}{12}\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\,\sqrt{1+4x}}\,dx-\frac{1969}{24}\int \frac{}{\sqrt{2-3x}\,\cdots}\\ &= \frac{5}{12}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}-\frac{\left(179\sqrt{\frac{11}{2}}\,\sqrt{5-2x}\right)\int \frac{1}{\sqrt{2-3x}\,\sqrt{\frac{10}{11}-\frac{4x}{11}}\,\sqrt{1+4x}}\,dx}{12\sqrt{-5+2x}}-\cdots\\ &= \frac{5}{12}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}+\frac{241\sqrt{11}\,\sqrt{-5+2x}\,E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)\middle|-\frac{1}{2}\right)}{36\sqrt{5-2x}}-\frac{17\cdots}{\cdots}\end{aligned}$$

Mathematica [A] time = 0.19, size = 115, normalized size = 0.88

$$\frac{-179\sqrt{66}\,\sqrt{5-2x}\ \text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{4x+1}\right),\frac{1}{3}\right)+30\sqrt{2-3x}\,\sqrt{4x+1}\,(2x-5)+241\sqrt{66}\,\sqrt{5-2x}\,E\left(\text{s}\cdots\right.}{72\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*(7 + 5*x))/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (30*Sqrt[2 - 3*x]*(-5 + 2*x)*Sqrt[1 + 4*x] + 241*Sqrt[66]*Sqrt[5 - 2*x]*Ell
ipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3] - 179*Sqrt[66]*Sqrt[5 - 2*x]*
EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(72*Sqrt[-5 + 2*x])
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(5x+7)\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{8x^2-18x-5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(8*x^2 - 18*x
 - 5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x+7)\sqrt{-3x+2}}{\sqrt{4x+1}\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((5*x + 7)*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

maple [A] time = 0.02, size = 140, normalized size = 1.07

$$-\frac{\sqrt{-3x+2}\,\sqrt{2x-5}\,\sqrt{4x+1}\,\left(-720x^3+2100x^2-630x-482\sqrt{11}\,\sqrt{-3x+2}\,\sqrt{-2x+5}\,\sqrt{4x+1}\,\operatorname{EllipticE}\left(\frac{2\sqrt{}}{}\right.\right.}{72\left(24x^3-70x^2+21x+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x+7)*(-3*x+2)^(1/2)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] -1/72*(-3*x+2)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(537*11^(1/2)*(-3*x+2)^(1/
2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/
2))-482*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticE(2/11
*(-33*x+22)^(1/2),1/2*I*2^(1/2))-720*x^3+2100*x^2-630*x-300)/(24*x^3-70*x^2
+21*x+10)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x+7)\sqrt{-3x+2}}{\sqrt{4x+1}\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((5*x + 7)*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2-3x}\,(5x+7)}{\sqrt{4x+1}\,\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(5*x + 7))/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(5*x + 7))/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2-3x}\,(5x+7)}{\sqrt{2x-5}\,\sqrt{4x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)*(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
[Out] Integral(sqrt(2 - 3*x)*(5*x + 7)/(sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

3.54 $\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx$

Optimal. Leaf size=47

$$\frac{\sqrt{\frac{11}{2}}\,\sqrt{5-2x}\,E\left(\sin^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{11}}\right)\middle|3\right)}{2\sqrt{2x-5}}$$

```
[Out] 1/4*EllipticE(1/11*(1+4*x)^(1/2)*11^(1/2),3^(1/2))*22^(1/2)*(5-2*x)^(1/2)/(
-5+2*x)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {114, 113}

$$\frac{\sqrt{\frac{11}{2}}\,\sqrt{5-2x}\,E\left(\sin^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{11}}\right)\middle|3\right)}{2\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (Sqrt[11/2]*Sqrt[5 - 2*x]*EllipticE[ArcSin[Sqrt[1 + 4*x]/Sqrt[11]], 3])/(2*
Sqrt[-5 + 2*x])
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] && !LtQ[-((b*c - a*d)/d), 0] && !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] && !LtQ[(b*c - a*d)/b, 0])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) && !LtQ[-((b*c - a*d)/d), 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx &= \frac{\sqrt{5-2x}\int \frac{\sqrt{\frac{8}{11}-\frac{12x}{11}}}{\sqrt{\frac{10}{11}-\frac{4x}{11}}\,\sqrt{1+4x}}\,dx}{\sqrt{2}\,\sqrt{-5+2x}}\\
&= \frac{\sqrt{\frac{11}{2}}\,\sqrt{5-2x}\,E\left(\sin^{-1}\left(\frac{\sqrt{1+4x}}{\sqrt{11}}\right)\middle|3\right)}{2\sqrt{-5+2x}}
\end{aligned}$$

Mathematica [B] time = 0.36, size = 111, normalized size = 2.36

$$-\frac{\frac{2(2x-5)(3x-2)}{\sqrt{2x+\frac{1}{2}}}+\sqrt{11}\sqrt{\frac{2x-5}{4x+1}}\sqrt{\frac{3x-2}{4x+1}}(4x+1)E\left(\sin^{-1}\left(\frac{\sqrt{\frac{11}{3}}}{\sqrt{4x+1}}\right)\middle|3\right)}{2\sqrt{2-3x}\sqrt{4x-10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] -1/2*((2*(-5 + 2*x)*(-2 + 3*x))/Sqrt[1/2 + 2*x] + Sqrt[11]*Sqrt[(-5 + 2*x)/
(1 + 4*x)]*Sqrt[(-2 + 3*x)/(1 + 4*x)]*(1 + 4*x)*EllipticE[ArcSin[Sqrt[11/3]
/Sqrt[1 + 4*x]], 3])/(Sqrt[2 - 3*x]*Sqrt[-10 + 4*x])
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{8x^2-18x-5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(8*x^2 - 18*x - 5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-3x+2}}{\sqrt{4x+1}\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

maple [C] time = 0.01, size = 55, normalized size = 1.17

$$\frac{\left(-\mathrm{EllipticE}\left(\frac{2\sqrt{-33x+22}}{11}, \frac{i\sqrt{2}}{2}\right)+\mathrm{EllipticF}\left(\frac{2\sqrt{-33x+22}}{11}, \frac{i\sqrt{2}}{2}\right)\right)\sqrt{-2x+5}\,\sqrt{11}}{2\sqrt{2x-5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x+2)^(1/2)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] 1/2*(EllipticF(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))-EllipticE(2/11*(-33*x+2
2)^(1/2),1/2*I*2^(1/2)))*(-2*x+5)^(1/2)*11^(1/2)/(2*x-5)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-3x+2}}{\sqrt{4x+1}\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{2-3\,x}}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
[Out] int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2-3x}}{\sqrt{2x-5}\,\sqrt{4x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
[Out] Integral(sqrt(2 - 3*x)/(sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

3.55 $\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)}\,dx$

Optimal. Leaf size=103

$$-\frac{\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{5\sqrt{2x-5}}-\frac{3\sqrt{5-2x}\,\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{5\sqrt{11}\sqrt{2x-5}}$$

```
[Out] -1/55*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*x)^(
1/2)/(-5+2*x)^(1/2)-3/55*EllipticPi(2/11*(2-3*x)^(1/2)*11^(1/2),55/124,1/2*
I*2^(1/2))*(5-2*x)^(1/2)*11^(1/2)/(-5+2*x)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {175, 121, 119, 168, 538, 537}

$$-\frac{\sqrt{\frac{6}{11}}\sqrt{5-2x}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{5\sqrt{2x-5}}-\frac{3\sqrt{5-2x}\,\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{5\sqrt{11}\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)),x]
```

```
[Out] -(Sqrt[6/11]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3]
)/(5*Sqrt[-5 + 2*x]) - (3*Sqrt[5 - 2*x]*EllipticPi[55/124, ArcSin[(2*Sqrt[2
 - 3*x])/Sqrt[11]], -1/2])/(5*Sqrt[11]*Sqrt[-5 + 2*x])
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 175

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))*Sqrt[(e_.) + (f_.)*(x_)]
*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/(Sqrt[c + d*x]*Sqr
```

```
t[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(b*c - a*d)/b, Int[1/((a + b*x)*Sq
rt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx = -\left(\frac{3}{5}\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx\right) + \frac{31}{5}\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}$$

$$= -\left(\frac{62}{5}\text{Subst}\left(\int \frac{1}{\left(31-5x^2\right)\sqrt{\frac{11}{3}-\frac{4x^2}{3}}\sqrt{-\frac{11}{3}-\frac{2x^2}{3}}}\,dx, x, \sqrt{2-3x}\right)\right) - \frac{\left(3\sqrt{\right.}{}$$

$$= -\frac{\sqrt{\frac{6}{11}}\sqrt{5-2x}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right)}{5\sqrt{-5+2x}} - \frac{\left(62\sqrt{\frac{3}{11}}\sqrt{5-2x}\right)\text{Subst}\left(\int -\right.}{5\sqrt{}}$$

$$= -\frac{\sqrt{\frac{6}{11}}\sqrt{5-2x}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{1+4x}\right)|\frac{1}{3}\right)}{5\sqrt{-5+2x}} - \frac{3\sqrt{5-2x}\,\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{2\sqrt{2-3}}{\sqrt{11}}\right.\right.}{5\sqrt{11}\sqrt{-5+2x}}$$

Mathematica [A] time = 0.42, size = 70, normalized size = 0.68

$$\frac{3\sqrt{5-2x}\left(\text{EllipticF}\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)-\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)\right)}{5\sqrt{22x-55}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)),x]
```

```
[Out] (3*Sqrt[5 - 2*x]*(EllipticF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] - Ell
ipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/(5*Sqrt[-55 + 2
2*x])
```

fricas [F] time = 1.26, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}}{40\,x^3-34\,x^2-151\,x-35}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(40*x^3 - 34*x^2 - 151*
x - 35), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-3\,x+2}}{(5\,x+7)\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(-3*x + 2)/((5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

maple [A] time = 0.02, size = 56, normalized size = 0.54

$$\frac{3\left(\mathrm{EllipticF}\left(\frac{2\sqrt{-33x+22}}{11},\frac{i\sqrt{2}}{2}\right)-\mathrm{EllipticPi}\left(\frac{2\sqrt{-33x+22}}{11},\frac{55}{124},\frac{i\sqrt{2}}{2}\right)\right)\sqrt{-2x+5}\,\sqrt{11}}{55\sqrt{2x-5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x+2)^(1/2)/(5*x+7)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] 3/55*(EllipticF(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))-EllipticPi(2/11*(-33*x
+22)^(1/2),55/124,1/2*I*2^(1/2)))*(-2*x+5)^(1/2)*11^(1/2)/(2*x-5)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-3\,x+2}}{(5\,x+7)\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(-3*x + 2)/((5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2-3\,x}}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\left(5\,x+7\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)),x)
```

```
[Out] int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2-3x}}{\sqrt{2x-5}\,\sqrt{4x+1}\,\left(5x+7\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)/(7+5*x)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)/(sqrt(2*x - 5)*sqrt(4*x + 1)*(5*x + 7)), x)
```

3.56 $\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^2}\,dx$

Optimal. Leaf size=189

$$-\frac{2\sqrt{\frac{6}{11}}\sqrt{5-2x}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{115\sqrt{2x-5}}-\frac{5\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{897(5x+7)}+\frac{2\sqrt{11}\sqrt{2x-5}\,E\left(\sin^{-1}\left(\frac{2\sqrt{2-}}{\sqrt{1}}\right.\right.}{897\sqrt{5-2x}}$$

```
[Out] -2/1265*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*x)
^(1/2)/(-5+2*x)^(1/2)-3571/1019590*EllipticPi(2/11*(2-3*x)^(1/2)*11^(1/2),5
5/124,1/2*I*2^(1/2))*(5-2*x)^(1/2)*11^(1/2)/(-5+2*x)^(1/2)+2/897*EllipticE(
2/11*(2-3*x)^(1/2)*11^(1/2),1/2*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^
(1/2)-5/897*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)
```

Rubi [A] time = 0.21, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {177, 1607, 168, 538, 537, 158, 114, 113, 121, 119}

$$-\frac{5\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{897(5x+7)}-\frac{2\sqrt{\frac{6}{11}}\sqrt{5-2x}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{115\sqrt{2x-5}}+\frac{2\sqrt{11}\sqrt{2x-5}\,E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right.}{897\sqrt{5-2x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2),x]
```

```
[Out] (-5*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(897*(7 + 5*x)) + (2*Sqrt[1
1]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(897
*Sqrt[5 - 2*x]) - (2*Sqrt[6/11]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*S
qrt[1 + 4*x]], 1/3])/(115*Sqrt[-5 + 2*x]) - (3571*Sqrt[5 - 2*x]*EllipticPi[
55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(92690*Sqrt[11]*Sqrt[-5
+ 2*x])
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
```

```
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 177

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.
)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((m + 1)*(b*e - a*f)*(b*g - a*h))
, x] + Dist[1/(2*(m + 1)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(
Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[2*a*c*f*h*(m + 1) - b*(d*e
*g + c*(2*m + 3)*(f*g + e*h)) + 2*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e
*h + c*f*h))*x - b*d*f*h*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, m}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 1607

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
```

```
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
 q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx = -\frac{5\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{897(7+5x)} - \frac{\int \frac{-479+336x+120x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx}{1794}$$

$$= -\frac{5\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{897(7+5x)} - \frac{\int \frac{\frac{168}{5}+24x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx}{1794} + \frac{3571\int \frac{}{\sqrt{2-3x}\,\sqrt{}}}{}$$

$$= -\frac{5\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{897(7+5x)} - \frac{2}{299}\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}}\,dx - \frac{6}{115}\int \frac{}{\sqrt{2}}$$

$$= -\frac{5\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{897(7+5x)} - \frac{\left(6\sqrt{\frac{2}{11}}\sqrt{5-2x}\right)\int \frac{1}{\sqrt{2-3x}\sqrt{\frac{10}{11}-\frac{4x}{11}}\sqrt{1+4x}}\,dx}{115\sqrt{-5+2x}}$$

$$= -\frac{5\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{897(7+5x)} + \frac{2\sqrt{11}\sqrt{-5+2x}\,E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)\middle|-\frac{1}{2}\right)}{897\sqrt{5-2x}}$$

Mathematica [A] time = 0.69, size = 130, normalized size = 0.69

$$\frac{-3\sqrt{55-22x}\left(-14508\,\text{EllipticF}\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)+6820E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)\middle|-\frac{1}{2}\right)+10713\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{2\sqrt{}}{\sqrt{}}\right.\right.\right.}{9176310\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2),x]
```

```
[Out] ((-51150*Sqrt[2 - 3*x]*(-5 + 2*x)*Sqrt[1 + 4*x])/(7 + 5*x) - 3*Sqrt[55 - 22
*x]*(6820*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] - 14508*Ellip
ticF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 10713*EllipticPi[55/124, A
rcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/(9176310*Sqrt[-5 + 2*x])
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{200x^4+110x^3-993x^2-1232x-245}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^2/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(200*x^4 + 110*x^3 - 99
3*x^2 - 1232*x - 245), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-3x+2}}{(5x+7)^2\sqrt{4x+1}\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^2/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(sqrt(-3*x + 2)/((5*x + 7)^2*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

maple [B] time = 0.02, size = 320, normalized size = 1.69

$$\frac{\sqrt{-3x+2}\,\sqrt{2x-5}\,\sqrt{4x+1}\,\left(-409200x^{3}+1193500x^{2}+34100\sqrt{11}\,\sqrt{-3x+2}\,\sqrt{-2x+5}\,\sqrt{4x+1}\,x\,\mathrm{Elliptic}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x+2)^(1/2)/(5*x+7)^2/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] 1/3058770*(-3*x+2)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(34100*11^(1/2)*(-3*x+
2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticE(2/11*(-33*x+22)^(1/2),1/2*I
*2^(1/2))*x+53565*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*Elli
pticPi(2/11*(-33*x+22)^(1/2),55/124,1/2*I*2^(1/2))*x-72540*11^(1/2)*(-3*x+2
)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(-33*x+22)^(1/2),1/2*I*
2^(1/2))*x+47740*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*Ellip
ticE(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))+74991*11^(1/2)*(-3*x+2)^(1/2)*(-2
*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticPi(2/11*(-33*x+22)^(1/2),55/124,1/2*I*2^(
1/2))-101556*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticF
(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))-409200*x^3+1193500*x^2-358050*x-17050
0)/(24*x^3-70*x^2+21*x+10)/(5*x+7)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-3\,x+2}}{(5\,x+7)^{2}\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^2/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sqrt(-3*x + 2)/((5*x + 7)^2*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2-3\,x}}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,{(5\,x+7)}^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^2),x)
```

```
[Out] int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2-3x}}{\sqrt{2x-5}\,\sqrt{4x+1}\,(5x+7)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)/(7+5*x)**2/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)/(sqrt(2*x - 5)*sqrt(4*x + 1)*(5*x + 7)**2), x)
```

3.57 $\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^3}\,dx$

Optimal. Leaf size=225

$$-\frac{13243\sqrt{5-2x}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{1065935\sqrt{66}\sqrt{2x-5}}-\frac{26825\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{33257172(5x+7)}-\frac{5\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x}}{1794(5x+7)^2}$$

```
[Out] -16369941/37802318840*EllipticPi(2/11*(2-3*x)^(1/2)*11^(1/2),55/124,1/2*I*2
^(1/2))*(5-2*x)^(1/2)*11^(1/2)/(-5+2*x)^(1/2)-13243/70351710*EllipticF(1/11
*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*x)^(1/2)/(-5+2*x)^(1/2)+
5365/16628586*EllipticE(2/11*(2-3*x)^(1/2)*11^(1/2),1/2*I*2^(1/2))*11^(1/2)
*(-5+2*x)^(1/2)/(5-2*x)^(1/2)-5/1794*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(
1/2)/(7+5*x)^2-26825/33257172*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7
+5*x)
```

Rubi [A] time = 0.31, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.314, Rules used = {177, 1604, 1607, 168, 538, 537, 158, 114, 113, 121, 119}

$$-\frac{26825\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{33257172(5x+7)}-\frac{5\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{1794(5x+7)^2}-\frac{13243\sqrt{5-2x}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{1065935\sqrt{66}\sqrt{2x-5}}+\frac{53}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^3),x]
```

```
[Out] (-5*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(1794*(7 + 5*x)^2) - (26825
*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(33257172*(7 + 5*x)) + (5365*S
qrt[11]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])
/(16628586*Sqrt[5 - 2*x]) - (13243*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11
]*Sqrt[1 + 4*x]], 1/3])/(1065935*Sqrt[66]*Sqrt[-5 + 2*x]) - (16369941*Sqrt[
5 - 2*x]*EllipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(343
6574440*Sqrt[11]*Sqrt[-5 + 2*x])
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
```

```
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 177

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.
)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((m + 1)*(b*e - a*f)*(b*g - a*h))
, x] + Dist[1/(2*(m + 1)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(
Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[2*a*c*f*h*(m + 1) - b*(d*e
*g + c*(2*m + 3)*(f*g + e*h)) + 2*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e
*h + c*f*h))*x - b*d*f*h*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, m}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 1604

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
```

```
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt
[e + f*x]*Sqrt[g + h*x])/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), x]
- Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m
 + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m +
 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
+ c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2
*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^
2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
+ c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 1607

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
 q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}\,dx = -\frac{5\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1794(7+5x)^2} - \frac{\int \frac{-1063+1372x-120x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,dx}{3588}$$

$$= -\frac{5\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1794(7+5x)^2} - \frac{26825\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{33257172(7+5x)} - \ldots$$

$$= -\frac{5\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1794(7+5x)^2} - \frac{26825\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{33257172(7+5x)} - \ldots$$

$$= -\frac{5\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1794(7+5x)^2} - \frac{26825\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{33257172(7+5x)} - \frac{5365\ldots}{\ldots}$$

$$= -\frac{5\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1794(7+5x)^2} - \frac{26825\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{33257172(7+5x)} - \frac{(545\ldots}{\ldots}$$

$$= -\frac{5\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{1794(7+5x)^2} - \frac{26825\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{33257172(7+5x)} + \frac{5365\ldots}{\ldots}$$

Mathematica [A] time = 0.46, size = 142, normalized size = 0.63

$$\frac{-\sqrt{55-22x}\,(5x+7)^2\left(-64043148\,\text{EllipticF}\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)+36589300E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)+491098\ldots}{113406956520\sqrt{2x-5}\,(5x+7\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^3),x]
```

```
[Out] (-17050*Sqrt[2 - 3*x]*(-5 + 2*x)*Sqrt[1 + 4*x]*(56093 + 26825*x) - Sqrt[55
 - 22*x]*(7 + 5*x)^2*(36589300*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]],
```

```
-1/2] - 64043148*EllipticF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 491
09823*EllipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/(11340
6956520*Sqrt[-5 + 2*x]*(7 + 5*x)^2)
```

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}}{1000\,x^5+1950\,x^4-4195\,x^3-13111\,x^2-9849\,x-1715},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^3/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(1000*x^5 + 1950*x^4 -
4195*x^3 - 13111*x^2 - 9849*x - 1715), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-3\,x+2}}{(5\,x+7)^3\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^3/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(sqrt(-3*x + 2)/((5*x + 7)^3*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

maple [B] time = 0.02, size = 461, normalized size = 2.05

$$-\frac{\sqrt{-3x+2}\,\sqrt{2x-5}\,\sqrt{4x+1}\,\left(10976790000x^4-9062381900x^3-914732500\sqrt{11}\,\sqrt{-3x+2}\,\sqrt{-2x+5}\,\sqrt{4x}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x+2)^(1/2)/(5*x+7)^3/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] -1/113406956520*(-3*x+2)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(1601078700*11^(
1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(-33*x+22)^
(1/2),1/2*I*2^(1/2))*x^2-914732500*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(
4*x+1)^(1/2)*EllipticE(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))*x^2-1227745575*
11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticPi(2/11*(-33*x
+22)^(1/2),55/124,1/2*I*2^(1/2))*x^2+4483020360*11^(1/2)*(-3*x+2)^(1/2)*(-2
*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))*x-
2561251000*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticE(2
/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))*x-3437687610*11^(1/2)*(-3*x+2)^(1/2)*(-
2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticPi(2/11*(-33*x+22)^(1/2),55/124,1/2*I*2^
(1/2))*x+3138114252*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*El
lipticF(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))-1792875700*11^(1/2)*(-3*x+2)^(
1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticE(2/11*(-33*x+22)^(1/2),1/2*I*2^(
1/2))-2406381327*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*Ellip
ticPi(2/11*(-33*x+22)^(1/2),55/124,1/2*I*2^(1/2))+10976790000*x^4-906238190
0*x^3-57342304250*x^2+24657761150*x+9563856500)/(24*x^3-70*x^2+21*x+10)/(5*
x+7)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-3\,x+2}}{(5\,x+7)^3\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^3/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(sqrt(-3*x + 2)/((5*x + 7)^3*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2-3\,x}}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,{\left(5\,x+7\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^3),x)
```

```
[Out] int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)/(7+5*x)**3/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Timed out
```

3.58 $\int \frac{\sqrt{c+dx}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal. Leaf size=293

$$\frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{h(de-cf)}{f(dg-ch)}\right)}{b\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}-\frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\Pi\left(-\frac{b(de-c}{(bc-ad}\right.}{b\sqrt{f}\sqrt{e+fx}\sqrt{g}}$$

```
[Out] 2*EllipticF(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g
))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g
))^(1/2)/b/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)-2*EllipticPi(f^(1/2)*(d*x+c)
^(1/2)/(c*f-d*e)^(1/2),-b*(-c*f+d*e)/(-a*d+b*c)/f,((-c*f+d*e)*h/f/(-c*h+d*g
))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g
))^(1/2)/b/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

Rubi [A] time = 0.50, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {175, 121, 120, 169, 538, 537}

$$\frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,F\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{b\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}-\frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\Pi\left(-\frac{b(de-cf)}{(bc-ad)f};\sin^{-1}\right.}{b\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/((a + b*x)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d
*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((
d*e - c*f)*h)/(f*(d*g - c*h))])/(b*Sqrt[f]*Sqrt[e + f*x]*Sqrt[g + h*x]) - (
2*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*
g - c*h)]*EllipticPi[-((b*(d*e - c*f))/((b*c - a*d)*f)), ArcSin[(Sqrt[f]*Sq
rt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(b*Sqrt
[f]*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d),
 0] && GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a +
b*x, e + f*x] && (PosQ[-((b*c - a*d)/d)] || NegQ[-((b*e - a*f)/f)])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
```

```
*x]
```

Rule 175

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))*Sqrt[(e_.) + (f_.)*(x_)]
*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/(Sqrt[c + d*x]*Sqr
t[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(b*c - a*d)/b, Int[1/((a + b*x)*Sq
rt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}}{(a+bx)\sqrt{e+fx}\,\sqrt{g+hx}}\,dx = \frac{d\int \frac{1}{\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+hx}}\,dx}{b} + \frac{(bc-ad)\int \frac{1}{(a+bx)\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+hx}}\,dx}{b}$$

$$= -\frac{(2(bc-ad))\,\text{Subst}\left(\int \frac{1}{\left(bc-ad-bx^2\right)\sqrt{e-\frac{cf}{d}+\frac{fx^2}{d}}\,\sqrt{g-\frac{ch}{d}+\frac{hx^2}{d}}}\,dx, x, \sqrt{c+dx}\right)}{b} + \frac{\left(d\sqrt{\right.}{}$$

$$= -\frac{\left(2(bc-ad)\sqrt{\frac{d(e+fx)}{de-cf}}\right)\text{Subst}\left(\int \frac{1}{\left(bc-ad-bx^2\right)\sqrt{1+\frac{fx^2}{d\left(e-\frac{cf}{d}\right)}}\,\sqrt{g-\frac{ch}{d}+\frac{hx^2}{d}}}\,dx, x, \sqrt{c+d}\right.}{b\sqrt{e+fx}}$$

$$= \frac{2\sqrt{-de+cf}\,\sqrt{\frac{d(e+fx)}{de-cf}}\,\sqrt{\frac{d(g+hx)}{dg-ch}}\,F\left(\sin^{-1}\left(\frac{\sqrt{f}\,\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{b\sqrt{f}\,\sqrt{e+fx}\,\sqrt{g+hx}} - \frac{\left(2(bc-ad)\sqrt{\right.}{}$$

$$= \frac{2\sqrt{-de+cf}\,\sqrt{\frac{d(e+fx)}{de-cf}}\,\sqrt{\frac{d(g+hx)}{dg-ch}}\,F\left(\sin^{-1}\left(\frac{\sqrt{f}\,\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{b\sqrt{f}\,\sqrt{e+fx}\,\sqrt{g+hx}} - \frac{2\sqrt{-de+cf}\,\sqrt{}}{}$$

Mathematica [C] time = 1.77, size = 202, normalized size = 0.69

$$-\frac{2i\sqrt{c+dx}\sqrt{\frac{d(g+hx)}{dg-ch}}\left(\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{f(c+dx)}{de-cf}}\right),\frac{deh-cfh}{dfg-cfh}\right)-\Pi\left(\frac{b(cf-de)}{(bc-ad)f};i\sinh^{-1}\left(\sqrt{\frac{f(c+dx)}{de-cf}}\right)|\frac{deh-cfh}{dfg-cfh}\right)\right)}{b\sqrt{e+fx}\sqrt{g+hx}\sqrt{\frac{f(c+dx)}{d(e+fx)}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/((a + b*x)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] ((-2*I)*Sqrt[c + d*x]*Sqrt[(d*(g + h*x))/(d*g - c*h)]*(EllipticF[I*ArcSinh[
Sqrt[(f*(c + d*x))/(d*e - c*f)]], (d*e*h - c*f*h)/(d*f*g - c*f*h)] - Ellipt
icPi[(b*(-(d*e) + c*f))/((b*c - a*d)*f), I*ArcSinh[Sqrt[(f*(c + d*x))/(d*e
- c*f)]], (d*e*h - c*f*h)/(d*f*g - c*f*h)]))/(b*Sqrt[(f*(c + d*x))/(d*(e +
f*x))]*Sqrt[e + f*x]*Sqrt[g + h*x])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="f
ricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="g
iac")
```

```
[Out] Timed out
```

maple [A] time = 0.05, size = 382, normalized size = 1.30

$$\frac{2\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}\,\sqrt{\frac{(dx+c)f}{cf-de}}\,\sqrt{-\frac{(hx+g)d}{ch-dg}}\,\sqrt{-\frac{(fx+e)d}{cf-de}}\left(cf\,\text{EllipticF}\left(\sqrt{\frac{(dx+c)f}{cf-de}},\sqrt{\frac{(cf-de)h}{(ch-dg)f}}\right)-cf\,\text{Ellipt}\right.}{\left(dfh\,x^3+cfh\,x^2+deh\,x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

```
[Out] 2*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/f/b*((d*x+c)/(c*f-d*e)*f)^(1/2)
*(-(h*x+g)*d/(c*h-d*g))^(1/2)*(-(f*x+e)/(c*f-d*e)*d)^(1/2)*(EllipticF(((d*x
+c)/(c*f-d*e)*f)^(1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*c*f-EllipticF(((d*x
+c)/(c*f-d*e)*f)^(1/2),((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*d*e-EllipticPi(((d*
x+c)/(c*f-d*e)*f)^(1/2),-(c*f-d*e)*b/f/(a*d-b*c),((c*f-d*e)*h/f/(c*h-d*g))^
(1/2))*c*f+EllipticPi(((d*x+c)/(c*f-d*e)*f)^(1/2),-(c*f-d*e)*b/f/(a*d-b*c),
((c*f-d*e)*h/f/(c*h-d*g))^(1/2))*d*e)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*
x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx+c}}{(bx+a)\sqrt{fx+e}\,\sqrt{hx+g}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(d*x + c)/((b*x + a)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c+d\,x}}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)),x)
```

```
[Out] int((c + d*x)^(1/2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx}}{\left(a + bx\right) \sqrt{e + fx} \sqrt{g + hx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x)/((a + b*x)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

3.59 $\int \frac{(c+dx)^{3/2}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal. Leaf size=449

$$\frac{2(bc-ad)\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\operatorname{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{h(de-cf)}{f(dg-ch)}\right)}{b^2\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}-\frac{2(bc-ad)\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{}}{b^2\sqrt{}}$$

```
[Out] 2*(-a*d+b*c)*EllipticF(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/
f/(-c*h+d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g
)/(-c*h+d*g))^(1/2)/b^2/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)-2*(-a*d+b*c)*El
lipticPi(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),-b*(-c*f+d*e)/(-a*d+b*c)/f,(
(-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1
/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)/b^2/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)+2*
d*EllipticE(h^(1/2)*(f*x+e)^(1/2)/(e*h-f*g)^(1/2),(-d*(-e*h+f*g)/(-c*f+d*e)
/h)^(1/2))*(e*h-f*g)^(1/2)*(d*x+c)^(1/2)*(f*(h*x+g)/(-e*h+f*g))^(1/2)/b/f/h
^(1/2)/(-f*(d*x+c)/(-c*f+d*e))^(1/2)/(h*x+g)^(1/2)
```

Rubi [A] time = 0.67, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {179, 121, 120, 169, 538, 537, 114, 113}

$$\frac{2(bc-ad)\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}F\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{b^2\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}-\frac{2(bc-ad)\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\Pi}{b^2\sqrt{f}\sqrt{e+f}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/((a + b*x)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*d*Sqrt[-(f*g) + e*h]*Sqrt[c + d*x]*Sqrt[(f*(g + h*x))/(f*g - e*h)]*Ellip
ticE[ArcSin[(Sqrt[h]*Sqrt[e + f*x])/Sqrt[-(f*g) + e*h]], -((d*(f*g - e*h))/
((d*e - c*f)*h))]])/(b*f*Sqrt[h]*Sqrt[-((f*(c + d*x))/(d*e - c*f))]*Sqrt[g +
 h*x]) + (2*(b*c - a*d)*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*
Sqrt[(d*(g + h*x))/(d*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sq
rt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(b^2*Sqrt[f]*Sqrt[e +
f*x]*Sqrt[g + h*x]) - (2*(b*c - a*d)*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/
(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h)]*EllipticPi[-((b*(d*e - c*f))/(
(b*c - a*d)*f)), ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e
- c*f)*h)/(f*(d*g - c*h))])/(b^2*Sqrt[f]*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
```

```
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d),
 0] && GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a +
b*x, e + f*x] && (PosQ[-((b*c - a*d)/d)] || NegQ[-((b*e - a*f)/f)])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 179

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_))/(Sqrt[(e_.) + (f
_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Int[ExpandIntegrand[1/(Sq
rt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), (a + b*x)^m*(c + d*x)^(n + 1/2),
x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IntegerQ[m] && IntegerQ[n +
 1/2]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{3/2}}{(a+bx)\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \left(\frac{d(bc-ad)}{b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}} + \frac{(bc-ad)^2}{b^2(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}} +$$

$$= \frac{d\int \frac{\sqrt{c+dx}}{\sqrt{e+fx}\sqrt{g+hx}}\,dx}{b} + \frac{(d(bc-ad))\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{b^2} + \frac{(bc-ad)^2\int \frac{}{(a}}{}$$

$$= -\frac{\left(2(bc-ad)^2\right)\operatorname{Subst}\left(\int \frac{1}{\left(bc-ad-bx^2\right)\sqrt{e-\frac{cf}{d}+\frac{fx^2}{d}}\sqrt{g-\frac{ch}{d}+\frac{hx^2}{d}}}\,dx, x, \sqrt{c+dx}\right)}{b^2} + $$

$$= \frac{2d\sqrt{-fg+eh}\sqrt{c+dx}\sqrt{\frac{f(g+hx)}{fg-eh}}\,E\left(\sin^{-1}\left(\frac{\sqrt{h}\sqrt{e+fx}}{\sqrt{-fg+eh}}\right)\middle|-\frac{d(fg-eh)}{(de-cf)h}\right)}{bf\sqrt{h}\sqrt{-\frac{f(c+dx)}{de-cf}}\sqrt{g+hx}} - \frac{\left(2(bc-\right.}{}$$

$$= \frac{2d\sqrt{-fg+eh}\sqrt{c+dx}\sqrt{\frac{f(g+hx)}{fg-eh}}\,E\left(\sin^{-1}\left(\frac{\sqrt{h}\sqrt{e+fx}}{\sqrt{-fg+eh}}\right)\middle|-\frac{d(fg-eh)}{(de-cf)h}\right)}{bf\sqrt{h}\sqrt{-\frac{f(c+dx)}{de-cf}}\sqrt{g+hx}} + \frac{2(bc-}{}$$

$$= \frac{2d\sqrt{-fg+eh}\sqrt{c+dx}\sqrt{\frac{f(g+hx)}{fg-eh}}\,E\left(\sin^{-1}\left(\frac{\sqrt{h}\sqrt{e+fx}}{\sqrt{-fg+eh}}\right)\middle|-\frac{d(fg-eh)}{(de-cf)h}\right)}{bf\sqrt{h}\sqrt{-\frac{f(c+dx)}{de-cf}}\sqrt{g+hx}} + \frac{2(bc-}{}$$

Mathematica [C] time = 7.74, size = 1198, normalized size = 2.67

$$-\frac{2\left(b^2d^2\sqrt{\frac{fg}{h}-e}\,he^3 - abd^2f\sqrt{\frac{fg}{h}-e}\,he^2 - b^2cdf\sqrt{\frac{fg}{h}-e}\,he^2 - 2b^2d^2\sqrt{\frac{fg}{h}-e}\,h(e+fx)e^2 - b^2d^2fg\sqrt{\frac{fg}{h}-e}\,e^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/((a + b*x)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (-2*(-(b^2*d^2*e^2*f*g*Sqrt[-e + (f*g)/h]) + b^2*c*d*e*f^2*g*Sqrt[-e + (f*g
)/h] + a*b*d^2*e*f^2*g*Sqrt[-e + (f*g)/h] - a*b*c*d*f^3*g*Sqrt[-e + (f*g)/h
] + b^2*d^2*e^3*Sqrt[-e + (f*g)/h]*h - b^2*c*d*e^2*f*Sqrt[-e + (f*g)/h]*h -
 a*b*d^2*e^2*f*Sqrt[-e + (f*g)/h]*h + a*b*c*d*e*f^2*Sqrt[-e + (f*g)/h]*h +
b^2*d^2*e*f*g*Sqrt[-e + (f*g)/h]*(e + f*x) - a*b*d^2*f^2*g*Sqrt[-e + (f*g)/
h]*(e + f*x) - 2*b^2*d^2*e^2*Sqrt[-e + (f*g)/h]*h*(e + f*x) + b^2*c*d*e*f*S
qrt[-e + (f*g)/h]*h*(e + f*x) + 2*a*b*d^2*e*f*Sqrt[-e + (f*g)/h]*h*(e + f*x
) - a*b*c*d*f^2*Sqrt[-e + (f*g)/h]*h*(e + f*x) + b^2*d^2*e*Sqrt[-e + (f*g)/
h]*h*(e + f*x)^2 - a*b*d^2*f*Sqrt[-e + (f*g)/h]*h*(e + f*x)^2 + I*b*d^2*(b*
e - a*f)*(f*g - e*h)*Sqrt[(f*(c + d*x))/(d*(e + f*x))]*(e + f*x)^(3/2)*Sqrt
[(f*(g + h*x))/(h*(e + f*x))]*EllipticE[I*ArcSinh[Sqrt[-e + (f*g)/h]/Sqrt[e
 + f*x]], ((d*e - c*f)*h)/(d*(-(f*g) + e*h))] - I*b*f*(a*d^2*(-(f*g) + e*h)
 + b*(d^2*e*g - 2*c*d*e*h + c^2*f*h))*Sqrt[(f*(c + d*x))/(d*(e + f*x))]*(e
+ f*x)^(3/2)*Sqrt[(f*(g + h*x))/(h*(e + f*x))]*EllipticF[I*ArcSinh[Sqrt[-e
```

```
+ (f*g)/h]/Sqrt[e + f*x]], ((d*e - c*f)*h)/(d*(-(f*g) + e*h))] + I*b^2*c^2*
f^2*h*Sqrt[(f*(c + d*x))/(d*(e + f*x))]*(e + f*x)^(3/2)*Sqrt[(f*(g + h*x))/
(h*(e + f*x))]*EllipticPi[-((b*e*h - a*f*h)/(b*f*g - b*e*h)), I*ArcSinh[Sqr
t[-e + (f*g)/h]/Sqrt[e + f*x]], ((d*e - c*f)*h)/(d*(-(f*g) + e*h))] - (2*I)
*a*b*c*d*f^2*h*Sqrt[(f*(c + d*x))/(d*(e + f*x))]*(e + f*x)^(3/2)*Sqrt[(f*(g
 + h*x))/(h*(e + f*x))]*EllipticPi[-((b*e*h - a*f*h)/(b*f*g - b*e*h)), I*Ar
cSinh[Sqrt[-e + (f*g)/h]/Sqrt[e + f*x]], ((d*e - c*f)*h)/(d*(-(f*g) + e*h))
] + I*a^2*d^2*f^2*h*Sqrt[(f*(c + d*x))/(d*(e + f*x))]*(e + f*x)^(3/2)*Sqrt[
(f*(g + h*x))/(h*(e + f*x))]*EllipticPi[-((b*e*h - a*f*h)/(b*f*g - b*e*h)),
 I*ArcSinh[Sqrt[-e + (f*g)/h]/Sqrt[e + f*x]], ((d*e - c*f)*h)/(d*(-(f*g) +
e*h))]))/(b^2*f^2*(-(b*e) + a*f)*Sqrt[-e + (f*g)/h]*h*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="f
ricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: AttributeError >> type
```

maple [B] time = 0.03, size = 968, normalized size = 2.16

$$-\frac{2\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}\,\sqrt{\frac{(dx+c)f}{cf-de}}\,\sqrt{-\frac{(hx+g)d}{ch-dg}}\,\sqrt{-\frac{(fx+e)d}{cf-de}}\left(acdfh\,\mathrm{EllipticF}\left(\sqrt{\frac{(dx+c)f}{cf-de}},\sqrt{\frac{(cf-de)h}{(ch-dg)f}}\right)-acdfh\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

```
[Out] -2*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/h/f/b^2*((d*x+c)/(c*f-d*e)*f)^
(1/2)*(-(h*x+g)/(c*h-d*g)*d)^(1/2)*(-(f*x+e)/(c*f-d*e)*d)^(1/2)*(EllipticF(
((d*x+c)/(c*f-d*e)*f)^(1/2),((c*f-d*e)/(c*h-d*g)/f*h)^(1/2))*a*c*d*f*h-Elli
pticF(((d*x+c)/(c*f-d*e)*f)^(1/2),((c*f-d*e)/(c*h-d*g)/f*h)^(1/2))*a*d^2*e*
h-2*EllipticF(((d*x+c)/(c*f-d*e)*f)^(1/2),((c*f-d*e)/(c*h-d*g)/f*h)^(1/2))*
b*c^2*f*h+2*EllipticF(((d*x+c)/(c*f-d*e)*f)^(1/2),((c*f-d*e)/(c*h-d*g)/f*h)
^(1/2))*b*c*d*e*h+EllipticF(((d*x+c)/(c*f-d*e)*f)^(1/2),((c*f-d*e)/(c*h-d*g
)/f*h)^(1/2))*b*c*d*f*g-EllipticF(((d*x+c)/(c*f-d*e)*f)^(1/2),((c*f-d*e)/(c
*h-d*g)/f*h)^(1/2))*b*d^2*e*g+EllipticE(((d*x+c)/(c*f-d*e)*f)^(1/2),((c*f-d
*e)/(c*h-d*g)/f*h)^(1/2))*b*c^2*f*h-EllipticE(((d*x+c)/(c*f-d*e)*f)^(1/2),(
(c*f-d*e)/(c*h-d*g)/f*h)^(1/2))*b*c*d*e*h-EllipticE(((d*x+c)/(c*f-d*e)*f)^(
1/2),((c*f-d*e)/(c*h-d*g)/f*h)^(1/2))*b*c*d*f*g+EllipticE(((d*x+c)/(c*f-d*e
)*f)^(1/2),((c*f-d*e)/(c*h-d*g)/f*h)^(1/2))*b*d^2*e*g-EllipticPi(((d*x+c)/(
c*f-d*e)*f)^(1/2),-(c*f-d*e)/(a*d-b*c)*b/f,((c*f-d*e)/(c*h-d*g)/f*h)^(1/2))
*a*c*d*f*h+EllipticPi(((d*x+c)/(c*f-d*e)*f)^(1/2),-(c*f-d*e)/(a*d-b*c)*b/f,
((c*f-d*e)/(c*h-d*g)/f*h)^(1/2))*a*d^2*e*h+EllipticPi(((d*x+c)/(c*f-d*e)*f)
```

```
^(1/2),-(c*f-d*e)/(a*d-b*c)*b/f,((c*f-d*e)/(c*h-d*g)/f*h)^(1/2))*b*c^2*f*h-
EllipticPi(((d*x+c)/(c*f-d*e)*f)^(1/2),-(c*f-d*e)/(a*d-b*c)*b/f,((c*f-d*e)/
(c*h-d*g)/f*h)^(1/2))*b*c*d*e*h)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c
*e*h*x+c*f*g*x+d*e*g*x+c*e*g)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{3}{2}}}{(bx+a)\sqrt{fx+e}\,\sqrt{hx+g}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate((d*x + c)^(3/2)/((b*x + a)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{3/2}}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)),x)
```

```
[Out] int((c + d*x)^(3/2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{3}{2}}}{(a+bx)\sqrt{e+fx}\,\sqrt{g+hx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] Integral((c + d*x)**(3/2)/((a + b*x)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

3.60 $\int \frac{(7+5x)^4}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$

Optimal. Leaf size=203

$$\frac{392989907\sqrt{5-2x}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{2016\sqrt{66}\sqrt{2x-5}}-\frac{25}{84}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2-\frac{305}{24}\sqrt{2-3x}\sqrt{2}$$

```
[Out] 392989907/133056*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2
)*(5-2*x)^(1/2)/(-5+2*x)^(1/2)-5109835/756*EllipticE(2/11*(2-3*x)^(1/2)*11^
(1/2),1/2*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1/2)-120355/288*(2-3*
x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)-305/24*(7+5*x)*(2-3*x)^(1/2)*(-5+2*x)
^(1/2)*(1+4*x)^(1/2)-25/84*(7+5*x)^2*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(
1/2)
```

Rubi [A] time = 0.22, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {167, 1600, 1615, 158, 114, 113, 121, 119}

$$-\frac{25}{84}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)^2-\frac{305}{24}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)-\frac{120355}{288}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x)^4/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (-120355*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/288 - (305*Sqrt[2 - 3*
x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x))/24 - (25*Sqrt[2 - 3*x]*Sqrt[-5 +
 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2)/84 - (5109835*Sqrt[11]*Sqrt[-5 + 2*x]*Elli
pticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(756*Sqrt[5 - 2*x]) + (392
989907*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(201
6*Sqrt[66]*Sqrt[-5 + 2*x])
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
```

```
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 167

```
Int[((a_.) + (b_.)*(x_))^(m_)/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*
(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(2*b^2*(a + b*x)^(m - 2)
*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(d*f*h*(2*m - 1)), x] - Dist[1/
(d*f*h*(2*m - 1)), Int[((a + b*x)^(m - 3)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt
[g + h*x]))*Simp[a*b^2*(d*e*g + c*f*g + c*e*h) + 2*b^3*c*e*g*(m - 2) - a^3*
d*f*h*(2*m - 1) + b*(2*a*b*(d*f*g + d*e*h + c*f*h) + b^2*(2*m - 3)*(d*e*g +
 c*f*g + c*e*h) - 3*a^2*d*f*h*(2*m - 1))*x - 2*b^2*(m - 1)*(3*a*d*f*h - b*(
d*f*g + d*e*h + c*f*h))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x] && IntegerQ[2*m] && GeQ[m, 2]
```

Rule 1600

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[(2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/
(d*f*h*(2*m + 3)), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(7+5x)^4}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = -\frac{25}{84}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2 + \frac{1}{168}\int \frac{(7+5x)(48949+13485}{\sqrt{2-3x}\sqrt{-5+2}}$$

$$= -\frac{305}{24}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x) - \frac{25}{84}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}($$

$$= -\frac{120355}{288}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} - \frac{305}{24}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+$$

$$= -\frac{120355}{288}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} - \frac{305}{24}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+$$

$$= -\frac{120355}{288}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} - \frac{305}{24}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+$$

$$= -\frac{120355}{288}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x} - \frac{305}{24}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+$$

Mathematica [A] time = 0.53, size = 125, normalized size = 0.62

$$\frac{392989907\sqrt{66}\sqrt{5-2x}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right) - 1650\sqrt{2-3x}\sqrt{4x+1}\left(1200x^3+10608x^2+500\right.}{133056\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x)^4/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (-1650*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(-210245 + 50078*x + 10608*x^2 + 1200*x^
3) - 449665480*Sqrt[66]*Sqrt[5 - 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 +
4*x]], 1/3] + 392989907*Sqrt[66]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*
Sqrt[1 + 4*x]], 1/3])/(133056*Sqrt[-5 + 2*x])
```

fricas [F] time = 1.40, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{(625\,x^4+3500\,x^3+7350\,x^2+6860\,x+2401)\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{24\,x^3-70\,x^2+21\,x+10}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^4/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(-(625*x^4 + 3500*x^3 + 7350*x^2 + 6860*x + 2401)*sqrt(4*x + 1)*sqr
t(2*x - 5)*sqrt(-3*x + 2)/(24*x^3 - 70*x^2 + 21*x + 10), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5\,x+7)^4}{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^4/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="giac")
```

[Out] integrate((5*x + 7)^4/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)

maple [A] time = 0.03, size = 150, normalized size = 0.74

$$\frac{\sqrt{-3x+2}\,\sqrt{2x-5}\,\sqrt{4x+1}\,\left(-23760000x^5-200138400x^4-900068400x^3+4611000900x^2-1569263850x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x+7)^4/(-3*x+2)^(1/2)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)

[Out] 1/133056*(-3*x+2)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(1178969721*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))-899330960*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticE(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))-23760000*x^5-200138400*x^4-900068400*x^3+4611000900*x^2-1569263850*x-693808500)/(24*x^3-70*x^2+21*x+10)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x+7)^4}{\sqrt{4x+1}\,\sqrt{2x-5}\,\sqrt{-3x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((7+5*x)^4/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="maxima")

[Out] integrate((5*x + 7)^4/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(5x+7)^4}{\sqrt{2-3x}\,\sqrt{4x+1}\,\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x + 7)^4/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)

[Out] int((5*x + 7)^4/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x+7)^4}{\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((7+5*x)**4/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)

[Out] Integral((5*x + 7)**4/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)), x)

3.61 $\int \frac{(7+5x)^3}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx$

Optimal. Leaf size=165

$$\frac{2474201\sqrt{5-2x}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{216\sqrt{66}\sqrt{2x-5}}-\frac{5}{12}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)-\frac{2135}{108}\sqrt{2-3x}\sqrt{2x-}$$

```
[Out] 2474201/14256*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(
5-2*x)^(1/2)/(-5+2*x)^(1/2)-487585/1296*EllipticE(2/11*(2-3*x)^(1/2)*11^(1/
2),1/2*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1/2)-2135/108*(2-3*x)^(1
/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)-5/12*(7+5*x)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*
(1+4*x)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {167, 1615, 158, 114, 113, 121, 119}

$$-\frac{5}{12}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}(5x+7)-\frac{2135}{108}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}+\frac{2474201\sqrt{5-2x}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right.\right.}{216\sqrt{66}\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x)^3/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (-2135*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/108 - (5*Sqrt[2 - 3*x]*S
qrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x))/12 - (487585*Sqrt[11]*Sqrt[-5 + 2*x]
*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(1296*Sqrt[5 - 2*x])
+ (2474201*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/
(216*Sqrt[66]*Sqrt[-5 + 2*x])
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
```

```
-(f/d)] || PosQ[-(f/b)]))
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 167

```
Int[((a_.) + (b_.)*(x_))^(m_)/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*
(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(2*b^2*(a + b*x)^(m - 2)
*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(d*f*h*(2*m - 1)), x] - Dist[1/
(d*f*h*(2*m - 1)), Int[((a + b*x)^(m - 3)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt
[g + h*x]))*Simp[a*b^2*(d*e*g + c*f*g + c*e*h) + 2*b^3*c*e*g*(m - 2) - a^3*
d*f*h*(2*m - 1) + b*(2*a*b*(d*f*g + d*e*h + c*f*h) + b^2*(2*m - 3)*(d*e*g +
 c*f*g + c*e*h) - 3*a^2*d*f*h*(2*m - 1))*x - 2*b^2*(m - 1)*(3*a*d*f*h - b*(
d*f*g + d*e*h + c*f*h))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x] && IntegerQ[2*m] && GeQ[m, 2]
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(7+5x)^3}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = -\frac{5}{12}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\,(7+5x)+\frac{1}{120}\int \frac{34985+104825x+8540}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+}}$$

$$= -\frac{2135}{108}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}-\frac{5}{12}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\,(7+5x)$$

$$= -\frac{2135}{108}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}-\frac{5}{12}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\,(7+5x)$$

$$= -\frac{2135}{108}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}-\frac{5}{12}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\,(7+5x)$$

$$= -\frac{2135}{108}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}-\frac{5}{12}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\,(7+5x)$$

Mathematica [A] time = 0.37, size = 120, normalized size = 0.73

$$\frac{4948402\sqrt{66}\sqrt{5-2x}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)-6600\sqrt{2-3x}\sqrt{4x+1}\left(18x^2+151x-490\right)-53634}{28512\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x)^3/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (-6600*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(-490 + 151*x + 18*x^2) - 5363435*Sqrt[6
6]*Sqrt[5 - 2*x]*EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3] + 4948402
*Sqrt[66]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(
28512*Sqrt[-5 + 2*x])
```

fricas [F] time = 1.41, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(125\,x^3+525\,x^2+735\,x+343\right)\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{24\,x^3-70\,x^2+21\,x+10},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^3/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(-(125*x^3 + 525*x^2 + 735*x + 343)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqr
t(-3*x + 2)/(24*x^3 - 70*x^2 + 21*x + 10), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5\,x+7)^3}{\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^3/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((5*x + 7)^3/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

maple [A] time = 0.02, size = 145, normalized size = 0.88

$$\frac{\sqrt{-3x+2}\sqrt{2x-5}\sqrt{4x+1}\left(-712800x^4-5682600x^3+22014300x^2-7088400x-5363435\sqrt{11}\sqrt{-3x+2}\sqrt{-}\right.}{342144x^3-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x+7)^3/(-3*x+2)^(1/2)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] 1/14256*(-3*x+2)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(7422603*11^(1/2)*(-3*x+
2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(-33*x+22)^(1/2),1/2*I
*2^(1/2))-5363435*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*Elli
pticE(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))-712800*x^4-5682600*x^3+22014300*
x^2-7088400*x-3234000)/(24*x^3-70*x^2+21*x+10)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x+7)^3}{\sqrt{4x+1}\,\sqrt{2x-5}\,\sqrt{-3x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^3/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((5*x + 7)^3/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(5x+7)^3}{\sqrt{2-3x}\,\sqrt{4x+1}\,\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + 7)^3/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
```

```
[Out] int((5*x + 7)^3/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x+7)^3}{\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**3/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral((5*x + 7)**3/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

3.62 $\int \frac{(7+5x)^2}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx$

Optimal. Leaf size=129

$$\frac{24353\sqrt{5-2x}\ \mathrm{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{36\sqrt{66}\sqrt{2x-5}}-\frac{25}{36}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}-\frac{2135\sqrt{11}\sqrt{2x-5}\,E\left(\sin^{-1}\left(\right.\right.}{108\sqrt{5-2x}}$$

```
[Out] 24353/2376*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2
*x)^(1/2)/(-5+2*x)^(1/2)-2135/108*EllipticE(2/11*(2-3*x)^(1/2)*11^(1/2),1/2
*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1/2)-25/36*(2-3*x)^(1/2)*(-5+2
*x)^(1/2)*(1+4*x)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {167, 24, 158, 114, 113, 121, 119}

$$-\frac{25}{36}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}+\frac{24353\sqrt{5-2x}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{36\sqrt{66}\sqrt{2x-5}}-\frac{2135\sqrt{11}\sqrt{2x-5}\,E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right.\right.}{108\sqrt{5-2x}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x)^2/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (-25*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/36 - (2135*Sqrt[11]*Sqrt[-
5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(108*Sqrt[5 -
 2*x]) + (24353*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1
/3])/(36*Sqrt[66]*Sqrt[-5 + 2*x])
```

Rule 24

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((A_.) + (B_.)*(v_) + (C_.)*(v_)^2), x_S
ymbol] :> Dist[1/b^2, Int[u*(a + b*v)^(m + 1)*Simp[b*B - a*C + b*C*v, x], x
], x] /; FreeQ[{a, b, A, B, C}, x] && EqQ[A*b^2 - a*b*B + a^2*C, 0] && LeQ[
m, -1]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
```

```
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 167

```
Int[((a_.) + (b_.)*(x_))^(m_)/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*
(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(2*b^2*(a + b*x)^(m - 2)
*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(d*f*h*(2*m - 1)), x] - Dist[1/
(d*f*h*(2*m - 1)), Int[((a + b*x)^(m - 3)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt
[g + h*x]))*Simp[a*b^2*(d*e*g + c*f*g + c*e*h) + 2*b^3*c*e*g*(m - 2) - a^3*
d*f*h*(2*m - 1) + b*(2*a*b*(d*f*g + d*e*h + c*f*h) + b^2*(2*m - 3)*(d*e*g +
 c*f*g + c*e*h) - 3*a^2*d*f*h*(2*m - 1))*x - 2*b^2*(m - 1)*(3*a*d*f*h - b*(
d*f*g + d*e*h + c*f*h))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x] && IntegerQ[2*m] && GeQ[m, 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{(7+5x)^2}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx &= -\frac{25}{36}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}+\frac{1}{72}\int \frac{21021+74795x+42700x^2}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+\cdots} \\
&= -\frac{25}{36}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}+\frac{\int \frac{75075+213500x}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx}{1800} \\
&= -\frac{25}{36}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}+\frac{2135}{36}\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\,\sqrt{1+4x}}\,dx+\frac{2435\cdots}{72} \\
&= -\frac{25}{36}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}+\frac{\left(24353\sqrt{5-2x}\right)\int \frac{1}{\sqrt{2-3x}\,\sqrt{\frac{10}{11}-\frac{4x}{11}}\,\sqrt{1+4x\cdots}}}{36\sqrt{22}\,\sqrt{-5+2x}} \\
&= -\frac{25}{36}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}-\frac{2135\sqrt{11}\,\sqrt{-5+2x}\,E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)\cdots\right.}{108\sqrt{5-2x}}
\end{aligned}$$

Mathematica [A] time = 0.26, size = 115, normalized size = 0.89

$$\frac{24353\sqrt{66}\,\sqrt{5-2x}\ \text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{4x+1}\right),\frac{1}{3}\right)+1650\sqrt{2-3x}\,\sqrt{4x+1}\,(5-2x)-23485\sqrt{66}\,\sqrt{5-2\cdots}}{2376\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x)^2/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (1650*Sqrt[2 - 3*x]*(5 - 2*x)*Sqrt[1 + 4*x] - 23485*Sqrt[66]*Sqrt[5 - 2*x]*
EllipticE[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3] + 24353*Sqrt[66]*Sqrt[5 -
2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])/(2376*Sqrt[-5 + 2*x]
)
```

fricas [F] time = 1.26, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(25\,x^2+70\,x+49\right)\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}}{24\,x^3-70\,x^2+21\,x+10},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^2/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(-(25*x^2 + 70*x + 49)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(
24*x^3 - 70*x^2 + 21*x + 10), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5\,x+7)^2}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^2/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((5*x + 7)^2/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

maple [A] time = 0.02, size = 140, normalized size = 1.09

$$\frac{\sqrt{-3x+2}\,\sqrt{2x-5}\,\sqrt{4x+1}\,\left(-39600x^3+115500x^2-34650x-46970\sqrt{11}\,\sqrt{-3x+2}\,\sqrt{-2x+5}\,\sqrt{4x+1}\,\text{Ellip}\right.}{57024x^3-166320x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x+7)^2/(-3*x+2)^(1/2)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] 1/2376*(-3*x+2)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(73059*11^(1/2)*(-3*x+2)^
(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(-33*x+22)^(1/2),1/2*I*2^
(1/2))-46970*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticE
(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))-39600*x^3+115500*x^2-34650*x-16500)/(
24*x^3-70*x^2+21*x+10)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5\,x+7)^2}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^2/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((5*x + 7)^2/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(5\,x+7)^2}{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + 7)^2/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
[Out] int((5*x + 7)^2/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x+7)^2}{\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**2/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
[Out] Integral((5*x + 7)**2/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

3.63 $\int \frac{7+5x}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx$

Optimal. Leaf size=98

$$\frac{13\sqrt{\frac{3}{22}}\,\sqrt{5-2x}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{4x+1}\right),\frac{1}{3}\right)}{\sqrt{2x-5}}-\frac{5\sqrt{11}\,\sqrt{2x-5}\,E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{6\sqrt{5-2x}}$$

```
[Out] 13/22*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*x)^(
1/2)/(-5+2*x)^(1/2)-5/6*EllipticE(2/11*(2-3*x)^(1/2)*11^(1/2),1/2*I*2^(1/2)
)*11^(1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {158, 114, 113, 121, 119}

$$\frac{13\sqrt{\frac{3}{22}}\,\sqrt{5-2x}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{2x-5}}-\frac{5\sqrt{11}\,\sqrt{2x-5}\,E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{6\sqrt{5-2x}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x)/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (-5*Sqrt[11]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -
1/2])/(6*Sqrt[5 - 2*x]) + (13*Sqrt[3/22]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqr
t[3/11]*Sqrt[1 + 4*x]], 1/3])/Sqrt[-5 + 2*x]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rubi steps

$$\int \frac{7+5x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{5}{2}\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}}\,dx + \frac{39}{2}\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$$

$$= \frac{\left(39\sqrt{5-2x}\right)\int \frac{1}{\sqrt{2-3x}\sqrt{\frac{10}{11}-\frac{4x}{11}}\sqrt{1+4x}}\,dx}{\sqrt{22}\sqrt{-5+2x}} + \frac{\left(5\sqrt{-5+2x}\right)\int \frac{\sqrt{\frac{15}{11}-\frac{6x}{11}}}{\sqrt{2-3x}\sqrt{\frac{3}{11}+\frac{12x}{11}}}\,d}{2\sqrt{5-2x}}$$

$$= -\frac{5\sqrt{11}\sqrt{-5+2x}\,E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{6\sqrt{5-2x}} + \frac{13\sqrt{\frac{3}{22}}\sqrt{5-2x}\,F\left(\sin^{-1}\left(\sqrt{\right.\right.}{\sqrt{-5+2x}}$$

Mathematica [A] time = 0.48, size = 187, normalized size = 1.91

$$\frac{-124\sqrt{22}\sqrt{\frac{2x-5}{4x+1}}\sqrt{\frac{3x-2}{4x+1}}(4x+1)^2\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{11}{3}}}{\sqrt{4x+1}}\right),3\right)+220\left(6x^2-19x+10\right)\sqrt{4x+1}+55\sqrt{22}\sqrt{\frac{2x-}{4x-}}}{132\sqrt{2-3x}\sqrt{2x-5}(4x+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x)/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (220*Sqrt[1 + 4*x]*(10 - 19*x + 6*x^2) + 55*Sqrt[22]*Sqrt[(-5 + 2*x)/(1 + 4
*x)]*Sqrt[(-2 + 3*x)/(1 + 4*x)]*(1 + 4*x)^2*EllipticE[ArcSin[Sqrt[11/3]/Sqr
t[1 + 4*x]], 3] - 124*Sqrt[22]*Sqrt[(-5 + 2*x)/(1 + 4*x)]*Sqrt[(-2 + 3*x)/(
1 + 4*x)]*(1 + 4*x)^2*EllipticF[ArcSin[Sqrt[11/3]/Sqrt[1 + 4*x]], 3])/(132*
Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*(1 + 4*x))
```

fricas [F] time = 1.22, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{(5x+7)\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{24x^3-70x^2+21x+10},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral(-(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(24*x^3 - 70
*x^2 + 21*x + 10), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5\,x+7}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((5*x + 7)/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

maple [A] time = 0.02, size = 57, normalized size = 0.58

$$-\frac{\left(-55\,\mathrm{EllipticE}\left(\frac{2\sqrt{-33x+22}}{11},\frac{i\sqrt{2}}{2}\right)+117\,\mathrm{EllipticF}\left(\frac{2\sqrt{-33x+22}}{11},\frac{i\sqrt{2}}{2}\right)\right)\sqrt{-2x+5}\,\sqrt{11}}{66\sqrt{2x-5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x+7)/(-3*x+2)^(1/2)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] -1/66*(117*EllipticF(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))-55*EllipticE(2/11
*(-33*x+22)^(1/2),1/2*I*2^(1/2)))*(-2*x+5)^(1/2)*11^(1/2)/(2*x-5)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5\,x+7}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((5*x + 7)/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x+7}{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + 7)/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
```

```
[Out] int((5*x + 7)/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x+7}{\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral((5*x + 7)/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

3.64 $\int \frac{1}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx$

Optimal. Leaf size=48

$$\frac{\sqrt{\frac{2}{33}}\,\sqrt{5-2x}\ \text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{4x+1}\right),\frac{1}{3}\right)}{\sqrt{2x-5}}$$

```
[Out] 1/33*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*x)^(1
/2)/(-5+2*x)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {121, 119}

$$\frac{\sqrt{\frac{2}{33}}\,\sqrt{5-2x}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{4x+1}\right)|\frac{1}{3}\right)}{\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (Sqrt[2/33]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3/11]*Sqrt[1 + 4*x]], 1/3])
/Sqrt[-5 + 2*x]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx &= \frac{\left(\sqrt{\frac{2}{11}}\,\sqrt{5-2x}\right)\int \frac{1}{\sqrt{2-3x}\,\sqrt{\frac{10}{11}-\frac{4x}{11}}\,\sqrt{1+4x}}\,dx}{\sqrt{-5+2x}}\\ &= \frac{\sqrt{\frac{2}{33}}\,\sqrt{5-2x}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{1+4x}\right)|\frac{1}{3}\right)}{\sqrt{-5+2x}}\end{aligned}$$

Mathematica [A] time = 0.11, size = 79, normalized size = 1.65

$$-\frac{\sqrt{\frac{3x-2}{4x+1}}(4x+1)\sqrt{\frac{4x-10}{44x+11}}\ \text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{11}{3}}}{\sqrt{4x+1}}\right),3\right)}{\sqrt{2-3x}\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] -((Sqrt[(-2 + 3*x)/(1 + 4*x)]*(1 + 4*x)*Sqrt[(-10 + 4*x)/(11 + 44*x)]*Ellip
ticF[ArcSin[Sqrt[11/3]/Sqrt[1 + 4*x]], 3])/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]))
```

fricas [F] time = 1.07, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{24x^3-70x^2+21x+10},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(-sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(24*x^3 - 70*x^2 + 21*
x + 10), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

maple [C] time = 0.02, size = 36, normalized size = 0.75

$$-\frac{\sqrt{-2x+5}\sqrt{11}\ \text{EllipticF}\left(\frac{2\sqrt{-33x+22}}{11},\frac{i\sqrt{2}}{2}\right)}{11\sqrt{2x-5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x+2)^(1/2)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] -1/11*EllipticF(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))*(-2*x+5)^(1/2)*11^(1/2
)/(2*x-5)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(1/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
[Out] int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
[Out] Integral(1/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

3.65 $\int \frac{1}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)}\,dx$

Optimal. Leaf size=51

$$-\frac{3\sqrt{5-2x}\,\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{31\sqrt{11}\,\sqrt{2x-5}}$$

```
[Out] -3/341*EllipticPi(2/11*(2-3*x)^(1/2)*11^(1/2),55/124,1/2*I*2^(1/2))*(5-2*x)
^(1/2)*11^(1/2)/(-5+2*x)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {168, 538, 537}

$$-\frac{3\sqrt{5-2x}\,\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{31\sqrt{11}\,\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)),x]
```

```
[Out] (-3*Sqrt[5 - 2*x]*EllipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1
/2])/(31*Sqrt[11]*Sqrt[-5 + 2*x])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}\,dx = -\left(2\,\text{Subst}\left(\int \frac{1}{(31-5x^2)\sqrt{\frac{11}{3}-\frac{4x^2}{3}}\sqrt{-\frac{11}{3}-\frac{2x^2}{3}}}\,dx, x, \sqrt{2-3x}\right.\right.$$

$$= -\frac{\left(2\sqrt{\frac{3}{11}}\sqrt{5-2x}\right)\text{Subst}\left(\int \frac{1}{(31-5x^2)\sqrt{\frac{11}{3}-\frac{4x^2}{3}}\sqrt{1+\frac{2x^2}{11}}}\,dx, x, \sqrt{2-3\right.}{\sqrt{-5+2x}}$$

$$= -\frac{3\sqrt{5-2x}\,\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)}{31\sqrt{11}\sqrt{-5+2x}}$$

Mathematica [A] time = 0.49, size = 99, normalized size = 1.94

$$-\frac{3(3x-2)\sqrt{\frac{8x^2-18x-5}{(2-3x)^2}}\left(\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{11}}{2\sqrt{2-3x}}\right),-2\right)-\Pi\left(\frac{124}{55};\sin^{-1}\left(\frac{\sqrt{11}}{2\sqrt{2-3x}}\right)\middle|-2\right)\right)}{31\sqrt{4x+1}\sqrt{11x-\frac{55}{2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)),x]
```

```
[Out] (-3*(-2 + 3*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*(EllipticF[ArcSin[Sqrt
[11]/(2*Sqrt[2 - 3*x])], -2] - EllipticPi[124/55, ArcSin[Sqrt[11]/(2*Sqrt[2
 - 3*x])], -2]))/(31*Sqrt[1 + 4*x]*Sqrt[-55/2 + 11*x])
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{120x^4-182x^3-385x^2+197x+70}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(-sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(120*x^4 - 182*x^3 - 3
85*x^2 + 197*x + 70), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(5x+7)\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(1/((5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

maple [A] time = 0.02, size = 37, normalized size = 0.73

$$-\frac{3\sqrt{-2x+5}\sqrt{11}\ \text{EllipticPi}\left(\frac{2\sqrt{-33x+22}}{11},\frac{55}{124},\frac{i\sqrt{2}}{2}\right)}{341\sqrt{2x-5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x+7)/(-3*x+2)^(1/2)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] -3/341*EllipticPi(2/11*(-33*x+22)^(1/2),55/124,1/2*I*2^(1/2))*(-2*x+5)^(1/2
)*11^(1/2)/(2*x-5)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(5x+7)\sqrt{4x+1}\,\sqrt{2x-5}\,\sqrt{-3x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(1/((5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{2-3x}\,\sqrt{4x+1}\,\sqrt{2x-5}\,(5x+7)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)),x)
```

```
[Out] int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(5x+7)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)*(5*x + 7)), x)
```

3.66 $\int \frac{1}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^2}\,dx$

Optimal. Leaf size=189

$$-\frac{2\sqrt{\frac{6}{11}}\,\sqrt{5-2x}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{4x+1}\right),\frac{1}{3}\right)}{713\sqrt{2x-5}}-\frac{25\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{27807(5x+7)}+\frac{10\sqrt{11}\,\sqrt{2x-5}\,E\left(\sin^{-1}\right.}{27807\sqrt{5-}}$$

```
[Out] -2/7843*EllipticF(1/11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*x)
^(1/2)/(-5+2*x)^(1/2)-8953/6321458*EllipticPi(2/11*(2-3*x)^(1/2)*11^(1/2),5
5/124,1/2*I*2^(1/2))*(5-2*x)^(1/2)*11^(1/2)/(-5+2*x)^(1/2)+10/27807*Ellipti
cE(2/11*(2-3*x)^(1/2)*11^(1/2),1/2*I*2^(1/2))*11^(1/2)*(-5+2*x)^(1/2)/(5-2*
x)^(1/2)-25/27807*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)
```

Rubi [A] time = 0.22, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {172, 1607, 168, 538, 537, 158, 114, 113, 121, 119}

$$-\frac{25\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{27807(5x+7)}-\frac{2\sqrt{\frac{6}{11}}\,\sqrt{5-2x}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\,\sqrt{4x+1}\right)|\frac{1}{3}\right)}{713\sqrt{2x-5}}+\frac{10\sqrt{11}\,\sqrt{2x-5}\,E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right.\right.}{27807\sqrt{5-2x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2),x]
```

```
[Out] (-25*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(27807*(7 + 5*x)) + (10*Sq
rt[11]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/
(27807*Sqrt[5 - 2*x]) - (2*Sqrt[6/11]*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sqrt[3
/11]*Sqrt[1 + 4*x]], 1/3])/(713*Sqrt[-5 + 2*x]) - (8953*Sqrt[5 - 2*x]*Ellip
ticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2])/(574678*Sqrt[11]*S
qrt[-5 + 2*x])
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
```

```
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 172

```
Int[((a_.) + (b_.)*(x_))^(m_)/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*
(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(b^2*(a + b*x)^(m + 1)*S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((m + 1)*(b*c - a*d)*(b*e - a*f)*
(b*g - a*h)), x] - Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)),
Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[2*
a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(
d*e*g + c*f*g + c*e*h) - 2*b*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h +
c*f*h))*x + d*f*h*(2*m + 5)*b^2*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 1607

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
```

```
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
 q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}dx = -\frac{25\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{27807(7+5x)} + \frac{\int \frac{7777-1680x-600x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)}dx}{55614}$$

$$= -\frac{25\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{27807(7+5x)} + \frac{\int \frac{-168-120x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}dx}{55614} + \cdots$$

$$= -\frac{25\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{27807(7+5x)} - \frac{10\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}}dx}{9269} - \frac{6}{713}\int \cdots$$

$$= -\frac{25\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{27807(7+5x)} - \frac{\left(6\sqrt{\frac{2}{11}}\sqrt{5-2x}\right)\int \frac{\cdots}{\sqrt{2-3x}\sqrt{\cdots}}}{713\sqrt{-5+2x}}$$

$$= -\frac{25\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{27807(7+5x)} + \frac{10\sqrt{11}\sqrt{-5+2x}E\left(\sin^{-1}\left(\cdots\right.\right.}{27807\sqrt{5-2x}}$$

Mathematica [A] time = 0.70, size = 130, normalized size = 0.69

$$\frac{-3\sqrt{55-22x}\left(-14508\,\text{EllipticF}\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)+6820E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)|-\frac{1}{2}\right)+26859\Pi\left(\frac{55}{124};\sin^{-1}\left(\cdots\right.\right.\right.}{56893122\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^2),x]
```

```
[Out] ((-51150*Sqrt[2 - 3*x]*(-5 + 2*x)*Sqrt[1 + 4*x])/(7 + 5*x) - 3*Sqrt[55 - 22
*x]*(6820*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] - 14508*Ellip
ticF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 26859*EllipticPi[55/124, A
rcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/(56893122*Sqrt[-5 + 2*x])
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{600x^5-70x^4-3199x^3-1710x^2+1729x+490},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)^2/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(-sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(600*x^5 - 70*x^4 - 31
99*x^3 - 1710*x^2 + 1729*x + 490), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(5x+7)^2\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)^2/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(1/((5*x + 7)^2*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

maple [B] time = 0.02, size = 320, normalized size = 1.69

$$-\frac{\sqrt{-3x+2}\,\sqrt{2x-5}\,\sqrt{4x+1}\,\left(409200x^{3}-1193500x^{2}-34100\sqrt{11}\,\sqrt{-3x+2}\,\sqrt{-2x+5}\,\sqrt{4x+1}\,x\,\mathrm{EllipticE}\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x+7)^2/(-3*x+2)^(1/2)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] -1/18964374*(-3*x+2)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(72540*11^(1/2)*(-3*
x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(-33*x+22)^(1/2),1/2
*I*2^(1/2))*x-34100*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*El
lipticE(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))*x-134295*11^(1/2)*(-3*x+2)^(1/
2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticPi(2/11*(-33*x+22)^(1/2),55/124,1/2
*I*2^(1/2))*x+101556*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*E
llipticF(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))-47740*11^(1/2)*(-3*x+2)^(1/2)
*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticE(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2)
)-188013*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticPi(2/
11*(-33*x+22)^(1/2),55/124,1/2*I*2^(1/2))+409200*x^3-1193500*x^2+358050*x+1
70500)/(24*x^3-70*x^2+21*x+10)/(5*x+7)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(5\,x+7)^{2}\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)^2/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate(1/((5*x + 7)^2*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,{(5\,x+7)}^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^2),x)
```

```
[Out] int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(5x+7)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)**2/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)*(5*x + 7)**2), x)
```

3.67 $\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}\,dx$

Optimal. Leaf size=225

$$-\frac{24007\sqrt{5-2x}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right),\frac{1}{3}\right)}{6608797\sqrt{66}\sqrt{2x-5}}-\frac{223825\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{1030972332(5x+7)}-\frac{25\sqrt{2-3x}\sqrt{2x-5}}{55614(5x+}$$

```
[Out] -48493305/234374376808*EllipticPi(2/11*(2-3*x)^(1/2)*11^(1/2),55/124,1/2*I*
2^(1/2))*(5-2*x)^(1/2)*11^(1/2)/(-5+2*x)^(1/2)-24007/436180602*EllipticF(1/
11*33^(1/2)*(1+4*x)^(1/2),1/3*3^(1/2))*66^(1/2)*(5-2*x)^(1/2)/(-5+2*x)^(1/2
)+44765/515486166*EllipticE(2/11*(2-3*x)^(1/2)*11^(1/2),1/2*I*2^(1/2))*11^(
1/2)*(-5+2*x)^(1/2)/(5-2*x)^(1/2)-25/55614*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+
4*x)^(1/2)/(7+5*x)^2-223825/1030972332*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)
^(1/2)/(7+5*x)
```

Rubi [A] time = 0.31, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {172, 1604, 1607, 168, 538, 537, 158, 114, 113, 121, 119}

$$-\frac{223825\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{1030972332(5x+7)}-\frac{25\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{55614(5x+7)^2}-\frac{24007\sqrt{5-2x}\,F\left(\sin^{-1}\left(\sqrt{\frac{3}{11}}\sqrt{4x+1}\right)|\frac{1}{3}\right)}{6608797\sqrt{66}\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^3),x]
```

```
[Out] (-25*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(55614*(7 + 5*x)^2) - (223
825*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(1030972332*(7 + 5*x)) + (4
4765*Sqrt[11]*Sqrt[-5 + 2*x]*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]],
-1/2])/(515486166*Sqrt[5 - 2*x]) - (24007*Sqrt[5 - 2*x]*EllipticF[ArcSin[Sq
rt[3/11]*Sqrt[1 + 4*x]], 1/3])/(6608797*Sqrt[66]*Sqrt[-5 + 2*x]) - (4849330
5*Sqrt[5 - 2*x]*EllipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2
])/(21306761528*Sqrt[11]*Sqrt[-5 + 2*x])
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 119

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[(b*c - a*d)/b,
```

```
 0] && GtQ[(b*e - a*f)/b, 0] && PosQ[-(b/d)] &&  !(SimplerQ[c + d*x, a + b*
x] && GtQ[(d*e - c*f)/d, 0] && GtQ[-(d/b), 0]) &&  !(SimplerQ[c + d*x, a +
b*x] && GtQ[(-(b*e) + a*f)/f, 0] && GtQ[-(f/b), 0]) &&  !(SimplerQ[e + f*x,
 a + b*x] && GtQ[(-(d*e) + c*f)/f, 0] && GtQ[(-(b*e) + a*f)/f, 0] && (PosQ[
-(f/d)] || PosQ[-(f/b)]))
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 172

```
Int[((a_.) + (b_.)*(x_))^(m_)/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*
(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(b^2*(a + b*x)^(m + 1)*S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((m + 1)*(b*c - a*d)*(b*e - a*f)*
(b*g - a*h)), x] - Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)),
Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[2*
a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(
d*e*g + c*f*g + c*e*h) - 2*b*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h +
c*f*h))*x + d*f*h*(2*m + 5)*b^2*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 1604

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt
[e + f*x]*Sqrt[g + h*x])/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), x]
- Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m
 + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m +
 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
+ c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2
*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^
2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
+ c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 1607

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Dist[PolynomialRem
ainder[Px, a + b*x, x], Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q
, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*
x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p,
 q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^3}\,dx = -\frac{25\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{55614(7+5x)^2} + \frac{\int \frac{16079-6860x+600x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^2}\,d}{111228}$$

$$= -\frac{25\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{55614(7+5x)^2} - \frac{223825\sqrt{2-3x}\sqrt{-5+2x}\sqrt{}}{1030972332(7+5x)}$$

$$= -\frac{25\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{55614(7+5x)^2} - \frac{223825\sqrt{2-3x}\sqrt{-5+2x}\sqrt{}}{1030972332(7+5x)}$$

$$= -\frac{25\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{55614(7+5x)^2} - \frac{223825\sqrt{2-3x}\sqrt{-5+2x}\sqrt{}}{1030972332(7+5x)}$$

$$= -\frac{25\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{55614(7+5x)^2} - \frac{223825\sqrt{2-3x}\sqrt{-5+2x}\sqrt{}}{1030972332(7+5x)}$$

$$= -\frac{25\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{55614(7+5x)^2} - \frac{223825\sqrt{2-3x}\sqrt{-5+2x}\sqrt{}}{1030972332(7+5x)}$$

Mathematica [A] time = 0.46, size = 142, normalized size = 0.63

$$\frac{-\sqrt{55-22x}(5x+7)^2\left(-116097852\,\text{EllipticF}\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right),-\frac{1}{2}\right)+61059460E\left(\sin^{-1}\left(\frac{2\sqrt{2-3x}}{\sqrt{11}}\right)\middle|-\frac{1}{2}\right)+145\right.}{703123130424\sqrt{2x-5}\,(5x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^3),x]
```

```
[Out] (-17050*Sqrt[2 - 3*x]*(-5 + 2*x)*Sqrt[1 + 4*x]*(81209 + 44765*x) - Sqrt[55
- 22*x]*(7 + 5*x)^2*(61059460*EllipticE[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]],
 -1/2] - 116097852*EllipticF[ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2] + 14
5479915*EllipticPi[55/124, ArcSin[(2*Sqrt[2 - 3*x])/Sqrt[11]], -1/2]))/(703
123130424*Sqrt[-5 + 2*x]*(7 + 5*x)^2)
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}}{3000\,x^6+3850\,x^5-16485\,x^4-30943\,x^3-3325\,x^2+14553\,x+3430},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)^3/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorit
hm="fricas")
```

```
[Out] integral(-sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(3000*x^6 + 3850*x^5 -
 16485*x^4 - 30943*x^3 - 3325*x^2 + 14553*x + 3430), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(5\,x+7)^3\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)^3/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(1/((5*x + 7)^3*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

maple [B] time = 0.02, size = 461, normalized size = 2.05

$$-\frac{\sqrt{-3x+2}\,\sqrt{2x-5}\,\sqrt{4x+1}\,\left(18317838000x^4-20196304700x^3-1526486500\sqrt{11}\,\sqrt{-3x+2}\,\sqrt{-2x+5}\,\sqrt{4x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x+7)^3/(-3*x+2)^(1/2)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] -1/703123130424*(-3*x+2)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(2902446300*11^(
1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(-33*x+22)^
(1/2),1/2*I*2^(1/2))*x^2-1526486500*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*
(4*x+1)^(1/2)*EllipticE(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))*x^2-3636997875
*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticPi(2/11*(-33*
x+22)^(1/2),55/124,1/2*I*2^(1/2))*x^2+8126849640*11^(1/2)*(-3*x+2)^(1/2)*(-
2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticF(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))*x
-4274162200*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticE(
2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))*x-10183594050*11^(1/2)*(-3*x+2)^(1/2)*
(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticPi(2/11*(-33*x+22)^(1/2),55/124,1/2*I*
2^(1/2))*x+5688794748*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*
EllipticF(2/11*(-33*x+22)^(1/2),1/2*I*2^(1/2))-2991913540*11^(1/2)*(-3*x+2)
^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*EllipticE(2/11*(-33*x+22)^(1/2),1/2*I*2
^(1/2))-7128515835*11^(1/2)*(-3*x+2)^(1/2)*(-2*x+5)^(1/2)*(4*x+1)^(1/2)*Ell
ipticPi(2/11*(-33*x+22)^(1/2),55/124,1/2*I*2^(1/2))+18317838000*x^4-2019630
4700*x^3-80894833250*x^2+36709314950*x+13846134500)/(24*x^3-70*x^2+21*x+10)
/(5*x+7)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(5\,x+7)^3\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)^3/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algorit
hm="maxima")
```

```
[Out] integrate(1/((5*x + 7)^3*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,{\left(5\,x+7\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^3),x)
```

```
[Out] int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)**3/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Timed out
```

3.68 $\int \frac{ci+dix}{\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+hx}}\,dx$

Optimal. Leaf size=137

$$\frac{2i\sqrt{c+dx}\,\sqrt{eh-fg}\,\sqrt{\frac{f(g+hx)}{fg-eh}}\,E\left(\sin^{-1}\left(\frac{\sqrt{h}\,\sqrt{e+fx}}{\sqrt{eh-fg}}\right)|-\frac{d(fg-eh)}{(de-cf)h}\right)}{f\sqrt{h}\,\sqrt{g+hx}\,\sqrt{-\frac{f(c+dx)}{de-cf}}}$$

```
[Out] 2*i*EllipticE(h^(1/2)*(f*x+e)^(1/2)/(e*h-f*g)^(1/2),(-d*(-e*h+f*g)/(-c*f+d*
e)/h)^(1/2))*(e*h-f*g)^(1/2)*(d*x+c)^(1/2)*(f*(h*x+g)/(-e*h+f*g))^(1/2)/f/h
^(1/2)/(-f*(d*x+c)/(-c*f+d*e))^(1/2)/(h*x+g)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {21, 114, 113}

$$\frac{2i\sqrt{c+dx}\,\sqrt{eh-fg}\,\sqrt{\frac{f(g+hx)}{fg-eh}}\,E\left(\sin^{-1}\left(\frac{\sqrt{h}\,\sqrt{e+fx}}{\sqrt{eh-fg}}\right)|-\frac{d(fg-eh)}{(de-cf)h}\right)}{f\sqrt{h}\,\sqrt{g+hx}\,\sqrt{-\frac{f(c+dx)}{de-cf}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*i + d*i*x)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*Sqrt[-(f*g) + e*h]*i*Sqrt[c + d*x]*Sqrt[(f*(g + h*x))/(f*g - e*h)]*Ellip
ticE[ArcSin[(Sqrt[h]*Sqrt[e + f*x])/Sqrt[-(f*g) + e*h]], -((d*(f*g - e*h))/
((d*e - c*f)*h))])/(f*Sqrt[h]*Sqrt[-((f*(c + d*x))/(d*e - c*f))]*Sqrt[g + h
*x])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rubi steps

$$\int \frac{68c+68dx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = 68\int \frac{\sqrt{c+dx}}{\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$= \frac{\left(68\sqrt{c+dx}\sqrt{\frac{f(g+hx)}{fg-eh}}\right)\int \frac{\sqrt{\frac{cf}{-de+cf}+\frac{dfx}{-de+cf}}}{\sqrt{e+fx}\sqrt{\frac{fg}{fg-eh}+\frac{fhx}{fg-eh}}}\,dx}{\sqrt{\frac{f(c+dx)}{-de+cf}}\sqrt{g+hx}}$$

$$= \frac{136\sqrt{-fg+eh}\sqrt{c+dx}\sqrt{\frac{f(g+hx)}{fg-eh}}\,E\left(\sin^{-1}\left(\frac{\sqrt{h}\sqrt{e+fx}}{\sqrt{-fg+eh}}\right)|-\frac{d(fg-eh)}{(de-cf)h}\right)}{f\sqrt{h}\sqrt{-\frac{f(c+dx)}{de-cf}}\sqrt{g+hx}}$$

Mathematica [C] time = 0.60, size = 180, normalized size = 1.31

$$-\frac{2ii\sqrt{c+dx}\sqrt{g+hx}\left(E\left(i\sinh^{-1}\left(\sqrt{\frac{f(c+dx)}{de-cf}}\right)|\frac{deh-cfh}{dfg-cfh}\right)-\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{f(c+dx)}{de-cf}}\right),\frac{deh-cfh}{dfg-cfh}\right)\right)}{h\sqrt{e+fx}\sqrt{\frac{f(c+dx)}{d(e+fx)}}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*i + d*i*x)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] ((-2*I)*i*Sqrt[c + d*x]*Sqrt[g + h*x]*(EllipticE[I*ArcSinh[Sqrt[(f*(c + d*x
))/(d*e - c*f)]], (d*e*h - c*f*h)/(d*f*g - c*f*h)] - EllipticF[I*ArcSinh[Sq
rt[(f*(c + d*x))/(d*e - c*f)]], (d*e*h - c*f*h)/(d*f*g - c*f*h)]))/(h*Sqrt[
(f*(c + d*x))/(d*(e + f*x))]*Sqrt[e + f*x]*Sqrt[(d*(g + h*x))/(d*g - c*h)])
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}\,i}{fhx^2+eg+\left(fg+eh\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*i*x+c*i)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorith
m="fricas")
```

```
[Out] integral(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)*i/(f*h*x^2 + e*g + (f*g
+ e*h)*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dix+ci}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*i*x+c*i)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate((d*i*x + c*i)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

maple [B] time = 0.03, size = 552, normalized size = 4.03

$$2\left(-c^{2} f h \operatorname{EllipticE}\left(\sqrt{\frac{(dx+c)f}{cf-de}}, \sqrt{\frac{(cf-de)h}{(ch-dg)f}}\right)+c^{2} f h \operatorname{EllipticF}\left(\sqrt{\frac{(dx+c)f}{cf-de}}, \sqrt{\frac{(cf-de)h}{(ch-dg)f}}\right)+cdeh \operatorname{EllipticE}\left(\sqrt{\frac{(dx+c)f}{cf-de}}, \sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*i*x+c*i)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

```
[Out] 2*i*(EllipticF(((d*x+c)/(c*f-d*e)*f)^(1/2),((c*f-d*e)/(c*h-d*g)/f*h)^(1/2))
*c^2*f*h-EllipticF(((d*x+c)/(c*f-d*e)*f)^(1/2),((c*f-d*e)/(c*h-d*g)/f*h)^(1
/2))*c*d*e*h-EllipticF(((d*x+c)/(c*f-d*e)*f)^(1/2),((c*f-d*e)/(c*h-d*g)/f*h
)^(1/2))*c*d*f*g+EllipticF(((d*x+c)/(c*f-d*e)*f)^(1/2),((c*f-d*e)/(c*h-d*g)
/f*h)^(1/2))*d^2*e*g-EllipticE(((d*x+c)/(c*f-d*e)*f)^(1/2),((c*f-d*e)/(c*h-
d*g)/f*h)^(1/2))*c^2*f*h+EllipticE(((d*x+c)/(c*f-d*e)*f)^(1/2),((c*f-d*e)/(
c*h-d*g)/f*h)^(1/2))*c*d*e*h+EllipticE(((d*x+c)/(c*f-d*e)*f)^(1/2),((c*f-d*
e)/(c*h-d*g)/f*h)^(1/2))*c*d*f*g-EllipticE(((d*x+c)/(c*f-d*e)*f)^(1/2),((c*
f-d*e)/(c*h-d*g)/f*h)^(1/2))*d^2*e*g)/d*(-(f*x+e)/(c*f-d*e)*d)^(1/2)*(-(h*x
+g)/(c*h-d*g)*d)^(1/2)*((d*x+c)/(c*f-d*e)*f)^(1/2)/h/f*(d*x+c)^(1/2)*(f*x+e
)^(1/2)*(h*x+g)^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*
g*x+d*e*g*x+c*e*g)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dix+ci}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*i*x+c*i)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate((d*i*x + c*i)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c\,i+d\,i\,x}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*i + d*i*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)),x)
```

```
[Out] int((c*i + d*i*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$i\int \frac{\sqrt{c+dx}}{\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*i*x+c*i)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] i*Integral(sqrt(c + d*x)/(sqrt(e + f*x)*sqrt(g + h*x)), x)
```

3.69 $\int \frac{a+bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal. Leaf size=284

$$\frac{2b\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\,E\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}\,h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}-\frac{2(bg-ah)\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\ \text{EllipticF}}{d\sqrt{f}\,h\sqrt{e+fx}\sqrt{g+h}}$$

```
[Out] 2*b*EllipticE(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d
*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(h*x+g)^(1/2)/d/h/
f^(1/2)/(f*x+e)^(1/2)/(d*(h*x+g)/(-c*h+d*g))^(1/2)-2*(-a*h+b*g)*EllipticF(f
^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*(c*
f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)/d/h/
f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {158, 114, 113, 121, 120}

$$\frac{2b\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\,E\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}\,h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}-\frac{2(bg-ah)\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,F\left(\sin^{-1}\right.}{d\sqrt{f}\,h\sqrt{e+fx}\sqrt{g+hx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*b*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[g + h*x]*Ellip
ticE[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f
*(d*g - c*h))])/(d*Sqrt[f]*h*Sqrt[e + f*x]*Sqrt[(d*(g + h*x))/(d*g - c*h)])
 - (2*Sqrt[-(d*e) + c*f]*(b*g - a*h)*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(
d*(g + h*x))/(d*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d
*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(d*Sqrt[f]*h*Sqrt[e + f*x]*S
qrt[g + h*x])
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
```

```
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d),
 0] && GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a +
b*x, e + f*x] && (PosQ[-((b*c - a*d)/d)] || NegQ[-((b*e - a*f)/f)])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rubi steps

$$\int \frac{a+bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{b\int \frac{\sqrt{g+hx}}{\sqrt{c+dx}\sqrt{e+fx}}\,dx}{h} + \frac{(-bg+ah)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{h}$$

$$= \frac{\left((-bg+ah)\sqrt{\frac{d(e+fx)}{de-cf}}\right)\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{g+hx}}\,dx}{h\sqrt{e+fx}} + \frac{\left(b\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}\right)}{h\sqrt{e+fx}}$$

$$= \frac{2b\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}\,E\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}\,h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}} + \frac{\left((-bg+ah)\right.}{}$$

$$= \frac{2b\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}\,E\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}\,h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}} - \frac{2\sqrt{-de+cf}}{}$$

Mathematica [C] time = 1.97, size = 319, normalized size = 1.12

$$-\frac{2\left(idh(c+dx)^{3/2}(be-af)\sqrt{\frac{d(e+fx)}{f(c+dx)}}\sqrt{\frac{d(g+hx)}{h(c+dx)}}\ \text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{de}{f}-c}}{\sqrt{c+dx}}\right),\frac{dfg-cfh}{deh-cfh}\right)-bd^2(e+fx)(g+hx)\sqrt{\frac{de}{f}-}\right.}{d^2fh\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\sqrt{\frac{de}{f}-}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (-2*(-(b*d^2*Sqrt[-c + (d*e)/f]*(e + f*x)*(g + h*x)) - I*b*(d*e - c*f)*h*(c
 + d*x)^(3/2)*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c +
d*x))]*EllipticE[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*
h)/(d*e*h - c*f*h)] + I*d*(b*e - a*f)*h*(c + d*x)^(3/2)*Sqrt[(d*(e + f*x))/
(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticF[I*ArcSinh[Sqrt[-
c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)]))/(d^2*Sqrt[-
c + (d*e)/f]*f*h*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
```

fricas [F] time = 1.12, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(bx+a)\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}{dfhx^3+ceg+\left(dfg+\left(de+cf\right)h\right)x^2+\left(ceh+\left(de+cf\right)g\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="f
ricas")
```

```
[Out] integral((b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)/(d*f*h*x^3 + c
*e*g + (d*f*g + (d*e + c*f)*h)*x^2 + (c*e*h + (d*e + c*f)*g)*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx+a}{\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate((b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

maple [B] time = 0.04, size = 559, normalized size = 1.97

$$2\left(acdfh\,\text{EllipticF}\left(\sqrt{\frac{(dx+c)f}{cf-de}},\sqrt{\frac{\left(cf-de\right)h}{\left(ch-dg\right)f}}\right)-a\,d^2eh\,\text{EllipticF}\left(\sqrt{\frac{(dx+c)f}{cf-de}},\sqrt{\frac{\left(cf-de\right)h}{\left(ch-dg\right)f}}\right)-b\,c^2fh\,\text{EllipticE}\left(\sqrt{\frac{(dx+}{cf-}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

```
[Out] 2*(EllipticF(((d*x+c)/(c*f-d*e)*f)^(1/2),((c*f-d*e)/(c*h-d*g)/f*h)^(1/2))*a
*c*d*f*h-EllipticF(((d*x+c)/(c*f-d*e)*f)^(1/2),((c*f-d*e)/(c*h-d*g)/f*h)^(1
/2))*a*d^2*e*h-EllipticF(((d*x+c)/(c*f-d*e)*f)^(1/2),((c*f-d*e)/(c*h-d*g)/f
*h)^(1/2))*b*c*d*f*g+EllipticF(((d*x+c)/(c*f-d*e)*f)^(1/2),((c*f-d*e)/(c*h-
d*g)/f*h)^(1/2))*b*d^2*e*g-EllipticE(((d*x+c)/(c*f-d*e)*f)^(1/2),((c*f-d*e)
/(c*h-d*g)/f*h)^(1/2))*b*c^2*f*h+EllipticE(((d*x+c)/(c*f-d*e)*f)^(1/2),((c*
f-d*e)/(c*h-d*g)/f*h)^(1/2))*b*c*d*e*h+EllipticE(((d*x+c)/(c*f-d*e)*f)^(1/2
),((c*f-d*e)/(c*h-d*g)/f*h)^(1/2))*b*c*d*f*g-EllipticE(((d*x+c)/(c*f-d*e)*f
)^(1/2),((c*f-d*e)/(c*h-d*g)/f*h)^(1/2))*b*d^2*e*g)*(-(f*x+e)/(c*f-d*e)*d)^
(1/2)*(-(h*x+g)/(c*h-d*g)*d)^(1/2)*((d*x+c)/(c*f-d*e)*f)^(1/2)/h/f/d^2*(d*x
+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*
x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx+a}{\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate((b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,x}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)),x)
```

```
[Out] int((a + b*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+hx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] Integral((a + b*x)/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

3.70 $\int \frac{1}{(a+bx)\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+hx}}\,dx$

Optimal. Leaf size=165

$$-\frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\Pi\left(-\frac{b(de-cf)}{(bc-ad)f};\sin^{-1}\left(\frac{\sqrt{f}\,\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{\sqrt{f}\,\sqrt{e+fx}\,\sqrt{g+hx}\,(bc-ad)}$$

```
[Out] -2*EllipticPi(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),-b*(-c*f+d*e)/(-a*d+b*c
)/f,((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e
))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)/(-a*d+b*c)/f^(1/2)/(f*x+e)^(1/2)/(h*x
+g)^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {169, 538, 537}

$$-\frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\Pi\left(-\frac{b(de-cf)}{(bc-ad)f};\sin^{-1}\left(\frac{\sqrt{f}\,\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{\sqrt{f}\,\sqrt{e+fx}\,\sqrt{g+hx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (-2*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(
d*g - c*h)]*EllipticPi[-((b*(d*e - c*f))/((b*c - a*d)*f)), ArcSin[(Sqrt[f]*
Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/((b*c
 - a*d)*Sqrt[f]*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\left(2\,\text{Subst}\left(\int \frac{1}{\left(bc-ad-bx^2\right)\sqrt{e-\frac{cf}{d}+\frac{fx^2}{d}}\sqrt{g-\frac{ch}{d}+\frac{hx^2}{d}}}\,dx, x, \sqrt{c\right.\right.$$

$$= -\frac{\left(2\sqrt{\frac{d(e+fx)}{de-cf}}\right)\text{Subst}\left(\int \frac{1}{\left(bc-ad-bx^2\right)\sqrt{1+\frac{fx^2}{d\left(e-\frac{cf}{d}\right)}}\sqrt{g-\frac{ch}{d}+\frac{hx^2}{d}}}\,dx, x, \sqrt{c+dx\right.}{\sqrt{e+fx}}$$

$$= -\frac{\left(2\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\right)\text{Subst}\left(\int \frac{1}{\left(bc-ad-bx^2\right)\sqrt{1+\frac{fx^2}{d\left(e-\frac{cf}{d}\right)}}\sqrt{1+\frac{hx^2}{d\left(g-\frac{ch}{d}\right)}}}\,dx, \right.}{\sqrt{e+fx}\sqrt{g+hx}}$$

$$= -\frac{2\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\Pi\left(-\frac{b(de-cf)}{(bc-ad)f};\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-c}{f(dg-}\right.}{(bc-ad)\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$

Mathematica [C] time = 1.40, size = 226, normalized size = 1.37

$$\frac{2i(c+dx)\sqrt{\frac{d(e+fx)}{f(c+dx)}}\sqrt{\frac{d(g+hx)}{h(c+dx)}}\left(\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{de}{f}-c}}{\sqrt{c+dx}}\right),\frac{dfg-cfh}{deh-cfh}\right)-\Pi\left(-\frac{bcf-adf}{bde-bcf};i\sinh^{-1}\left(\frac{\sqrt{\frac{de}{f}-c}}{\sqrt{c+dx}}\right)|\frac{dfg-cfh}{deh-cfh}\right)\right)}{\sqrt{e+fx}\sqrt{g+hx}\,(ad-bc)\sqrt{\frac{de}{f}-c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] ((2*I)*(c + d*x)*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c
 + d*x))]*(EllipticF[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g -
c*f*h)/(d*e*h - c*f*h)] - EllipticPi[-((b*c*f - a*d*f)/(b*d*e - b*c*f)), I*
ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)]
))/((-(b*c) + a*d)*Sqrt[-c + (d*e)/f]*Sqrt[e + f*x]*Sqrt[g + h*x])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm=
"giac")
```

```
[Out] Exception raised: AttributeError >> type
```

maple [A] time = 0.03, size = 223, normalized size = 1.35

$$\frac{2\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}\,\sqrt{\frac{(dx+c)f}{cf-de}}\,\sqrt{-\frac{\left(hx+g\right)d}{ch-dg}}\,\sqrt{-\frac{\left(fx+e\right)d}{cf-de}}\,\left(cf-de\right)\mathrm{EllipticPi}\left(\sqrt{\frac{(dx+c)f}{cf-de}},-\frac{\left(cf-de\right)b}{(ad-bc)f},\sqrt{\frac{(cf-}{(ch-}}\right.}{(ad-bc)\left(dfh\,x^{3}+cfh\,x^{2}+deh\,x^{2}+dfg\,x^{2}+cehx+cfgx+degx+ceg\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

```
[Out] 2*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/f*((d*x+c)/(c*f-d*e)*f)^(1/2)*(
-(h*x+g)/(c*h-d*g)*d)^(1/2)*(-(f*x+e)/(c*f-d*e)*d)^(1/2)*EllipticPi(((d*x+c
)/(c*f-d*e)*f)^(1/2),-(c*f-d*e)/(a*d-b*c)*b/f,((c*f-d*e)/(c*h-d*g)/f*h)^(1/
2))*(c*f-d*e)/(a*d-b*c)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*
f*g*x+d*e*g*x+c*e*g)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm=
"maxima")
```

```
[Out] integrate(1/((b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\left(a+b\,x\right)\,\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)*(c + d*x)^(1/2)),x)
```

```
[Out] int(1/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)*(c + d*x)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+hx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

3.71 $\int \frac{1}{(a+bx)(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal. Leaf size=393

$$\frac{2d^2\sqrt{e+fx}\sqrt{g+hx}}{\sqrt{c+dx}\,(bc-ad)(de-cf)(dg-ch)}-\frac{2d\sqrt{h}\sqrt{c+dx}\sqrt{eh-fg}\sqrt{\frac{f(g+hx)}{fg-eh}}\,E\left(\sin^{-1}\left(\frac{\sqrt{h}\sqrt{e+fx}}{\sqrt{eh-fg}}\right)|-\frac{d(fg-eh)}{(de-cf)h}\right)}{\sqrt{g+hx}\,(bc-ad)(de-cf)(dg-ch)\sqrt{-\frac{f(c+dx)}{de-cf}}}-\frac{2b\sqrt{cf}}{}$$

```
[Out] 2*d^2*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/(-c*f+d*e)/(-c*h+d*g)/(d*x+c)^
(1/2)-2*b*EllipticPi(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),-b*(-c*f+d*e)/(-
a*d+b*c)/f,((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-
c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)/(-a*d+b*c)^2/f^(1/2)/(f*x+e)^(
1/2)/(h*x+g)^(1/2)-2*d*EllipticE(h^(1/2)*(f*x+e)^(1/2)/(e*h-f*g)^(1/2),(-d*
(-e*h+f*g)/(-c*f+d*e)/h)^(1/2))*h^(1/2)*(e*h-f*g)^(1/2)*(d*x+c)^(1/2)*(f*(h
*x+g)/(-e*h+f*g))^(1/2)/(-a*d+b*c)/(-c*f+d*e)/(-c*h+d*g)/(-f*(d*x+c)/(-c*f+
d*e))^(1/2)/(h*x+g)^(1/2)
```

Rubi [A] time = 0.62, antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {179, 104, 21, 114, 113, 169, 538, 537}

$$\frac{2d^2\sqrt{e+fx}\sqrt{g+hx}}{\sqrt{c+dx}\,(bc-ad)(de-cf)(dg-ch)}-\frac{2d\sqrt{h}\sqrt{c+dx}\sqrt{eh-fg}\sqrt{\frac{f(g+hx)}{fg-eh}}\,E\left(\sin^{-1}\left(\frac{\sqrt{h}\sqrt{e+fx}}{\sqrt{eh-fg}}\right)|-\frac{d(fg-eh)}{(de-cf)h}\right)}{\sqrt{g+hx}\,(bc-ad)(de-cf)(dg-ch)\sqrt{-\frac{f(c+dx)}{de-cf}}}-\frac{2b\sqrt{cf}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*d^2*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*d)*(d*e - c*f)*(d*g - c*h)*Sq
rt[c + d*x]) - (2*d*Sqrt[h]*Sqrt[-(f*g) + e*h]*Sqrt[c + d*x]*Sqrt[(f*(g + h
*x))/(f*g - e*h)]*EllipticE[ArcSin[(Sqrt[h]*Sqrt[e + f*x])/Sqrt[-(f*g) + e*
h]], -((d*(f*g - e*h))/((d*e - c*f)*h))])/((b*c - a*d)*(d*e - c*f)*(d*g - c
*h)*Sqrt[-((f*(c + d*x))/(d*e - c*f))]*Sqrt[g + h*x]) - (2*b*Sqrt[-(d*e) +
c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h)]*Ellipt
icPi[-((b*(d*e - c*f))/((b*c - a*d)*f)), ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqr
t[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/((b*c - a*d)^2*Sqrt[f]*
Sqrt[e + f*x]*Sqrt[g + h*x])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 179

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_))/(Sqrt[(e_.) + (f
_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Int[ExpandIntegrand[1/(Sq
rt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), (a + b*x)^m*(c + d*x)^(n + 1/2),
x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IntegerQ[m] && IntegerQ[n +
 1/2]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \left(-\frac{d}{(bc-ad)(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}} + \frac{b}{(bc-ad)(a+bx)\sqrt{c+\ldots}}\right.$$

$$= \frac{b\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{bc-ad} - \frac{d\int \frac{1}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{bc-ad}$$

$$= \frac{2d^2\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(de-cf)(dg-ch)\sqrt{c+dx}} - \frac{(2b)\,\mathrm{Subst}\left(\int \frac{}{\left(bc-ad-bx^2\right)\sqrt{e-\ldots}}\ldots\right.}{bc\ldots}$$

$$= \frac{2d^2\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(de-cf)(dg-ch)\sqrt{c+dx}} - \frac{(dfh)\int \frac{\sqrt{c+dx}}{\sqrt{e+fx}\sqrt{g+hx}}\,dx}{(bc-ad)(de-cf)(dg-ch)} - \ldots$$

$$= \frac{2d^2\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(de-cf)(dg-ch)\sqrt{c+dx}} - \frac{\left(2b\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\right)\mathrm{Subs}\ldots}{(b\ldots}$$

$$= \frac{2d^2\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(de-cf)(dg-ch)\sqrt{c+dx}} - \frac{2d\sqrt{h}\sqrt{-fg+eh}\sqrt{c+dx}\sqrt{\ldots}}{(bc-ad)(de-cf)\ldots}$$

Mathematica [C] time = 4.05, size = 322, normalized size = 0.82

$$\frac{2i(c+dx)\sqrt{\frac{d(e+fx)}{f(c+dx)}}\sqrt{\frac{d(g+hx)}{h(c+dx)}}\left((adf-2bcf+bde)\,\mathrm{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{dg}{h}-c}}{\sqrt{c+dx}}\right),\frac{deh-cfh}{dfg-cfh}\right)+f(bc-ad)E\left(i\sinh^{-1}\left(\ldots\right.\right.\right.}{\sqrt{e+fx}\sqrt{g+hx}(bc-ad)^2(cf-de)\sqrt{\frac{dg}{h}-c\ldots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(c + d*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] ((2*I)*(c + d*x)*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c
 + d*x))]*((b*c - a*d)*f*EllipticE[I*ArcSinh[Sqrt[-c + (d*g)/h]/Sqrt[c + d*
x]], (d*e*h - c*f*h)/(d*f*g - c*f*h)] + (b*d*e - 2*b*c*f + a*d*f)*EllipticF
[I*ArcSinh[Sqrt[-c + (d*g)/h]/Sqrt[c + d*x]], (d*e*h - c*f*h)/(d*f*g - c*f*
h)] + b*(-(d*e) + c*f)*EllipticPi[-((b*c*h - a*d*h)/(b*d*g - b*c*h)), I*Arc
Sinh[Sqrt[-c + (d*g)/h]/Sqrt[c + d*x]], (d*e*h - c*f*h)/(d*f*g - c*f*h)]))/
((b*c - a*d)^2*(-(d*e) + c*f)*Sqrt[-c + (d*g)/h]*Sqrt[e + f*x]*Sqrt[g + h*x
])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm=
"giac")
```

```
[Out] Timed out
```

maple [B] time = 0.08, size = 2842, normalized size = 7.23

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

```
[Out] -2*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)*(-EllipticF(((d*x+c)/(c*f-d*e)
*f)^(1/2),((c*f-d*e)/(c*h-d*g)/f*h)^(1/2))*b*c^2*d*f^2*g*((d*x+c)/(c*f-d*e)
*f)^(1/2)*(-(h*x+g)/(c*h-d*g)*d)^(1/2)*(-(f*x+e)/(c*f-d*e)*d)^(1/2)+Ellipti
cE(((d*x+c)/(c*f-d*e)*f)^(1/2),((c*f-d*e)/(c*h-d*g)/f*h)^(1/2))*a*c^2*d*f^2
*h*((d*x+c)/(c*f-d*e)*f)^(1/2)*(-(h*x+g)/(c*h-d*g)*d)^(1/2)*(-(f*x+e)/(c*f-
d*e)*d)^(1/2)-EllipticE(((d*x+c)/(c*f-d*e)*f)^(1/2),((c*f-d*e)/(c*h-d*g)/f*
h)^(1/2))*a*c*d^2*f^2*g*((d*x+c)/(c*f-d*e)*f)^(1/2)*(-(h*x+g)/(c*h-d*g)*d)^
(1/2)*(-(f*x+e)/(c*f-d*e)*d)^(1/2)+EllipticE(((d*x+c)/(c*f-d*e)*f)^(1/2),((
c*f-d*e)/(c*h-d*g)/f*h)^(1/2))*a*d^3*e*f*g*((d*x+c)/(c*f-d*e)*f)^(1/2)*(-(h
*x+g)/(c*h-d*g)*d)^(1/2)*(-(f*x+e)/(c*f-d*e)*d)^(1/2)+EllipticE(((d*x+c)/(c
*f-d*e)*f)^(1/2),((c*f-d*e)/(c*h-d*g)/f*h)^(1/2))*b*c^2*d*f^2*g*((d*x+c)/(c
*f-d*e)*f)^(1/2)*(-(h*x+g)/(c*h-d*g)*d)^(1/2)*(-(f*x+e)/(c*f-d*e)*d)^(1/2)-
EllipticPi(((d*x+c)/(c*f-d*e)*f)^(1/2),-(c*f-d*e)/(a*d-b*c)*b/f,((c*f-d*e)/
(c*h-d*g)/f*h)^(1/2))*b*c^2*d*f^2*g*((d*x+c)/(c*f-d*e)*f)^(1/2)*(-(h*x+g)/(
c*h-d*g)*d)^(1/2)*(-(f*x+e)/(c*f-d*e)*d)^(1/2)+EllipticPi(((d*x+c)/(c*f-d*e
)*f)^(1/2),-(c*f-d*e)/(a*d-b*c)*b/f,((c*f-d*e)/(c*h-d*g)/f*h)^(1/2))*b*c*d^
2*e^2*h*((d*x+c)/(c*f-d*e)*f)^(1/2)*(-(h*x+g)/(c*h-d*g)*d)^(1/2)*(-(f*x+e)/
(c*f-d*e)*d)^(1/2)-EllipticF(((d*x+c)/(c*f-d*e)*f)^(1/2),((c*f-d*e)/(c*h-d*
g)/f*h)^(1/2))*a*c^2*d*f^2*h*((d*x+c)/(c*f-d*e)*f)^(1/2)*(-(h*x+g)/(c*h-d*g
)*d)^(1/2)*(-(f*x+e)/(c*f-d*e)*d)^(1/2)+EllipticF(((d*x+c)/(c*f-d*e)*f)^(1/
2),((c*f-d*e)/(c*h-d*g)/f*h)^(1/2))*a*c*d^2*f^2*g*((d*x+c)/(c*f-d*e)*f)^(1/
2)*(-(h*x+g)/(c*h-d*g)*d)^(1/2)*(-(f*x+e)/(c*f-d*e)*d)^(1/2)-EllipticF(((d*
x+c)/(c*f-d*e)*f)^(1/2),((c*f-d*e)/(c*h-d*g)/f*h)^(1/2))*a*d^3*e*f*g*((d*x+
c)/(c*f-d*e)*f)^(1/2)*(-(h*x+g)/(c*h-d*g)*d)^(1/2)*(-(f*x+e)/(c*f-d*e)*d)^(
1/2)+EllipticF(((d*x+c)/(c*f-d*e)*f)^(1/2),((c*f-d*e)/(c*h-d*g)/f*h)^(1/2))
*a*c*d^2*e*f*h*((d*x+c)/(c*f-d*e)*f)^(1/2)*(-(h*x+g)/(c*h-d*g)*d)^(1/2)*(-(
f*x+e)/(c*f-d*e)*d)^(1/2)-EllipticF(((d*x+c)/(c*f-d*e)*f)^(1/2),((c*f-d*e)/
(c*h-d*g)/f*h)^(1/2))*b*c^2*d*e*f*h*((d*x+c)/(c*f-d*e)*f)^(1/2)*(-(h*x+g)/(
c*h-d*g)*d)^(1/2)*(-(f*x+e)/(c*f-d*e)*d)^(1/2)+EllipticF(((d*x+c)/(c*f-d*e)
*f)^(1/2),((c*f-d*e)/(c*h-d*g)/f*h)^(1/2))*b*c*d^2*e*f*g*((d*x+c)/(c*f-d*e)
*f)^(1/2)*(-(h*x+g)/(c*h-d*g)*d)^(1/2)*(-(f*x+e)/(c*f-d*e)*d)^(1/2)-Ellipti
cE(((d*x+c)/(c*f-d*e)*f)^(1/2),((c*f-d*e)/(c*h-d*g)/f*h)^(1/2))*a*c*d^2*e*f
*h*((d*x+c)/(c*f-d*e)*f)^(1/2)*(-(h*x+g)/(c*h-d*g)*d)^(1/2)*(-(f*x+e)/(c*f-
d*e)*d)^(1/2)+EllipticE(((d*x+c)/(c*f-d*e)*f)^(1/2),((c*f-d*e)/(c*h-d*g)/f*
h)^(1/2))*b*c^2*d*e*f*h*((d*x+c)/(c*f-d*e)*f)^(1/2)*(-(h*x+g)/(c*h-d*g)*d)^
(1/2)*(-(f*x+e)/(c*f-d*e)*d)^(1/2)-EllipticE(((d*x+c)/(c*f-d*e)*f)^(1/2),((
c*f-d*e)/(c*h-d*g)/f*h)^(1/2))*b*c*d^2*e*f*g*((d*x+c)/(c*f-d*e)*f)^(1/2)*(-
(h*x+g)/(c*h-d*g)*d)^(1/2)*(-(f*x+e)/(c*f-d*e)*d)^(1/2)-2*EllipticPi(((d*x+
c)/(c*f-d*e)*f)^(1/2),-(c*f-d*e)/(a*d-b*c)*b/f,((c*f-d*e)/(c*h-d*g)/f*h)^(1
/2))*b*c^2*d*e*f*h*((d*x+c)/(c*f-d*e)*f)^(1/2)*(-(h*x+g)/(c*h-d*g)*d)^(1/2)
```

```
*(-(f*x+e)/(c*f-d*e)*d)^(1/2)+2*EllipticPi(((d*x+c)/(c*f-d*e)*f)^(1/2),-(c*
f-d*e)/(a*d-b*c)*b/f,((c*f-d*e)/(c*h-d*g)/f*h)^(1/2))*b*c*d^2*e*f*g*((d*x+c
)/(c*f-d*e)*f)^(1/2)*(-(h*x+g)/(c*h-d*g)*d)^(1/2)*(-(f*x+e)/(c*f-d*e)*d)^(1
/2)+x*a*d^3*e*f*h-x*b*c*d^2*f^2*g-x^2*b*c*d^2*f^2*h+x^2*a*d^3*f^2*h-b*c*d^2
*e*f*g+x*a*d^3*f^2*g+a*d^3*e*f*g+EllipticPi(((d*x+c)/(c*f-d*e)*f)^(1/2),-(c
*f-d*e)/(a*d-b*c)*b/f,((c*f-d*e)/(c*h-d*g)/f*h)^(1/2))*b*c^3*f^2*h*((d*x+c)
/(c*f-d*e)*f)^(1/2)*(-(h*x+g)/(c*h-d*g)*d)^(1/2)*(-(f*x+e)/(c*f-d*e)*d)^(1/
2)-EllipticPi(((d*x+c)/(c*f-d*e)*f)^(1/2),-(c*f-d*e)/(a*d-b*c)*b/f,((c*f-d*
e)/(c*h-d*g)/f*h)^(1/2))*b*d^3*e^2*g*((d*x+c)/(c*f-d*e)*f)^(1/2)*(-(h*x+g)/
(c*h-d*g)*d)^(1/2)*(-(f*x+e)/(c*f-d*e)*d)^(1/2)+EllipticF(((d*x+c)/(c*f-d*e
)*f)^(1/2),((c*f-d*e)/(c*h-d*g)/f*h)^(1/2))*b*c^3*f^2*h*((d*x+c)/(c*f-d*e)*
f)^(1/2)*(-(h*x+g)/(c*h-d*g)*d)^(1/2)*(-(f*x+e)/(c*f-d*e)*d)^(1/2)-Elliptic
E(((d*x+c)/(c*f-d*e)*f)^(1/2),((c*f-d*e)/(c*h-d*g)/f*h)^(1/2))*b*c^3*f^2*h*
((d*x+c)/(c*f-d*e)*f)^(1/2)*(-(h*x+g)/(c*h-d*g)*d)^(1/2)*(-(f*x+e)/(c*f-d*e
)*d)^(1/2)-x*b*c*d^2*e*f*h)/f/(a*d-b*c)^2/(c*h-d*g)/(c*f-d*e)/(d*f*h*x^3+c*
f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)(dx+c)^{\frac{3}{2}}\sqrt{fx+e}\,\sqrt{hx+g}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm=
"maxima")
```

```
[Out] integrate(1/((b*x + a)*(d*x + c)^(3/2)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\left(a+b\,x\right)\,{\left(c+d\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)*(c + d*x)^(3/2)),x)
```

```
[Out] int(1/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)*(c + d*x)^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**(3/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] Timed out
```

3.72 $\int \frac{1}{(a+bx)(c+dx)^{5/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal. Leaf size=875

$$-\frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\Pi\left(-\frac{b(de-cf)}{(bc-ad)f};\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)b^2}{(bc-ad)^3\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}-\frac{2d\sqrt{h}\sqrt{eh-fg}\sqrt{c+dx}\sqrt{\frac{f(g+hx)}{fg-eh}}}{(bc-ad)^2(de-cf)(dg-}$$

```
[Out] 2/3*d^2*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/(-c*f+d*e)/(-c*h+d*g)/(d*x+c
)^(3/2)+2*b*d^2*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)^2/(-c*f+d*e)/(-c*h+d
*g)/(d*x+c)^(1/2)-4/3*d^2*(-2*c*f*h+d*e*h+d*f*g)*(f*x+e)^(1/2)*(h*x+g)^(1/2
)/(-a*d+b*c)/(-c*f+d*e)^2/(-c*h+d*g)^2/(d*x+c)^(1/2)+4/3*d*(-2*c*f*h+d*e*h+
d*f*g)*EllipticE(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*
h+d*g))^(1/2))*f^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c
)/(c*f-d*e)^(3/2)/(-c*h+d*g)^2/(f*x+e)^(1/2)/(d*(h*x+g)/(-c*h+d*g))^(1/2)-2
/3*(-3*c*f*h+d*e*h+2*d*f*g)*EllipticF(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2)
,((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*f^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d
*(h*x+g)/(-c*h+d*g))^(1/2)/(-a*d+b*c)/(c*f-d*e)^(3/2)/(-c*h+d*g)/(f*x+e)^(1
/2)/(h*x+g)^(1/2)-2*b^2*EllipticPi(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),-b
*(-c*f+d*e)/(-a*d+b*c)/f,((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))*(c*f-d*e)^(1/2)
*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)/(-a*d+b*c)^3/f^(
1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)-2*b*d*EllipticE(h^(1/2)*(f*x+e)^(1/2)/(e*h
-f*g)^(1/2),(-d*(-e*h+f*g)/(-c*f+d*e)/h)^(1/2))*h^(1/2)*(e*h-f*g)^(1/2)*(d*
x+c)^(1/2)*(f*(h*x+g)/(-e*h+f*g))^(1/2)/(-a*d+b*c)^2/(-c*f+d*e)/(-c*h+d*g)/
(-f*(d*x+c)/(-c*f+d*e))^(1/2)/(h*x+g)^(1/2)
```

Rubi [A] time = 1.34, antiderivative size = 875, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 12, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.343, Rules used = {179, 104, 152, 158, 114, 113, 121, 120, 21, 169, 538, 537}

$$-\frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\Pi\left(-\frac{b(de-cf)}{(bc-ad)f};\sin^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)b^2}{(bc-ad)^3\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}-\frac{2d\sqrt{h}\sqrt{eh-fg}\sqrt{c+dx}\sqrt{\frac{f(g+hx)}{fg-eh}}}{(bc-ad)^2(de-cf)(dg-}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)^(5/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*d^2*Sqrt[e + f*x]*Sqrt[g + h*x])/(3*(b*c - a*d)*(d*e - c*f)*(d*g - c*h)*
(c + d*x)^(3/2)) + (2*b*d^2*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*d)^2*(d*
e - c*f)*(d*g - c*h)*Sqrt[c + d*x]) - (4*d^2*(d*f*g + d*e*h - 2*c*f*h)*Sqrt
[e + f*x]*Sqrt[g + h*x])/(3*(b*c - a*d)*(d*e - c*f)^2*(d*g - c*h)^2*Sqrt[c
+ d*x]) + (4*d*Sqrt[f]*(d*f*g + d*e*h - 2*c*f*h)*Sqrt[(d*(e + f*x))/(d*e -
c*f)]*Sqrt[g + h*x]*EllipticE[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) +
c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))]])/(3*(b*c - a*d)*(-(d*e) + c*f)^(3/2
)*(d*g - c*h)^2*Sqrt[e + f*x]*Sqrt[(d*(g + h*x))/(d*g - c*h)]) - (2*b*d*Sqr
t[h]*Sqrt[-(f*g) + e*h]*Sqrt[c + d*x]*Sqrt[(f*(g + h*x))/(f*g - e*h)]*Ellip
ticE[ArcSin[(Sqrt[h]*Sqrt[e + f*x])/Sqrt[-(f*g) + e*h]], -((d*(f*g - e*h))/
((d*e - c*f)*h))]])/((b*c - a*d)^2*(d*e - c*f)*(d*g - c*h)*Sqrt[-((f*(c + d*
x))/(d*e - c*f))]*Sqrt[g + h*x]) - (2*Sqrt[f]*(2*d*f*g + d*e*h - 3*c*f*h)*S
qrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h)]*EllipticF[Ar
cSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g -
 c*h))]])/(3*(b*c - a*d)*(-(d*e) + c*f)^(3/2)*(d*g - c*h)*Sqrt[e + f*x]*Sqrt
[g + h*x]) - (2*b^2*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt
[(d*(g + h*x))/(d*g - c*h)]*EllipticPi[-((b*(d*e - c*f))/((b*c - a*d)*f)),
```

```
ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g
 - c*h))])/((b*c - a*d)^3*Sqrt[f]*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 113

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2*Rt[-((b*e - a*f)/d), 2]*EllipticE[ArcSin[Sqrt[a +
 b*x]/Rt[-((b*c - a*d)/d), 2]], (f*(b*c - a*d))/(d*(b*e - a*f))])/b, x] /;
FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f),
 0] &&  !LtQ[-((b*c - a*d)/d), 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-
(d/(b*c - a*d)), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[(Sqrt[e + f*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)])/(Sqr
t[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]), Int[Sqrt[(b*e)/(b*e - a*f) + (
b*f*x)/(b*e - a*f)]/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c -
a*d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-((b*c - a*d)/d), 0]
```

Rule 120

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Simp[(2*Rt[-(b/d), 2]*EllipticF[ArcSin[Sqrt[a + b*x]/(Rt
[-(b/d), 2]*Sqrt[(b*c - a*d)/b])], (f*(b*c - a*d))/(d*(b*e - a*f))])/(b*Sqr
t[(b*e - a*f)/b]), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d),
 0] && GtQ[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a +
b*x, e + f*x] && (PosQ[-((b*c - a*d)/d)] || NegQ[-((b*e - a*f)/f)])
```

Rule 121

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_Symbol] :> Dist[Sqrt[(b*(c + d*x))/(b*c - a*d)]/Sqrt[c + d*x], Int[
1/(Sqrt[a + b*x]*Sqrt[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)]*Sqrt[e + f*x
]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Si
mplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
```

```
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 158

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_Symbol] :> Dist[h/f, Int[Sqrt[e + f*x]/(Sqrt[a
+ b*x]*Sqrt[c + d*x]), x], x] + Dist[(f*g - e*h)/f, Int[1/(Sqrt[a + b*x]*Sq
rt[c + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 SimplerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 179

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_))/(Sqrt[(e_.) + (f
_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Int[ExpandIntegrand[1/(Sq
rt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), (a + b*x)^m*(c + d*x)^(n + 1/2),
x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IntegerQ[m] && IntegerQ[n +
 1/2]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)(c+dx)^{5/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int\Bigg(-\frac{d}{(bc-ad)(c+dx)^{5/2}\sqrt{e+fx}\sqrt{g+hx}} - \frac{bd}{(bc-ad)^2(c+dx)^{3/2}\sqrt{}}$$

$$= \frac{b^2\int\frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{(bc-ad)^2} - \frac{(bd)\int\frac{1}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{(bc-ad)^2} - \frac{d}{}$$

$$= \frac{2d^2\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(de-cf)(dg-ch)(c+dx)^{3/2}} + \frac{2bd^2\sqrt{e+fx}\sqrt{g+}}{(bc-ad)^2(de-cf)(dg-c}$$

$$= \frac{2d^2\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(de-cf)(dg-ch)(c+dx)^{3/2}} + \frac{2bd^2\sqrt{e+fx}\sqrt{g+}}{(bc-ad)^2(de-cf)(dg-c}$$

$$= \frac{2d^2\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(de-cf)(dg-ch)(c+dx)^{3/2}} + \frac{2bd^2\sqrt{e+fx}\sqrt{g+}}{(bc-ad)^2(de-cf)(dg-c}$$

$$= \frac{2d^2\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(de-cf)(dg-ch)(c+dx)^{3/2}} + \frac{2bd^2\sqrt{e+fx}\sqrt{g+}}{(bc-ad)^2(de-cf)(dg-c}$$

$$= \frac{2d^2\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(de-cf)(dg-ch)(c+dx)^{3/2}} + \frac{2bd^2\sqrt{e+fx}\sqrt{g+}}{(bc-ad)^2(de-cf)(dg-c}$$

$$= \frac{2d^2\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(de-cf)(dg-ch)(c+dx)^{3/2}} + \frac{2bd^2\sqrt{e+fx}\sqrt{g+}}{(bc-ad)^2(de-cf)(dg-c}$$

Mathematica [C] time = 15.77, size = 721, normalized size = 0.82

$$2\Bigg(d^2(e+fx)(g+hx)(bc-ad)\sqrt{\frac{dg}{h}-c}\,\Big((c+dx)\Big(2ad(-2cfh+deh+dfg)+b\Big(7c^2fh-5cd(eh+fg)+3d^2eg\Big)\Big)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(c + d*x)^(5/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*(d^2*(b*c - a*d)*Sqrt[-c + (d*g)/h]*(e + f*x)*(g + h*x)*((b*c - a*d)*(-(
d*e) + c*f)*(-(d*g) + c*h) + (2*a*d*(d*f*g + d*e*h - 2*c*f*h) + b*(3*d^2*e*
g + 7*c^2*f*h - 5*c*d*(f*g + e*h)))*(c + d*x)) - (c + d*x)*(d^2*(b*c - a*d)
*Sqrt[-c + (d*g)/h]*(2*a*d*(d*f*g + d*e*h - 2*c*f*h) + b*(3*d^2*e*g + 7*c^2
```

```
*f*h - 5*c*d*(f*g + e*h)))*(e + f*x)*(g + h*x) + I*(d*g - c*h)*(c + d*x)^(3
/2)*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*((b
*c - a*d)*f*(2*a*d*(d*f*g + d*e*h - 2*c*f*h) + b*(3*d^2*e*g + 7*c^2*f*h - 5
*c*d*(f*g + e*h)))*EllipticE[I*ArcSinh[Sqrt[-c + (d*g)/h]/Sqrt[c + d*x]], (
d*e*h - c*f*h)/(d*f*g - c*f*h)] + (a^2*d^2*f*(2*d*f*g + d*e*h - 3*c*f*h) +
b^2*(3*d^3*e^2*g - 9*c^3*f^2*h - 3*c*d^2*e*(3*f*g + e*h) + 2*c^2*d*f*(4*f*g
 + 5*e*h)) + a*b*d*f*(3*d^2*e*g + 9*c^2*f*h - c*d*(7*f*g + 5*e*h)))*Ellipti
cF[I*ArcSinh[Sqrt[-c + (d*g)/h]/Sqrt[c + d*x]], (d*e*h - c*f*h)/(d*f*g - c*
f*h)] - 3*b^2*(d*e - c*f)^2*(d*g - c*h)*EllipticPi[-((b*c*h - a*d*h)/(b*d*g
 - b*c*h)), I*ArcSinh[Sqrt[-c + (d*g)/h]/Sqrt[c + d*x]], (d*e*h - c*f*h)/(d
*f*g - c*f*h)]))))/(3*(b*c - a*d)^3*(d*e - c*f)^2*Sqrt[-c + (d*g)/h]*(d*g -
 c*h)^2*(c + d*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(5/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(5/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm=
"giac")
```

```
[Out] Timed out
```

maple [B] time = 0.28, size = 17330, normalized size = 19.81

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^(5/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)(dx+c)^{\frac{5}{2}}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(5/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm=
"maxima")
```

```
[Out] integrate(1/((b*x + a)*(d*x + c)^(5/2)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\left(a+b\,x\right)\,{\left(c+d\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)*(c + d*x)^(5/2)),x)
[Out] int(1/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)*(c + d*x)^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**(5/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
[Out] Timed out
```

3.73 $\int \frac{1}{(a+bx)\sqrt{c+dx}\,\sqrt{1-fx}\,\sqrt{1+fx}}\,dx$

Optimal. Leaf size=74

$$-\frac{2\sqrt{\frac{f(c+dx)}{cf+d}}\,\Pi\left(\frac{2b}{b+af};\sin^{-1}\left(\frac{\sqrt{1-fx}}{\sqrt{2}}\right)|\frac{2d}{d+cf}\right)}{(af+b)\sqrt{c+dx}}$$

```
[Out] -2*EllipticPi(1/2*(-f*x+1)^(1/2)*2^(1/2),2*b/(a*f+b),2^(1/2)*(d/(c*f+d))^(1
/2))*(f*(d*x+c)/(c*f+d))^(1/2)/(a*f+b)/(d*x+c)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {168, 538, 537}

$$-\frac{2\sqrt{\frac{f(c+dx)}{cf+d}}\,\Pi\left(\frac{2b}{b+af};\sin^{-1}\left(\frac{\sqrt{1-fx}}{\sqrt{2}}\right)|\frac{2d}{d+cf}\right)}{(af+b)\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[1 - f*x]*Sqrt[1 + f*x]),x]
```

```
[Out] (-2*Sqrt[(f*(c + d*x))/(d + c*f)]*EllipticPi[(2*b)/(b + a*f), ArcSin[Sqrt[1
 - f*x]/Sqrt[2]], (2*d)/(d + c*f)])/((b + a*f)*Sqrt[c + d*x])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-fx}\sqrt{1+fx}}\,dx = -\left(2\,\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\left(b+af-bx^2\right)\sqrt{c+\frac{d}{f}-\frac{dx^2}{f}}}\,dx, x, \sqrt{1-fx}\right)\right)$$

$$= -\frac{\left(2\sqrt{\frac{f(c+dx)}{d+cf}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\left(b+af-bx^2\right)\sqrt{1-\frac{dx^2}{\left(c+\frac{d}{f}\right)f}}}\,dx, x, \sqrt{1-fx}\right)}{\sqrt{c+dx}}$$

$$= -\frac{2\sqrt{\frac{f(c+dx)}{d+cf}}\,\Pi\left(\frac{2b}{b+af};\sin^{-1}\left(\frac{\sqrt{1-fx}}{\sqrt{2}}\right)|\frac{2d}{d+cf}\right)}{(b+af)\sqrt{c+dx}}$$

Mathematica [C] time = 0.91, size = 203, normalized size = 2.74

$$\frac{2i(c+dx)\sqrt{\frac{d(fx-1)}{f(c+dx)}}\sqrt{\frac{dfx+d}{cf+dfx}}\left(\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{cf+d}{f}}}{\sqrt{c+dx}}\right),\frac{cf-d}{cf+d}\right)-\Pi\left(\frac{bcf-adf}{bd+bcf};i\sinh^{-1}\left(\frac{\sqrt{-\frac{d+cf}{f}}}{\sqrt{c+dx}}\right)|\frac{cf-d}{d+cf}\right)\right)}{\sqrt{1-f^2x^2}\sqrt{-\frac{cf+d}{f}}(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[1 - f*x]*Sqrt[1 + f*x]),x]
```

```
[Out] ((2*I)*(c + d*x)*Sqrt[(d*(-1 + f*x))/(f*(c + d*x))]*Sqrt[(d + d*f*x)/(c*f +
 d*f*x)]*(EllipticF[I*ArcSinh[Sqrt[-((d + c*f)/f)]/Sqrt[c + d*x]], (-d + c*
f)/(d + c*f)] - EllipticPi[(b*c*f - a*d*f)/(b*d + b*c*f), I*ArcSinh[Sqrt[-(
(d + c*f)/f)]/Sqrt[c + d*x]], (-d + c*f)/(d + c*f)]))/((-(b*c) + a*d)*Sqrt[
-((d + c*f)/f)]*Sqrt[1 - f^2*x^2])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f*x+1)^(1/2)/(f*x+1)^(1/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)\sqrt{dx+c}\sqrt{fx+1}\sqrt{-fx+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f*x+1)^(1/2)/(f*x+1)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(1/((b*x + a)*sqrt(d*x + c)*sqrt(f*x + 1)*sqrt(-f*x + 1)), x)
```

maple [B] time = 0.10, size = 184, normalized size = 2.49

$$-\frac{2\left(cf-d\right)\sqrt{-\frac{\left(fx+1\right)d}{cf-d}}\,\sqrt{-\frac{\left(fx-1\right)d}{cf+d}}\,\sqrt{\frac{\left(dx+c\right)f}{cf-d}}\,\sqrt{fx+1}\,\sqrt{-fx+1}\,\sqrt{dx+c}\,\operatorname{EllipticPi}\left(\sqrt{\frac{\left(dx+c\right)f}{cf-d}},-\frac{\left(cf-d\right)b}{\left(ad-bc\right)f},\sqrt{\cdots}\right.}{\left(ad-bc\right)\left(d\,f^{2}x^{3}+c\,f^{2}x^{2}-dx-c\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^(1/2)/(-f*x+1)^(1/2)/(f*x+1)^(1/2),x)
```

```
[Out] -2*(c*f-d)*EllipticPi(((d*x+c)*f/(c*f-d))^(1/2),-(c*f-d)*b/f/(a*d-b*c),((c*
f-d)/(c*f+d))^(1/2))*(-(f*x+1)*d/(c*f-d))^(1/2)*(-(f*x-1)*d/(c*f+d))^(1/2)*
((d*x+c)*f/(c*f-d))^(1/2)*(f*x+1)^(1/2)*(-f*x+1)^(1/2)*(d*x+c)^(1/2)/f/(a*d
-b*c)/(d*f^2*x^3+c*f^2*x^2-d*x-c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)\sqrt{dx+c}\,\sqrt{fx+1}\,\sqrt{-fx+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f*x+1)^(1/2)/(f*x+1)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(1/((b*x + a)*sqrt(d*x + c)*sqrt(f*x + 1)*sqrt(-f*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{1-f\,x}\,\sqrt{f\,x+1}\,\left(a+b\,x\right)\,\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - f*x)^(1/2)*(f*x + 1)^(1/2)*(a + b*x)*(c + d*x)^(1/2)),x)
```

```
[Out] int(1/((1 - f*x)^(1/2)*(f*x + 1)^(1/2)*(a + b*x)*(c + d*x)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\,\sqrt{-fx+1}\,\sqrt{fx+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**(1/2)/(-f*x+1)**(1/2)/(f*x+1)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x)*sqrt(c + d*x)*sqrt(-f*x + 1)*sqrt(f*x + 1)), x)
```

3.74 $\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^2x^2}}\,dx$

Optimal. Leaf size=74

$$-\frac{2\sqrt{\frac{f(c+dx)}{cf+d}}\,\Pi\left(\frac{2b}{b+af};\sin^{-1}\left(\frac{\sqrt{1-fx}}{\sqrt{2}}\right)|\frac{2d}{d+cf}\right)}{(af+b)\sqrt{c+dx}}$$

```
[Out] -2*EllipticPi(1/2*(-f*x+1)^(1/2)*2^(1/2),2*b/(a*f+b),2^(1/2)*(d/(c*f+d))^(1
/2))*(f*(d*x+c)/(c*f+d))^(1/2)/(a*f+b)/(d*x+c)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {932, 168, 538, 537}

$$-\frac{2\sqrt{\frac{f(c+dx)}{cf+d}}\,\Pi\left(\frac{2b}{b+af};\sin^{-1}\left(\frac{\sqrt{1-fx}}{\sqrt{2}}\right)|\frac{2d}{d+cf}\right)}{(af+b)\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[1 - f^2*x^2]),x]
```

```
[Out] (-2*Sqrt[(f*(c + d*x))/(d + c*f)]*EllipticPi[(2*b)/(b + a*f), ArcSin[Sqrt[1
 - f*x]/Sqrt[2]], (2*d)/(d + c*f)])/((b + a*f)*Sqrt[c + d*x])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 932

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*
x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^2x^2}}\,dx = \int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-fx}\sqrt{1+fx}}\,dx$$

$$= -\left(2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\left(b+af-bx^2\right)\sqrt{c+\frac{d}{f}-\frac{dx^2}{f}}}\,dx, x, \sqrt{1-fx}\right)\right)$$

$$= -\frac{\left(2\sqrt{\frac{f(c+dx)}{d+cf}}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\left(b+af-bx^2\right)\sqrt{1-\frac{dx^2}{\left(c+\frac{d}{f}\right)f}}}\,dx, x, \sqrt{1-fx}\right)}{\sqrt{c+dx}}$$

$$= -\frac{2\sqrt{\frac{f(c+dx)}{d+cf}}\,\Pi\left(\frac{2b}{b+af};\sin^{-1}\left(\frac{\sqrt{1-fx}}{\sqrt{2}}\right)|\frac{2d}{d+cf}\right)}{(b+af)\sqrt{c+dx}}$$

Mathematica [C] time = 0.17, size = 203, normalized size = 2.74

$$\frac{2i(c+dx)\sqrt{\frac{d(fx-1)}{f(c+dx)}}\sqrt{\frac{dfx+d}{cf+dfx}}\left(\mathrm{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{cf+d}{f}}}{\sqrt{c+dx}}\right),\frac{cf-d}{cf+d}\right)-\Pi\left(\frac{bcf-adf}{bd+bcf};i\sinh^{-1}\left(\frac{\sqrt{-\frac{d+cf}{f}}}{\sqrt{c+dx}}\right)|\frac{cf-d}{d+cf}\right)\right)}{\sqrt{1-f^2x^2}\sqrt{-\frac{cf+d}{f}}\,(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[1 - f^2*x^2]),x]
```

```
[Out] ((2*I)*(c + d*x)*Sqrt[(d*(-1 + f*x))/(f*(c + d*x))]*Sqrt[(d + d*f*x)/(c*f +
 d*f*x)]*(EllipticF[I*ArcSinh[Sqrt[-((d + c*f)/f)]/Sqrt[c + d*x]], (-d + c*
f)/(d + c*f)] - EllipticPi[(b*c*f - a*d*f)/(b*d + b*c*f), I*ArcSinh[Sqrt[-(
(d + c*f)/f)]/Sqrt[c + d*x]], (-d + c*f)/(d + c*f)]))/((-(b*c) + a*d)*Sqrt[
-((d + c*f)/f)]*Sqrt[1 - f^2*x^2])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-f^2x^2+1}\,(bx+a)\sqrt{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-f^2*x^2 + 1)*(b*x + a)*sqrt(d*x + c)), x)
```

maple [B] time = 0.03, size = 181, normalized size = 2.45

$$-\frac{2\left(cf-d\right)\sqrt{-\frac{\left(fx+1\right)d}{cf-d}}\,\sqrt{-\frac{\left(fx-1\right)d}{cf+d}}\,\sqrt{\frac{(dx+c)f}{cf-d}}\,\sqrt{-f^2x^2+1}\,\sqrt{dx+c}\,\operatorname{EllipticPi}\left(\sqrt{\frac{(dx+c)f}{cf-d}},-\frac{\left(cf-d\right)b}{(ad-bc)f},\sqrt{\frac{cf-d}{cf+d}}\right)}{(ad-bc)\left(d\,f^2x^3+c\,f^2x^2-dx-c\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^(1/2)/(-f^2*x^2+1)^(1/2),x)
```

```
[Out] -2*(c*f-d)*EllipticPi(((d*x+c)/(c*f-d)*f)^(1/2),-(c*f-d)/(a*d-b*c)*b/f,((c*
f-d)/(c*f+d))^(1/2))*(-(f*x+1)/(c*f-d)*d)^(1/2)*(-(f*x-1)/(c*f+d)*d)^(1/2)*
((d*x+c)/(c*f-d)*f)^(1/2)*(-f^2*x^2+1)^(1/2)*(d*x+c)^(1/2)/f/(a*d-b*c)/(d*f
^2*x^3+c*f^2*x^2-d*x-c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-f^2x^2+1}\,(bx+a)\sqrt{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-f^2*x^2 + 1)*(b*x + a)*sqrt(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{\sqrt{1-f^2\,x^2}\,\left(a+b\,x\right)\,\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - f^2*x^2)^(1/2)*(a + b*x)*(c + d*x)^(1/2)),x)
```

```
[Out] int(1/((1 - f^2*x^2)^(1/2)*(a + b*x)*(c + d*x)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-\left(fx-1\right)\left(fx+1\right)}\,\left(a+bx\right)\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**(1/2)/(-f**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(f*x - 1)*(f*x + 1))*(a + b*x)*sqrt(c + d*x)), x)
```

3.75 $\int \frac{1}{(a+bx)\sqrt{c+dx}\,\sqrt{1-f^2x}\,\sqrt{1+f^2x}}\,dx$

Optimal. Leaf size=86

$$-\frac{2\sqrt{\frac{f^2(c+dx)}{cf^2+d}}\,\Pi\left(\frac{2b}{af^2+b};\sin^{-1}\left(\frac{\sqrt{1-f^2x}}{\sqrt{2}}\right)|\frac{2d}{cf^2+d}\right)}{\left(af^2+b\right)\sqrt{c+dx}}$$

```
[Out] -2*EllipticPi(1/2*(-f^2*x+1)^(1/2)*2^(1/2),2*b/(a*f^2+b),2^(1/2)*(d/(c*f^2+
d))^(1/2))*(f^2*(d*x+c)/(c*f^2+d))^(1/2)/(a*f^2+b)/(d*x+c)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {168, 538, 537}

$$-\frac{2\sqrt{\frac{f^2(c+dx)}{cf^2+d}}\,\Pi\left(\frac{2b}{af^2+b};\sin^{-1}\left(\frac{\sqrt{1-f^2x}}{\sqrt{2}}\right)|\frac{2d}{cf^2+d}\right)}{\left(af^2+b\right)\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[1 - f^2*x]*Sqrt[1 + f^2*x]),x]
```

```
[Out] (-2*Sqrt[(f^2*(c + d*x))/(d + c*f^2)]*EllipticPi[(2*b)/(b + a*f^2), ArcSin[
Sqrt[1 - f^2*x]/Sqrt[2]], (2*d)/(d + c*f^2)])/((b + a*f^2)*Sqrt[c + d*x])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^2x}\sqrt{1+f^2x}}\,dx = -\left(2\,\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\left(b+af^2-bx^2\right)\sqrt{c+\frac{d}{f^2}-\frac{dx^2}{f^2}}}\,dx, x, \sqrt{1-\right.\right.$$

$$= -\frac{\left(2\sqrt{\frac{f^2(c+dx)}{d+cf^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\left(b+af^2-bx^2\right)\sqrt{1-\frac{dx^2}{\left(c+\frac{d}{f^2}\right)f^2}}}\,dx, x, \sqrt{1-f^2\right.}{\sqrt{c+dx}}$$

$$= -\frac{2\sqrt{\frac{f^2(c+dx)}{d+cf^2}}\,\Pi\left(\frac{2b}{b+af^2};\sin^{-1}\left(\frac{\sqrt{1-f^2x}}{\sqrt{2}}\right)|\frac{2d}{d+cf^2}\right)}{\left(b+af^2\right)\sqrt{c+dx}}$$

Mathematica [C] time = 0.86, size = 218, normalized size = 2.53

$$\frac{2i(c+dx)\sqrt{\frac{d(f^2x-1)}{f^2(c+dx)}}\sqrt{\frac{d(f^2x+1)}{f^2(c+dx)}}\left(\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-c-\frac{d}{f^2}}}{\sqrt{c+dx}}\right),\frac{cf^2-d}{cf^2+d}\right)-\Pi\left(\frac{(bc-ad)f^2}{b\left(cf^2+d\right)};i\sinh^{-1}\left(\frac{\sqrt{-c-\frac{d}{f^2}}}{\sqrt{c+dx}}\right)|\frac{cf^2-d}{cf^2+d}\right)\right)}{\sqrt{1-f^4x^2}\sqrt{-c-\frac{d}{f^2}}\,(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[1 - f^2*x]*Sqrt[1 + f^2*x]),x]
```

```
[Out] ((2*I)*(c + d*x)*Sqrt[(d*(-1 + f^2*x))/(f^2*(c + d*x))]*Sqrt[(d*(1 + f^2*x)
)/(f^2*(c + d*x))]*(EllipticF[I*ArcSinh[Sqrt[-c - d/f^2]/Sqrt[c + d*x]], (-
d + c*f^2)/(d + c*f^2)] - EllipticPi[((b*c - a*d)*f^2)/(b*(d + c*f^2)), I*A
rcSinh[Sqrt[-c - d/f^2]/Sqrt[c + d*x]], (-d + c*f^2)/(d + c*f^2)]))/((-(b*c
) + a*d)*Sqrt[-c - d/f^2]*Sqrt[1 - f^4*x^2])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f^2*x+1)^(1/2)/(f^2*x+1)^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f^2*x+1)^(1/2)/(f^2*x+1)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.15, size = 212, normalized size = 2.47

$$-\frac{2\left(c\,f^2-d\right)\sqrt{-\frac{\left(f^2x+1\right)d}{c\,f^2-d}}\sqrt{-\frac{\left(f^2x-1\right)d}{c\,f^2+d}}\sqrt{\frac{(dx+c)f^2}{c\,f^2-d}}\sqrt{f^2x+1}\sqrt{-f^2x+1}\sqrt{dx+c}\;\text{EllipticPi}\left(\sqrt{\frac{(dx+c)f^2}{c\,f^2-d}},-\frac{\left(c\,f^2-d\right)b}{(ad-bc)f^2}\right.}{(ad-bc)\left(d\,f^4x^3+c\,f^4x^2-dx-c\right)f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^(1/2)/(-f^2*x+1)^(1/2)/(f^2*x+1)^(1/2),x)
```

```
[Out] -2*(c*f^2-d)*EllipticPi(((d*x+c)*f^2/(c*f^2-d))^(1/2),-(c*f^2-d)*b/f^2/(a*d
-b*c),((c*f^2-d)/(c*f^2+d))^(1/2))*(-(f^2*x+1)*d/(c*f^2-d))^(1/2)*(-(f^2*x-
1)*d/(c*f^2+d))^(1/2)*((d*x+c)*f^2/(c*f^2-d))^(1/2)*(f^2*x+1)^(1/2)*(-f^2*x
+1)^(1/2)*(d*x+c)^(1/2)/f^2/(a*d-b*c)/(d*f^4*x^3+c*f^4*x^2-d*x-c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{f^2x+1}\,\sqrt{-f^2x+1}\,(bx+a)\sqrt{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f^2*x+1)^(1/2)/(f^2*x+1)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(1/(sqrt(f^2*x + 1)*sqrt(-f^2*x + 1)*(b*x + a)*sqrt(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x)\,\sqrt{1-f^2\,x}\,\sqrt{x\,f^2+1}\,\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)*(1 - f^2*x)^(1/2)*(f^2*x + 1)^(1/2)*(c + d*x)^(1/2)),x)
```

```
[Out] int(1/((a + b*x)*(1 - f^2*x)^(1/2)*(f^2*x + 1)^(1/2)*(c + d*x)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)\,\sqrt{c+dx}\,\sqrt{-f^2x+1}\,\sqrt{f^2x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**(1/2)/(-f**2*x+1)**(1/2)/(f**2*x+1)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x)*sqrt(c + d*x)*sqrt(-f**2*x + 1)*sqrt(f**2*x + 1)), x)
```

3.76 $\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^4x^2}}\,dx$

Optimal. Leaf size=86

$$-\frac{2\sqrt{\frac{f^2(c+dx)}{cf^2+d}}\,\Pi\left(\frac{2b}{af^2+b};\sin^{-1}\left(\frac{\sqrt{1-f^2x}}{\sqrt{2}}\right)|\frac{2d}{cf^2+d}\right)}{\left(af^2+b\right)\sqrt{c+dx}}$$

```
[Out] -2*EllipticPi(1/2*(-f^2*x+1)^(1/2)*2^(1/2),2*b/(a*f^2+b),2^(1/2)*(d/(c*f^2+
d))^(1/2))*(f^2*(d*x+c)/(c*f^2+d))^(1/2)/(a*f^2+b)/(d*x+c)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {932, 168, 538, 537}

$$-\frac{2\sqrt{\frac{f^2(c+dx)}{cf^2+d}}\,\Pi\left(\frac{2b}{af^2+b};\sin^{-1}\left(\frac{\sqrt{1-f^2x}}{\sqrt{2}}\right)|\frac{2d}{cf^2+d}\right)}{\left(af^2+b\right)\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[1 - f^4*x^2]),x]
```

```
[Out] (-2*Sqrt[(f^2*(c + d*x))/(d + c*f^2)]*EllipticPi[(2*b)/(b + a*f^2), ArcSin[
Sqrt[1 - f^2*x]/Sqrt[2]], (2*d)/(d + c*f^2)])/((b + a*f^2)*Sqrt[c + d*x])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 932

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*
x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^4x^2}}\,dx = \int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{1-f^2x}\sqrt{1+f^2x}}\,dx$$

$$= -\left(2\,\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\left(b+af^2-bx^2\right)\sqrt{c+\frac{d}{f^2}-\frac{dx^2}{f^2}}}\,dx, x, \sqrt{1-f^2x}\right)\right)$$

$$= -\frac{\left(2\sqrt{\frac{f^2(c+dx)}{d+cf^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}\left(b+af^2-bx^2\right)\sqrt{1-\frac{dx^2}{\left(c+\frac{d}{f^2}\right)f^2}}}\,dx, x, \sqrt{1-f^2x}\right)}{\sqrt{c+dx}}$$

$$= -\frac{2\sqrt{\frac{f^2(c+dx)}{d+cf^2}}\,\Pi\left(\frac{2b}{b+af^2};\sin^{-1}\left(\frac{\sqrt{1-f^2x}}{\sqrt{2}}\right)|\frac{2d}{d+cf^2}\right)}{\left(b+af^2\right)\sqrt{c+dx}}$$

Mathematica [C] time = 0.16, size = 218, normalized size = 2.53

$$\frac{2i(c+dx)\sqrt{\frac{d\left(f^2x-1\right)}{f^2(c+dx)}}\sqrt{\frac{d\left(f^2x+1\right)}{f^2(c+dx)}}\left(\text{EllipticF}\left(i\sinh^{-1}\left(\frac{\sqrt{-c-\frac{d}{f^2}}}{\sqrt{c+dx}}\right),\frac{cf^2-d}{cf^2+d}\right)-\Pi\left(\frac{(bc-ad)f^2}{b\left(cf^2+d\right)};i\sinh^{-1}\left(\frac{\sqrt{-c-\frac{d}{f^2}}}{\sqrt{c+dx}}\right)|\frac{cf^2-d}{cf^2+d}\right)\right)}{\sqrt{1-f^4x^2}\sqrt{-c-\frac{d}{f^2}}\,(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*Sqrt[c + d*x]*Sqrt[1 - f^4*x^2]),x]
```

```
[Out] ((2*I)*(c + d*x)*Sqrt[(d*(-1 + f^2*x))/(f^2*(c + d*x))]*Sqrt[(d*(1 + f^2*x)
)/(f^2*(c + d*x))]*(EllipticF[I*ArcSinh[Sqrt[-c - d/f^2]/Sqrt[c + d*x]], (-
d + c*f^2)/(d + c*f^2)] - EllipticPi[((b*c - a*d)*f^2)/(b*(d + c*f^2)), I*A
rcSinh[Sqrt[-c - d/f^2]/Sqrt[c + d*x]], (-d + c*f^2)/(d + c*f^2)]))/((-(b*c
) + a*d)*Sqrt[-c - d/f^2]*Sqrt[1 - f^4*x^2])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f^4*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-f^4x^2+1}\,(bx+a)\sqrt{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f^4*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-f^4*x^2 + 1)*(b*x + a)*sqrt(d*x + c)), x)
```

maple [B] time = 0.03, size = 205, normalized size = 2.38

$$-\frac{2\left(c f^2-d\right)\sqrt{-\frac{\left(f^2x+1\right)d}{c f^2-d}}\,\sqrt{-\frac{\left(f^2x-1\right)d}{c f^2+d}}\,\sqrt{\frac{(dx+c)f^2}{c f^2-d}}\,\sqrt{-f^4x^2+1}\,\sqrt{dx+c}\,\mathrm{EllipticPi}\left(\sqrt{\frac{(dx+c)f^2}{c f^2-d}},-\frac{\left(c f^2-d\right)b}{(ad-bc)f^2},\sqrt{\frac{c f^2-d}{c f^2+d}}\right)}{(ad-bc)\left(d f^4x^3+c f^4x^2-dx-c\right)f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^(1/2)/(-f^4*x^2+1)^(1/2),x)
```

```
[Out] -2*(c*f^2-d)*EllipticPi(((d*x+c)/(c*f^2-d)*f^2)^(1/2),-(c*f^2-d)/(a*d-b*c)*
b/f^2,((c*f^2-d)/(c*f^2+d))^(1/2))*(-(f^2*x+1)/(c*f^2-d)*d)^(1/2)*(-(f^2*x-
1)/(c*f^2+d)*d)^(1/2)*((d*x+c)/(c*f^2-d)*f^2)^(1/2)*(-f^4*x^2+1)^(1/2)*(d*x
+c)^(1/2)/f^2/(a*d-b*c)/(d*f^4*x^3+c*f^4*x^2-d*x-c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-f^4x^2+1}\,(bx+a)\sqrt{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2)/(-f^4*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-f^4*x^2 + 1)*(b*x + a)*sqrt(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{1-f^4\,x^2}\,\left(a+b\,x\right)\,\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - f^4*x^2)^(1/2)*(a + b*x)*(c + d*x)^(1/2)),x)
```

```
[Out] int(1/((1 - f^4*x^2)^(1/2)*(a + b*x)*(c + d*x)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\left(f^2x-1\right)\left(f^2x+1\right)}\,(a+bx)\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**(1/2)/(-f**4*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(f**2*x - 1)*(f**2*x + 1))*(a + b*x)*sqrt(c + d*x)), x)
```

3.77 $\int \sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^{5/2}\,dx$

Optimal. Leaf size=471

$$-\frac{245264762213\sqrt{\frac{11}{23}}\sqrt{5x+7}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{99532800\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}+\frac{1}{25}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(5x+7)^{7/2}-\frac{42}{}$$

```
[Out] -83363/34560*(7+5*x)^(3/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)-427/2
400*(7+5*x)^(5/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)+1/25*(7+5*x)^(
7/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)-57691792727443/213497856000
*(2-3*x)*EllipticPi(1/23*253^(1/2)*(7+5*x)^(1/2)/(2-3*x)^(1/2),-69/55,1/39*
I*897^(1/2))*((5-2*x)/(2-3*x))^(1/2)*((-1-4*x)/(2-3*x))^(1/2)*429^(1/2)/(-5
+2*x)^(1/2)/(1+4*x)^(1/2)-1450582567/3686400*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7
+5*x)^(1/2)/(-5+2*x)^(1/2)-70489981/1658880*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1
+4*x)^(1/2)*(7+5*x)^(1/2)-245264762213/2289254400*(1/(4+2*(1+4*x)/(2-3*x)))
^(1/2)*(4+2*(1+4*x)/(2-3*x))^(1/2)*EllipticF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^
(1/2)/(4+2*(1+4*x)/(2-3*x))^(1/2),1/23*I*897^(1/2))*253^(1/2)*(7+5*x)^(1/2)
/(-5+2*x)^(1/2)/((7+5*x)/(5-2*x))^(1/2)+1450582567/7372800*EllipticE(1/23*8
97^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),1/39*I*897^(1/2))*429^(1/2)*(2-3*x)^(
1/2)*((7+5*x)/(5-2*x))^(1/2)/((2-3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A] time = 0.67, antiderivative size = 471, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.270$, Rules used = {161, 1600, 1602, 1598, 170, 418, 165, 537, 176, 424}

$$\frac{1}{25}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(5x+7)^{7/2}-\frac{427\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(5x+7)^{5/2}}{2400}-\frac{83363\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x}}{34560}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2),x]
```

```
[Out] (-1450582567*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(3686400*Sqrt[-5 +
2*x]) - (70489981*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])
/1658880 - (83363*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2
))/34560 - (427*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2))
/2400 + (Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(7/2))/25 + (
1450582567*Sqrt[143/3]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[Ar
cSin[(Sqrt[39/23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(2457600*Sqrt[(2
 - 3*x)/(5 - 2*x)]*Sqrt[7 + 5*x]) - (245264762213*Sqrt[11/23]*Sqrt[7 + 5*x]
*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(9953280
0*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]) - (57691792727443*(2 - 3*x)*Sqr
t[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((1 + 4*x)/(2 - 3*x))]*EllipticPi[-69/55, ArcS
in[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -23/39])/(497664000*Sqrt[429
]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Rule 161

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[(2*(a + b*x)^(m + 1)*Sqrt[
c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*(2*m + 5)), x] + Dist[1/(b*(2*m +
5)), Int[((a + b*x)^m*Simp[3*b*c*e*g - a*(d*e*g + c*f*g + c*e*h) + 2*(b*(d*
e*g + c*f*g + c*e*h) - a*(d*f*g + d*e*h + c*f*h))*x - (3*a*d*f*h - b*(d*f*g
 + d*e*h + c*f*h))*x^2, x])/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] &&  !LtQ[m,
```

```
-1]
```

Rule 165

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*(a + b*x)*Sqrt[((b*g -
 a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*
g - e*h)*(a + b*x))])/(Sqrt[c + d*x]*Sqrt[e + f*x]), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + ((b*c - a*d)*x^2)/(d*g - c*h)]*Sqrt[1 + ((b*e - a*f)*x^2)/(f*g
- e*h)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 1598

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
```

```
h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 1600

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[(2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/
(d*f*h*(2*m + 3)), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1602

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[(C*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*f*h*Sqrt[c + d*
x]), x] + (Dist[1/(2*b*d*f*h), Int[(1*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f
*h) + (2*b*B*d*f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x])/(Sqrt[
a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(C*(d*e
- c*f)*(d*g - c*h))/(2*b*d*f*h), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$\int \sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^{5/2}\,dx = \frac{1}{25}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^{7/2} + \frac{1}{50}\int \frac{(7+5x)^{5/2}\left(-3\right.}{\sqrt{2-3x}\,\sqrt{-\ldots}}$$

$$= -\frac{427\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^{5/2}}{2400} + \frac{1}{25}\sqrt{2-3x}\,\sqrt{-5+\ldots}$$

$$= -\frac{83363\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^{3/2}}{34560} - \frac{427\sqrt{2-3x}\,\sqrt{-\ldots}}{}$$

$$= -\frac{70489981\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{1658880} - \frac{83363\sqrt{2-3x}\ldots}{}$$

$$= -\frac{1450582567\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{3686400\sqrt{-5+2x}} - \frac{70489981\sqrt{2-3x}\,\sqrt{-\ldots}}{165\ldots}$$

$$= -\frac{1450582567\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{3686400\sqrt{-5+2x}} - \frac{70489981\sqrt{2-3x}\,\sqrt{-\ldots}}{165\ldots}$$

$$= -\frac{1450582567\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{3686400\sqrt{-5+2x}} - \frac{70489981\sqrt{2-3x}\,\sqrt{-\ldots}}{165\ldots}$$

$$= -\frac{1450582567\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{3686400\sqrt{-5+2x}} - \frac{70489981\sqrt{2-3x}\,\sqrt{-\ldots}}{165\ldots}$$

Mathematica [A] time = 4.60, size = 350, normalized size = 0.74

$$-\frac{\sqrt{2x-5}\,\sqrt{4x+1}\left(-62507925572\sqrt{682}\sqrt{\frac{8x^2-18x-5}{(2-3x)^2}}\left(15x^2+11x-14\right)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{2x-5}{3x-2}}\right),\frac{39}{62}\right)+78\ldots\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2),x]
```

```
[Out] -1/398131200*(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(78331458618*Sqrt[682]*Sqrt[(-5
- 18*x + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticE[ArcSin[Sqrt[31
/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] - 62507925572*Sqrt[682]*Sqrt[(-5
- 18*x + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticF[ArcSin[Sqrt[31
/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + Sqrt[(7 + 5*x)/(-2 + 3*x)]*(6*(
1118234665415 + 5225923788019*x + 2861488598626*x^2 - 795166559320*x^3 - 84
9459145920*x^4 - 288728294400*x^5 + 71414784000*x^6 + 39813120000*x^7) - 60
033082963*Sqrt[682]*(2 - 3*x)^2*Sqrt[(1 + 4*x)/(-2 + 3*x)]*Sqrt[(-35 - 11*x
 + 10*x^2)/(2 - 3*x)^2]*EllipticPi[117/62, ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*
x)/(-2 + 3*x)]], 39/62])))/(Sqrt[2 - 3*x]*Sqrt[7 + 5*x]*Sqrt[(7 + 5*x)/(-2
+ 3*x)]*(-5 - 18*x + 8*x^2))
```

fricas [F] time = 1.27, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(25\,x^2+70\,x+49\right)\sqrt{5\,x+7}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(5/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((25*x^2 + 70*x + 49)*sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqr
t(-3*x + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (5x+7)^{\frac{5}{2}}\sqrt{4x+1}\,\sqrt{2x-5}\,\sqrt{-3x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(5/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((5*x + 7)^(5/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

maple [B] time = 0.08, size = 895, normalized size = 1.90

$$\frac{\sqrt{5x+7}\,\sqrt{-3x+2}\,\sqrt{2x-5}\,\sqrt{4x+1}\,\left(170798284800000x^{7}+306369423360000x^{6}-1238644382976000x^{5}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x+7)^(5/2)*(-3*x+2)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2),x)
```

```
[Out] 1/284663808000*(5*x+7)^(1/2)*(-3*x+2)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(62
433731183120*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*
x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x^2*EllipticF(1/31*31^(1/2)*11^(1/2)*((
5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-684857410441904*11^(1/2)*((5*
x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+
1))^(1/2)*x^2*EllipticPi(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),124
/55,1/39*31^(1/2)*78^(1/2))-896111886589920*11^(1/2)*((5*x+7)/(4*x+1))^(1/2
)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x^2*Elli
pticE(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2)
)+31216865591560*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)
/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x*EllipticF(1/31*31^(1/2)*11^(1/2)*
((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-342428705220952*11^(1/2)*((
5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*
x+1))^(1/2)*x*EllipticPi(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),124
/55,1/39*31^(1/2)*78^(1/2))-448055943294960*11^(1/2)*((5*x+7)/(4*x+1))^(1/2
)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x*Ellipt
icE(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+
3902108198945*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4
*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticF(1/31*31^(1/2)*11^(1/2)*((5*x
+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-42803588152619*11^(1/2)*((5*x+7)
/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^
(1/2)*EllipticPi(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),124/55,1/39
*31^(1/2)*78^(1/2))-56006992911870*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)
*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticE(1/31*31
^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+17079828480
0000*x^7+306369423360000*x^6-1238644382976000*x^5-3644179735996800*x^4-3411
264539482800*x^3+18436555308411240*x^2+15642366908265240*x-167654655564396
0)/(120*x^4-182*x^3-385*x^2+197*x+70)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (5x+7)^{\frac{5}{2}}\sqrt{4x+1}\,\sqrt{2x-5}\,\sqrt{-3x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(5/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((5*x + 7)^(5/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,{\left(5\,x+7\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(5/2),x)
```

```
[Out] int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**(5/2)*(2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2),x)
```

```
[Out] Timed out
```

3.78 $\int \sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^{3/2}\,dx$

Optimal. Leaf size=429

$$-\frac{982275517\sqrt{\frac{11}{23}}\sqrt{5x+7}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{4147200\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}+\frac{1}{20}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(5x+7)^{5/2}-\frac{427\sqrt{2}}{}$$

```
[Out] -427/1440*(7+5*x)^(3/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)+1/20*(7+
5*x)^(5/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)-145131624827/88957440
00*(2-3*x)*EllipticPi(1/23*253^(1/2)*(7+5*x)^(1/2)/(2-3*x)^(1/2),-69/55,1/3
9*I*897^(1/2))*((5-2*x)/(2-3*x))^(1/2)*((-1-4*x)/(2-3*x))^(1/2)*429^(1/2)/(
-5+2*x)^(1/2)/(1+4*x)^(1/2)-1471781/51200*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*
x)^(1/2)/(-5+2*x)^(1/2)-267029/69120*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(
1/2)*(7+5*x)^(1/2)-982275517/95385600*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*
(1+4*x)/(2-3*x))^(1/2)*EllipticF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(
1+4*x)/(2-3*x))^(1/2),1/23*I*897^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1
/2)/((7+5*x)/(5-2*x))^(1/2)+1471781/102400*EllipticE(1/23*897^(1/2)*(1+4*x)
^(1/2)/(-5+2*x)^(1/2),1/39*I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5
-2*x))^(1/2)/((2-3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A] time = 0.54, antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.270$, Rules used = {161, 1600, 1602, 1598, 170, 418, 165, 537, 176, 424}

$$\frac{1}{20}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(5x+7)^{5/2}-\frac{427\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(5x+7)^{3/2}}{1440}-\frac{267029\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{}}{69120}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2),x]
```

```
[Out] (-1471781*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(51200*Sqrt[-5 + 2*x])
 - (267029*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/69120
- (427*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2))/1440 + (
Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2))/20 + (1471781*S
qrt[429]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/
23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(102400*Sqrt[(2 - 3*x)/(5 - 2*
x)]*Sqrt[7 + 5*x]) - (982275517*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[
Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(4147200*Sqrt[-5 + 2*x]*Sq
rt[(7 + 5*x)/(5 - 2*x)]) - (145131624827*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)
]*Sqrt[-((1 + 4*x)/(2 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[
7 + 5*x])/Sqrt[2 - 3*x]], -23/39])/(20736000*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[
1 + 4*x])
```

Rule 161

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[(2*(a + b*x)^(m + 1)*Sqrt[
c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*(2*m + 5)), x] + Dist[1/(b*(2*m +
5)), Int[((a + b*x)^m*Simp[3*b*c*e*g - a*(d*e*g + c*f*g + c*e*h) + 2*(b*(d*
e*g + c*f*g + c*e*h) - a*(d*f*g + d*e*h + c*f*h))*x - (3*a*d*f*h - b*(d*f*g
 + d*e*h + c*f*h))*x^2, x])/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] &&  !LtQ[m,
-1]
```

Rule 165

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*(a + b*x)*Sqrt[((b*g -
 a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*
g - e*h)*(a + b*x))])/(Sqrt[c + d*x]*Sqrt[e + f*x]), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + ((b*c - a*d)*x^2)/(d*g - c*h)]*Sqrt[1 + ((b*e - a*f)*x^2)/(f*g
- e*h)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 1598

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 1600

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[(2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/
(d*f*h*(2*m + 3)), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1602

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[(C*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*f*h*Sqrt[c + d*
x]), x] + (Dist[1/(2*b*d*f*h), Int[(1*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f
*h) + (2*b*B*d*f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x])/(Sqrt[
a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(C*(d*e
- c*f)*(d*g - c*h))/(2*b*d*f*h), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$\int \sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^{3/2}\,dx = \frac{1}{20}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^{5/2}+\frac{1}{40}\int \frac{(7+5x)^{3/2}\,(}{\sqrt{2-3x}\,}$$

$$= -\frac{427\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^{3/2}}{1440}+\frac{1}{20}\sqrt{2-3x}\,\sqrt{-5}$$

$$= -\frac{267029\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{69120}-\frac{427\sqrt{2-3x}\,}{}$$

$$= -\frac{1471781\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{51200\sqrt{-5+2x}}-\frac{267029\sqrt{2-3x}\,\sqrt{-5+}}{6912}$$

$$= -\frac{1471781\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{51200\sqrt{-5+2x}}-\frac{267029\sqrt{2-3x}\,\sqrt{-5+}}{6912}$$

$$= -\frac{1471781\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{51200\sqrt{-5+2x}}-\frac{267029\sqrt{2-3x}\,\sqrt{-5+}}{6912}$$

$$= -\frac{1471781\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{51200\sqrt{-5+2x}}-\frac{267029\sqrt{2-3x}\,\sqrt{-5+}}{6912}$$

Mathematica [A] time = 4.33, size = 345, normalized size = 0.80

$$-\frac{\sqrt{2x-5}\sqrt{4x+1}\left(-5426733148\sqrt{682}\sqrt{\frac{8x^2-18x-5}{(2-3x)^2}}\left(15x^2+11x-14\right)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{2x-5}{3x-2}}\right),\frac{39}{62}\right)+739\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2),x]
```

```
[Out] -1/514252800*(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7391284182*Sqrt[682]*Sqrt[(-5 -
 18*x + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticE[ArcSin[Sqrt[31/
39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] - 5426733148*Sqrt[682]*Sqrt[(-5 -
18*x + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticF[ArcSin[Sqrt[31/3
9]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + Sqrt[(7 + 5*x)/(-2 + 3*x)]*(186*(
3497259535 + 16491468251*x + 9107809874*x^2 - 4479491480*x^3 - 3503236800*x
^4 + 40320000*x^5 + 414720000*x^6) - 4681665317*Sqrt[682]*(2 - 3*x)^2*Sqrt[
(1 + 4*x)/(-2 + 3*x)]*Sqrt[(-35 - 11*x + 10*x^2)/(2 - 3*x)^2]*EllipticPi[11
7/62, ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62])))/(Sqrt[2 -
3*x]*Sqrt[7 + 5*x]*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x + 8*x^2))
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left((5x+7)^{\frac{3}{2}}\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(3/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(5x+7)^{\frac{3}{2}}\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(3/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

maple [B] time = 0.02, size = 890, normalized size = 2.07

$$\frac{\sqrt{5x+7}\sqrt{-3x+2}\sqrt{2x-5}\sqrt{4x+1}\left(1779148800000x^6+172972800000x^5-15028885872000x^4-19217018\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x+7)^(3/2)*(-3*x+2)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2),x)
```

```
[Out] 1/11860992000*(5*x+7)^(1/2)*(-3*x+2)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(201
797192080*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1
))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x^2*EllipticF(1/31*31^(1/2)*11^(1/2)*((5*x
+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-1722852836656*11^(1/2)*((5*x+7)/
(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(
1/2)*x^2*EllipticPi(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),124/55,1
```

```
/39*31^(1/2)*78^(1/2))-2727622291680*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/
2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x^2*EllipticE(1
/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+10089
8596040*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))
^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x*EllipticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/
(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-861426418328*11^(1/2)*((5*x+7)/(4*x+
1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*
x*EllipticPi(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),124/55,1/39*31^
(1/2)*78^(1/2))-1363811145840*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(
1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x*EllipticE(1/31*31^(1
/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+12612324505*11
^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3
*x-2)/(4*x+1))^(1/2)*EllipticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/
2),1/39*31^(1/2)*78^(1/2))-107678302291*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^
(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticPi(1
/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*78^(1/2)
)-170476393230*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(
4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticE(1/31*31^(1/2)*11^(1/2)*((5*
x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+1779148800000*x^6+172972800000*
x^5-15028885872000*x^4-19217018449200*x^3+57824907614760*x^2+50120755215960
*x-50630167988400)/(120*x^4-182*x^3-385*x^2+197*x+70)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (5x+7)^{\frac{3}{2}} \sqrt{4x+1}\,\sqrt{2x-5}\,\sqrt{-3x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(3/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{2-3x}\,\sqrt{4x+1}\,\sqrt{2x-5}\,(5x+7)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(3/2),x)
```

```
[Out] int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**(3/2)*(2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2),x)
```

```
[Out] Timed out
```

3.79 $\int \sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}\,dx$

Optimal. Leaf size=391

$$-\frac{1368371\sqrt{\frac{11}{23}}\sqrt{5x+7}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{43200\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}-\frac{1}{9}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}\,(2-3x)^{3/2}+\frac{23}{240}\sqrt{2x-5}$$

```
[Out] -65750101/92664000*(2-3*x)*EllipticPi(1/23*253^(1/2)*(7+5*x)^(1/2)/(2-3*x)^
(1/2),-69/55,1/39*I*897^(1/2))*((5-2*x)/(2-3*x))^(1/2)*((-1-4*x)/(2-3*x))^(
1/2)*429^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)-13027/4800*(2-3*x)^(1/2)*(1+4*x
)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)-1/9*(2-3*x)^(3/2)*(-5+2*x)^(1/2)*(1+4*
x)^(1/2)*(7+5*x)^(1/2)+23/240*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7
+5*x)^(1/2)-1368371/993600*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)/(2-
3*x))^(1/2)*EllipticF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/(2-3
*x))^(1/2),1/23*I*897^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7+5*x
)/(5-2*x))^(1/2)+13027/9600*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x)
^(1/2),1/39*I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/2)/((
2-3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.270$, Rules used = {161, 1600, 1602, 1598, 170, 418, 165, 537, 176, 424}

$$-\frac{1}{9}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}\,(2-3x)^{3/2}+\frac{23}{240}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}\sqrt{2-3x}-\frac{13027\sqrt{4x+1}\sqrt{5x+7}\sqrt{2-3x}}{4800\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x],x]
```

```
[Out] (-13027*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(4800*Sqrt[-5 + 2*x]) +
(23*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/240 - ((2 - 3
*x)^(3/2)*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/9 + (13027*Sqrt[143/3
]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqr
t[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(3200*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[
7 + 5*x]) - (1368371*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*
x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(43200*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/
(5 - 2*x)]) - (65750101*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((1 + 4*x
)/(2 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2
- 3*x]], -23/39])/(216000*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Rule 161

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[(2*(a + b*x)^(m + 1)*Sqrt[
c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*(2*m + 5)), x] + Dist[1/(b*(2*m +
5)), Int[((a + b*x)^m*Simp[3*b*c*e*g - a*(d*e*g + c*f*g + c*e*h) + 2*(b*(d*
e*g + c*f*g + c*e*h) - a*(d*f*g + d*e*h + c*f*h))*x - (3*a*d*f*h - b*(d*f*g
 + d*e*h + c*f*h))*x^2, x])/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] &&  !LtQ[m,
-1]
```

Rule 165

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*(a + b*x)*Sqrt[((b*g -
```

```
 a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*
g - e*h)*(a + b*x))])/(Sqrt[c + d*x]*Sqrt[e + f*x]), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + ((b*c - a*d)*x^2)/(d*g - c*h)]*Sqrt[1 + ((b*e - a*f)*x^2)/(f*g
- e*h)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 1598

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 1600

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
```

```
ymbol] :> Simp[(2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/
(d*f*h*(2*m + 3)), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1602

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[(C*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*f*h*Sqrt[c + d*
x]), x] + (Dist[1/(2*b*d*f*h), Int[(1*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f
*h) + (2*b*B*d*f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x])/(Sqrt[
a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(C*(d*e
- c*f)*(d*g - c*h))/(2*b*d*f*h), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$\int \sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}\,dx = -\frac{1}{9}(2-3x)^{3/2}\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x} - \frac{1}{18}\int \frac{\sqrt{2-3x}\,(617+\cdots}{\sqrt{-5+2x}\,\sqrt{1\cdots}}$$

$$= \frac{23}{240}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x} - \frac{1}{9}(2-3x)^{3/2}\sqrt{-5+2x}\,\cdots$$

$$= -\frac{13027\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{4800\sqrt{-5+2x}} + \frac{23}{240}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4\cdots}$$

$$= -\frac{13027\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{4800\sqrt{-5+2x}} + \frac{23}{240}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4\cdots}$$

$$= -\frac{13027\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{4800\sqrt{-5+2x}} + \frac{23}{240}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4\cdots}$$

$$= -\frac{13027\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{4800\sqrt{-5+2x}} + \frac{23}{240}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4\cdots}$$

Mathematica [A] time = 4.27, size = 340, normalized size = 0.87

$$-\frac{\sqrt{2x-5}\,\sqrt{4x+1}\left(-4532324\sqrt{682}\sqrt{\frac{8x^2-18x-5}{(2-3x)^2}}\left(15x^2+11x-14\right)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{2x-5}{3x-2}}\right),\frac{39}{62}\right)+726906\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x],x]
```

```
[Out] -1/5356800*(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7269066*Sqrt[682]*Sqrt[(-5 - 18*x
 + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticE[ArcSin[Sqrt[31/39]*S
qrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] - 4532324*Sqrt[682]*Sqrt[(-5 - 18*x + 8
*x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticF[ArcSin[Sqrt[31/39]*Sqrt[
(-5 + 2*x)/(-2 + 3*x)]], 39/62] + Sqrt[(7 + 5*x)/(-2 + 3*x)]*(186*(3848705
+ 17658613*x + 7278862*x^2 - 7723240*x^3 - 2184000*x^4 + 1152000*x^5) - 212
0971*Sqrt[682]*(2 - 3*x)^2*Sqrt[(1 + 4*x)/(-2 + 3*x)]*Sqrt[(-35 - 11*x + 10
*x^2)/(2 - 3*x)^2]*EllipticPi[117/62, ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-
2 + 3*x)]], 39/62])))/(Sqrt[2 - 3*x]*Sqrt[7 + 5*x]*Sqrt[(7 + 5*x)/(-2 + 3*x
)]*(-5 - 18*x + 8*x^2))
```

fricas [F] time = 1.18, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{5\,x+7}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{5\,x+7}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

maple [B] time = 0.02, size = 885, normalized size = 2.26

$$-\frac{\sqrt{-3x+2}\;\sqrt{2x-5}\;\sqrt{4x+1}\;\sqrt{5x+7}\;\left(-4942080000x^5+9369360000x^4+33132699600x^3+2682519840\sqrt{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x+2)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(5*x+7)^(1/2),x)
```

```
[Out] -1/123552000*(-3*x+2)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(5*x+7)^(1/2)*(7805
17328*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(
1/2)*((3*x-2)/(4*x+1))^(1/2)*x^2*EllipticPi(1/31*31^(1/2)*11^(1/2)*((5*x+7)
/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*78^(1/2))+2682519840*11^(1/2)*((5*x+7)
/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^
(1/2)*x^2*EllipticE(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^
(1/2)*78^(1/2))-454813040*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)
*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x^2*EllipticF(1/31*31^(1/2
)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+390258664*11^(1/
2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2
)/(4*x+1))^(1/2)*x*EllipticPi(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2
),124/55,1/39*31^(1/2)*78^(1/2))+1341259920*11^(1/2)*((5*x+7)/(4*x+1))^(1/2
)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x*Ellipt
icE(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-
227406520*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1
```

```
))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x*EllipticF(1/31*31^(1/2)*11^(1/2)*((5*x+7
)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+48782333*11^(1/2)*((5*x+7)/(4*x+1)
)^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*El
lipticPi(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),124/55,1/39*31^(1/2
)*78^(1/2))+167657490*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2
*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticE(1/31*31^(1/2)*11^(1/
2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-28425815*11^(1/2)*((5*x+
7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1)
)^(1/2)*EllipticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1
/2)*78^(1/2))-4942080000*x^5+9369360000*x^4+33132699600*x^3-49668641880*x^2
-55468893480*x+48037189200)/(120*x^4-182*x^3-385*x^2+197*x+70)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{5\,x+7}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{5\,x+7}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(1/2),x)
```

```
[Out] int((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{5x+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)*(7+5*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7), x)
```

3.80 $\int \frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{\sqrt{7+5x}}\,dx$

Optimal. Leaf size=351

$$-\frac{20057\sqrt{\frac{11}{23}}\sqrt{5x+7}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{1800\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}+\frac{1}{10}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{5x+7}-\frac{427\sqrt{2-3x}}{600}$$

```
[Out] 1008833/3861000*(2-3*x)*EllipticPi(1/23*253^(1/2)*(7+5*x)^(1/2)/(2-3*x)^(1/
2),-69/55,1/39*I*897^(1/2))*((5-2*x)/(2-3*x))^(1/2)*((-1-4*x)/(2-3*x))^(1/2
)*429^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)-427/600*(2-3*x)^(1/2)*(1+4*x)^(1/2
)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)+1/10*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1
/2)*(7+5*x)^(1/2)-20057/41400*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)/
(2-3*x))^(1/2)*EllipticF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/(
2-3*x))^(1/2),1/23*I*897^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7+
5*x)/(5-2*x))^(1/2)+427/1200*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x
)^(1/2),1/39*I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/2)/(
(2-3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {161, 1602, 1598, 170, 418, 165, 537, 176, 424}

$$\frac{1}{10}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{5x+7}-\frac{427\sqrt{2-3x}\,\sqrt{4x+1}\,\sqrt{5x+7}}{600\sqrt{2x-5}}-\frac{20057\sqrt{\frac{11}{23}}\sqrt{5x+7}\,F\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right.\right.}{1800\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/Sqrt[7 + 5*x],x]
```

```
[Out] (-427*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(600*Sqrt[-5 + 2*x]) + (Sq
rt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/10 + (427*Sqrt[143/
3]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sq
rt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(400*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[
7 + 5*x]) - (20057*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x]
/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(1800*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5
- 2*x)]) + (1008833*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((1 + 4*x)/(2
 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*
x]], -23/39])/(9000*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Rule 161

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[(2*(a + b*x)^(m + 1)*Sqrt[
c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*(2*m + 5)), x] + Dist[1/(b*(2*m +
5)), Int[((a + b*x)^m*Simp[3*b*c*e*g - a*(d*e*g + c*f*g + c*e*h) + 2*(b*(d*
e*g + c*f*g + c*e*h) - a*(d*f*g + d*e*h + c*f*h))*x - (3*a*d*f*h - b*(d*f*g
 + d*e*h + c*f*h))*x^2, x])/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] &&  !LtQ[m,
-1]
```

Rule 165

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*(a + b*x)*Sqrt[((b*g -
 a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*
```

```
g - e*h)*(a + b*x))]])/(Sqrt[c + d*x]*Sqrt[e + f*x]), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + ((b*c - a*d)*x^2)/(d*g - c*h)]*Sqrt[1 + ((b*e - a*f)*x^2)/(f*g
- e*h)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 1598

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 1602

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[(C*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*f*h*Sqrt[c + d*
```

```
x]), x] + (Dist[1/(2*b*d*f*h), Int[(1*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f
*h) + (2*b*B*d*f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x])/(Sqrt[
a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(C*(d*e
- c*f)*(d*g - c*h))/(2*b*d*f*h), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{\sqrt{7+5x}}\,dx = \frac{1}{10}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x} + \frac{1}{20}\int \frac{-3-1190x+85}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+}}$$

$$= -\frac{427\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{600\sqrt{-5+2x}} + \frac{1}{10}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5}$$

$$= -\frac{427\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{600\sqrt{-5+2x}} + \frac{1}{10}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5}$$

$$= -\frac{427\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{600\sqrt{-5+2x}} + \frac{1}{10}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5}$$

$$= -\frac{427\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{600\sqrt{-5+2x}} + \frac{1}{10}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5}$$

Mathematica [A] time = 3.07, size = 347, normalized size = 0.99

$$\frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{5x+7}\left(\frac{117924\sqrt{682}\,(3x-2)(5x+7)\sqrt{\frac{8x^2-18x-5}{(2-3x)^2}}\ \text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{2x-5}{3x-2}}\right),\frac{39}{62}\right)-238266\sqrt{682}\,(3x-2)(5x+7)\sqrt{\frac{8x^2}{(}}}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/Sqrt[7 + 5*x],x]
```

```
[Out] (Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]*(66960*(2 - 3*x) + (-238266*Sqr
t[682]*(-2 + 3*x)*(7 + 5*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticE
[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + 117924*Sqrt[682]
*(-2 + 3*x)*(7 + 5*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticF[ArcSi
n[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] - 7*Sqrt[(7 + 5*x)/(-2 +
 3*x)]*(-102114*(-35 - 151*x - 34*x^2 + 40*x^3) + 13947*Sqrt[682]*(2 - 3*x)
^2*Sqrt[(1 + 4*x)/(-2 + 3*x)]*Sqrt[(-35 - 11*x + 10*x^2)/(2 - 3*x)^2]*Ellip
ticPi[117/62, ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62]))/((2
 - 3*x)*((7 + 5*x)/(-2 + 3*x))^(3/2)*(5 + 18*x - 8*x^2))))/(669600*Sqrt[2 -
 3*x])
```

fricas [F] time = 1.26, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}}{\sqrt{5\,x+7}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/sqrt(5*x + 7), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}}{\sqrt{5\,x+7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/sqrt(5*x + 7), x)
```

maple [B] time = 0.04, size = 880, normalized size = 2.51

$$\sqrt{-3x+2}\,\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{5x+7}\left(61776000x^4-313513200x^3-29309280\sqrt{11}\,\sqrt{\frac{5x+7}{4x+1}}\,\sqrt{3}\,\sqrt{13}\,\sqrt{\frac{2x-5}{4x+1}}\,\sqrt{\frac{3}{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x+2)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)/(5*x+7)^(1/2),x)
```

```
[Out] 1/5148000*(-3*x+2)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(5*x+7)^(1/2)*(1532168
0*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)
*((3*x-2)/(4*x+1))^(1/2)*x^2*EllipticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x
+1))^(1/2),1/39*31^(1/2)*78^(1/2))+11975824*11^(1/2)*((5*x+7)/(4*x+1))^(1/2
)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x^2*Elli
pticPi(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*
78^(1/2))-29309280*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-
5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x^2*EllipticE(1/31*31^(1/2)*11^(1
/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+7660840*11^(1/2)*((5*x+
7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1)
)^(1/2)*x*EllipticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^
(1/2)*78^(1/2))+5987912*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*(
(2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x*EllipticPi(1/31*31^(1/2)*1
1^(1/2)*((5*x+7)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*78^(1/2))-14654640*11^
(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*
x-2)/(4*x+1))^(1/2)*x*EllipticE(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1
/2),1/39*31^(1/2)*78^(1/2))+957605*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)
*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticF(1/31*31
^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+748489*11^(
1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x
-2)/(4*x+1))^(1/2)*EllipticPi(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2
),124/55,1/39*31^(1/2)*78^(1/2))-1831830*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3
^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticE(1
/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-31351
3200*x^3+61776000*x^4+190149960*x^2+709583160*x-476876400)/(120*x^4-182*x^3
-385*x^2+197*x+70)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}}{\sqrt{5\,x+7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/sqrt(5*x + 7), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}{\sqrt{5\,x+7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^(1/2),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{\sqrt{5x+7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)/sqrt(5*x + 7), x)
```

3.81 $\int \frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{(7+5x)^{3/2}}\,dx$

Optimal. Leaf size=349

$$\frac{296\sqrt{\frac{11}{23}}\sqrt{5x+7}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{75\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}+\frac{6\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{25\sqrt{2x-5}}-\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{5\sqrt{5x+7}}-$$

```
[Out] -26474/160875*(2-3*x)*EllipticPi(1/23*253^(1/2)*(7+5*x)^(1/2)/(2-3*x)^(1/2)
,-69/55,1/39*I*897^(1/2))*((5-2*x)/(2-3*x))^(1/2)*((-1-4*x)/(2-3*x))^(1/2)*
429^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)-2/5*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+
4*x)^(1/2)/(7+5*x)^(1/2)+6/25*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5
+2*x)^(1/2)+296/1725*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)/(2-3*x))^
(1/2)*EllipticF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/(2-3*x))^(
1/2),1/23*I*897^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7+5*x)/(5-2
*x))^(1/2)-3/25*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),1/39*
I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/2)/((2-3*x)/(5-2*
x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {160, 1602, 1598, 170, 418, 165, 537, 176, 424}

$$\frac{6\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{25\sqrt{2x-5}}-\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{5\sqrt{5x+7}}+\frac{296\sqrt{\frac{11}{23}}\sqrt{5x+7}\,F\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{75\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}-\frac{3\sqrt{429}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(5*Sqrt[7 + 5*x]) + (6*Sqrt
[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(25*Sqrt[-5 + 2*x]) - (3*Sqrt[429]*S
qrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqrt[1
 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(25*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7 + 5
*x]) + (296*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[
2]*Sqrt[2 - 3*x])], -39/23])/(75*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 - 2*x)])
- (26474*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((1 + 4*x)/(2 - 3*x))]*E
llipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -23/39
])/(375*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Rule 160

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[((a + b*x)^(m + 1)*Sqrt[c
+ d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*(m + 1)), x] - Dist[1/(2*b*(m + 1)),
 Int[((a + b*x)^(m + 1)*Simp[d*e*g + c*f*g + c*e*h + 2*(d*f*g + d*e*h + c*f
*h)*x + 3*d*f*h*x^2, x])/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 165

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*(a + b*x)*Sqrt[((b*g -
 a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*
g - e*h)*(a + b*x))])/(Sqrt[c + d*x]*Sqrt[e + f*x]), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + ((b*c - a*d)*x^2)/(d*g - c*h)]*Sqrt[1 + ((b*e - a*f)*x^2)/(f*g
```

```
- e*h)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 1598

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 1602

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[(C*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*f*h*Sqrt[c + d*
x]), x] + (Dist[1/(2*b*d*f*h), Int[(1*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f
*h) + (2*b*B*d*f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x])/(Sqrt[
```

```
a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(C*(d*e
- c*f)*(d*g - c*h))/(2*b*d*f*h), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{3/2}}\,dx = -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{5\sqrt{7+5x}} + \frac{1}{5}\int \frac{-21+140x-72x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}$$

$$= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{5\sqrt{7+5x}} + \frac{6\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{25\sqrt{-5+2x}} - \int \frac{-12}{\sqrt{2-3x}\sqrt{-5}}$$

$$= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{5\sqrt{7+5x}} + \frac{6\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{25\sqrt{-5+2x}} - \frac{427}{75}\int \frac{}{\sqrt{-}}$$

$$= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{5\sqrt{7+5x}} + \frac{6\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{25\sqrt{-5+2x}} - \frac{3\sqrt{429}\sqrt{2}}{}$$

$$= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{5\sqrt{7+5x}} + \frac{6\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{25\sqrt{-5+2x}} - \frac{3\sqrt{429}\sqrt{2}}{}$$

Mathematica [A] time = 2.98, size = 330, normalized size = 0.95

$$-\frac{\sqrt{2x-5}\sqrt{4x+1}\left(262\sqrt{682}\sqrt{\frac{8x^2-18x-5}{(2-3x)^2}}\left(15x^2+11x-14\right)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{2x-5}{3x-2}}\right),\frac{39}{62}\right)-558\sqrt{682}\sqrt{\frac{8x}{}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^(3/2),x]
```

```
[Out] -1/4650*(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(-558*Sqrt[682]*Sqrt[(-5 - 18*x + 8*x
^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticE[ArcSin[Sqrt[31/39]*Sqrt[(-
5 + 2*x)/(-2 + 3*x)]], 39/62] + 262*Sqrt[682]*Sqrt[(-5 - 18*x + 8*x^2)/(2 -
 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticF[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)
/(-2 + 3*x)]], 39/62] + Sqrt[(7 + 5*x)/(-2 + 3*x)]*(186*(-415 - 1569*x + 39
4*x^2 + 120*x^3) - 427*Sqrt[682]*(2 - 3*x)^2*Sqrt[(1 + 4*x)/(-2 + 3*x)]*Sqr
t[(-35 - 11*x + 10*x^2)/(2 - 3*x)^2]*EllipticPi[117/62, ArcSin[Sqrt[31/39]*
Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62]))/(Sqrt[2 - 3*x]*Sqrt[7 + 5*x]*Sqrt[(
7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x + 8*x^2))
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{5\,x+7}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}}{25\,x^2+70\,x+49}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(25*x^2 +
 70*x + 49), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^(3/2), x)
```

maple [B] time = 0.05, size = 875, normalized size = 2.51

$$-\frac{\sqrt{-3x+2}\,\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{5x+7}\left(-514800x^{3}-205920\sqrt{11}\,\sqrt{\frac{5x+7}{4x+1}}\,\sqrt{3}\,\sqrt{13}\,\sqrt{\frac{2x-5}{4x+1}}\,\sqrt{\frac{3x-2}{4x+1}}\,x^{2}\,\mathrm{Elliptic}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x+2)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)/(5*x+7)^(3/2),x)
```

```
[Out] -1/107250*(-3*x+2)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(5*x+7)^(1/2)*(157520*
11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*(
(3*x-2)/(4*x+1))^(1/2)*x^2*EllipticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1
))^(1/2),1/39*31^(1/2)*78^(1/2))+157136*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^
(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x^2*Elliptic
Pi(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*78^(
1/2))-205920*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*
x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x^2*EllipticE(1/31*31^(1/2)*11^(1/2)*((
5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+78760*11^(1/2)*((5*x+7)/(4*x+
1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*
x*EllipticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78
^(1/2))+78568*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4
*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x*EllipticPi(1/31*31^(1/2)*11^(1/2)*((
5*x+7)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*78^(1/2))-102960*11^(1/2)*((5*x+
7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1)
)^(1/2)*x*EllipticE(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^
(1/2)*78^(1/2))+9845*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*
x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticF(1/31*31^(1/2)*11^(1/2
)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+9821*11^(1/2)*((5*x+7)/(4
*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/
2)*EllipticPi(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),124/55,1/39*31
^(1/2)*78^(1/2))-12870*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((
2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticE(1/31*31^(1/2)*11^(1
/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-514800*x^3-274560*x^2+5
173740*x-3174600)/(120*x^4-182*x^3-385*x^2+197*x+70)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^(3/2), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}{{\left(5\,x+7\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^(3/2),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^(3/2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**(3/2),x)
```

```
[Out] Timed out
```

3.82 $\int \frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{(7+5x)^{5/2}}\,dx$

Optimal. Leaf size=391

$$-\frac{496\sqrt{\frac{11}{23}}\sqrt{5x+7}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{1725\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}-\frac{35812\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{2085525\sqrt{2x-5}}+\frac{17906\sqrt{2-3x}\sqrt{2x}}{417105\sqrt{5x}}$$

```
[Out] -2/15*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2)+496/53625*(2
-3*x)*EllipticPi(1/23*253^(1/2)*(7+5*x)^(1/2)/(2-3*x)^(1/2),-69/55,1/39*I*8
97^(1/2))*((5-2*x)/(2-3*x))^(1/2)*((-1-4*x)/(2-3*x))^(1/2)*429^(1/2)/(-5+2*
x)^(1/2)/(1+4*x)^(1/2)+17906/417105*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1
/2)/(7+5*x)^(1/2)-35812/2085525*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(
-5+2*x)^(1/2)-496/39675*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)/(2-3*x
))^(1/2)*EllipticF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/(2-3*x)
)^(1/2),1/23*I*897^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7+5*x)/(
5-2*x))^(1/2)+17906/2085525*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x)
^(1/2),1/39*I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/2)/((
2-3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.270$, Rules used = {160, 1604, 1602, 1598, 170, 418, 165, 537, 176, 424}

$$-\frac{35812\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{2085525\sqrt{2x-5}}+\frac{17906\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{417105\sqrt{5x+7}}-\frac{2\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{15(5x+7)^{3/2}}-\frac{496\sqrt{\frac{11}{23}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^(5/2),x]
```

```
[Out] (-2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(15*(7 + 5*x)^(3/2)) + (179
06*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(417105*Sqrt[7 + 5*x]) - (35
812*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(2085525*Sqrt[-5 + 2*x]) + (
17906*Sqrt[11/39]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[
(Sqrt[39/23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(53475*Sqrt[(2 - 3*x)
/(5 - 2*x)]*Sqrt[7 + 5*x]) - (496*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTa
n[Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(1725*Sqrt[-5 + 2*x]*Sqr
t[(7 + 5*x)/(5 - 2*x)]) + (496*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((
1 + 4*x)/(2 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/
Sqrt[2 - 3*x]], -23/39])/(125*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Rule 160

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[((a + b*x)^(m + 1)*Sqrt[c
+ d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*(m + 1)), x] - Dist[1/(2*b*(m + 1)),
 Int[((a + b*x)^(m + 1)*Simp[d*e*g + c*f*g + c*e*h + 2*(d*f*g + d*e*h + c*f
*h)*x + 3*d*f*h*x^2, x])/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 165

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*(a + b*x)*Sqrt[((b*g -
 a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*
```

```
g - e*h)*(a + b*x))])/(Sqrt[c + d*x]*Sqrt[e + f*x]), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + ((b*c - a*d)*x^2)/(d*g - c*h)]*Sqrt[1 + ((b*e - a*f)*x^2)/(f*g
- e*h)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 1598

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 1602

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[(C*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*f*h*Sqrt[c + d*
```

```
x]), x] + (Dist[1/(2*b*d*f*h), Int[(1*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f
*h) + (2*b*B*d*f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x])/(Sqrt[
a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(C*(d*e
- c*f)*(d*g - c*h))/(2*b*d*f*h), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rule 1604

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt
[e + f*x]*Sqrt[g + h*x])/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), x]
- Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m
 + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m +
 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
+ c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2
*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^
2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
+ c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{(7+5x)^{5/2}}\,dx = -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{15(7+5x)^{3/2}}+\frac{1}{15}\int \frac{-21+140x-72x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\,(7+ \cdots}$$

$$= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{15(7+5x)^{3/2}}+\frac{17906\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{417105\sqrt{7+5x}}+\int \cdots$$

$$= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{15(7+5x)^{3/2}}+\frac{17906\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{417105\sqrt{7+5x}}-\frac{35\cdots}{\cdots}$$

$$= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{15(7+5x)^{3/2}}+\frac{17906\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{417105\sqrt{7+5x}}-\frac{35\cdots}{\cdots}$$

$$= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{15(7+5x)^{3/2}}+\frac{17906\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{417105\sqrt{7+5x}}-\frac{35\cdots}{\cdots}$$

$$= -\frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{15(7+5x)^{3/2}}+\frac{17906\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{417105\sqrt{7+5x}}-\frac{35\cdots}{\cdots}$$

Mathematica [A] time = 3.23, size = 366, normalized size = 0.94

$$\frac{\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}\left(\frac{30\sqrt{6-9x}\,(44765x+34864)}{(5x+7)^2}-\frac{2\left(-37053\sqrt{682}\,(3x-2)(5x+7)\sqrt{\frac{8x^2-18x-5}{(2-3x)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{2x-5}{3x-2}}\right),\frac{39}{62}\right)\cdots\right.}{\cdots}\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^(5/2),x]
```

```
[Out] (Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]*((30*Sqrt[6 - 9*x]*(34864 + 447
65*x))/(7 + 5*x)^2 - (2*(80577*Sqrt[682]*(-2 + 3*x)*(7 + 5*x)*Sqrt[(-5 - 18
*x + 8*x^2)/(2 - 3*x)^2]*EllipticE[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 +
 3*x)]], 39/62] - 37053*Sqrt[682]*(-2 + 3*x)*(7 + 5*x)*Sqrt[(-5 - 18*x + 8*
x^2)/(2 - 3*x)^2]*EllipticF[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]]
, 39/62] + Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-241731*(-35 - 151*x - 34*x^2 + 40*x
^3) + 48438*Sqrt[682]*(2 - 3*x)^2*Sqrt[(1 + 4*x)/(-2 + 3*x)]*Sqrt[(-35 - 11
*x + 10*x^2)/(2 - 3*x)^2]*EllipticPi[117/62, ArcSin[Sqrt[31/39]*Sqrt[(-5 +
2*x)/(-2 + 3*x)]], 39/62])))/(Sqrt[3]*(2 - 3*x)^(3/2)*((7 + 5*x)/(-2 + 3*x)
)^(3/2)*(5 + 18*x - 8*x^2))))/(6256575*Sqrt[3])
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{5\,x+7}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}}{125\,x^3+525\,x^2+735\,x+343},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(125*x^3
+ 525*x^2 + 735*x + 343), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^(5/2), x)
```

maple [B] time = 0.05, size = 1149, normalized size = 2.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x+2)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)/(5*x+7)^(5/2),x)
```

```
[Out] 2/114703875*(24389200*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2
*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticF(1/31*31^(1/2)*11^(1/
2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))*x^3+23614560*11^(1/2)*((
5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*
x+1))^(1/2)*EllipticPi(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),124/5
5,1/39*31^(1/2)*78^(1/2))*x^3-39393200*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(
1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticE(1/3
1*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))*x^3+463
39480*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(
1/2)*((3*x-2)/(4*x+1))^(1/2)*x^2*EllipticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/
(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+44867664*11^(1/2)*((5*x+7)/(4*x+1))^
(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x^2*
EllipticPi(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),124/55,1/39*31^(1
/2)*78^(1/2))-74847080*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((
```

```
2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x^2*EllipticE(1/31*31^(1/2)*1
1^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+18596765*11^(1/2)*(
(5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4
*x+1))^(1/2)*x*EllipticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/3
9*31^(1/2)*78^(1/2))+18006102*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(
1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x*EllipticPi(1/31*31^(
1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*78^(1/2))-300373
15*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2
)*((3*x-2)/(4*x+1))^(1/2)*x*EllipticE(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+
1))^(1/2),1/39*31^(1/2)*78^(1/2))+2134055*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*
3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticF(
1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+2066
274*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/
2)*((3*x-2)/(4*x+1))^(1/2)*EllipticPi(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+
1))^(1/2),124/55,1/39*31^(1/2)*78^(1/2))-3446905*11^(1/2)*((5*x+7)/(4*x+1))
^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*Ell
ipticE(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2
))-523292550*x^3+1135538635*x^2+779434810*x-869257400)*(4*x+1)^(1/2)*(2*x-5
)^(1/2)*(-3*x+2)^(1/2)/(120*x^4-182*x^3-385*x^2+197*x+70)/(5*x+7)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{4x+1}\,\sqrt{2x-5}\,\sqrt{-3x+2}}{(5x+7)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2-3x}\,\sqrt{4x+1}\,\sqrt{2x-5}}{(5x+7)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^(5/2),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**(5/2),x)
```

```
[Out] Timed out
```

3.83 $\int \frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{(7+5x)^{7/2}}\,dx$

Optimal. Leaf size=330

$$-\frac{48884\sqrt{\frac{11}{23}}\,\sqrt{5x+7}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\,\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{9593415\sqrt{2x-5}\,\sqrt{\frac{5x+7}{5-2x}}}-\frac{2852696\sqrt{2-3x}\,\sqrt{4x+1}\,\sqrt{5x+7}}{11598438735\sqrt{2x-5}}+\frac{1426348\sqrt{2-3x}}{231968774}$$

```
[Out] -2/25*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2)+17906/208552
5*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2)+1426348/23196877
47*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2)-2852696/1159843
8735*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)-48884/2206485
45*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)/(2-3*x))^(1/2)*EllipticF((1
+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/(2-3*x))^(1/2),1/23*I*897^(1
/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7+5*x)/(5-2*x))^(1/2)+1426348
/11598438735*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),1/39*I*8
97^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/2)/((2-3*x)/(5-2*x))
^(1/2)/(7+5*x)^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used = {160, 1604, 1599, 1602, 12, 170, 418, 176, 424}

$$-\frac{2852696\sqrt{2-3x}\,\sqrt{4x+1}\,\sqrt{5x+7}}{11598438735\sqrt{2x-5}}+\frac{1426348\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{2319687747\sqrt{5x+7}}+\frac{17906\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{2085525(5x+7)^{3/2}}-2\sqrt{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^(7/2),x]
```

```
[Out] (-2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(25*(7 + 5*x)^(5/2)) + (179
06*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(2085525*(7 + 5*x)^(3/2)) +
(1426348*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(2319687747*Sqrt[7 + 5
*x]) - (2852696*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(11598438735*Sqr
t[-5 + 2*x]) + (1426348*Sqrt[11/39]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]
*EllipticE[ArcSin[(Sqrt[39/23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(29
7395865*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7 + 5*x]) - (48884*Sqrt[11/23]*Sqrt[
7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/
(9593415*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 - 2*x)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 160

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[((a + b*x)^(m + 1)*Sqrt[c
+ d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*(m + 1)), x] - Dist[1/(2*b*(m + 1)),
 Int[((a + b*x)^(m + 1)*Simp[d*e*g + c*f*g + c*e*h + 2*(d*f*g + d*e*h + c*f
*h)*x + 3*d*f*h*x^2, x])/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1599

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
(A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]
)/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), x] - Dist[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sq
rt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d*
f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*e
*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
- b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rule 1602

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[(C*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*f*h*Sqrt[c + d*
x]), x] + (Dist[1/(2*b*d*f*h), Int[(1*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f
*h) + (2*b*B*d*f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x])/(Sqrt[
a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(C*(d*e
- c*f)*(d*g - c*h))/(2*b*d*f*h), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rule 1604

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt
[e + f*x]*Sqrt[g + h*x])/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), x]
- Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m
 + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m +
 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
+ c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2
*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^
2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
+ c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{(7+5x)^{7/2}}\,dx = -\frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{25(7+5x)^{5/2}} + \frac{1}{25}\int \frac{-21+140x-72x^2}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)$$

$$= -\frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{25(7+5x)^{5/2}} + \frac{17906\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{2085525(7+5x)^{3/2}} + \frac{\int \frac{}{\sqrt{2}}$$

$$= -\frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{25(7+5x)^{5/2}} + \frac{17906\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{2085525(7+5x)^{3/2}} + 1426$$

$$= -\frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{25(7+5x)^{5/2}} + \frac{17906\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{2085525(7+5x)^{3/2}} + 1426$$

$$= -\frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{25(7+5x)^{5/2}} + \frac{17906\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{2085525(7+5x)^{3/2}} + 1426$$

$$= -\frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{25(7+5x)^{5/2}} + \frac{17906\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{2085525(7+5x)^{3/2}} + 1426$$

$$= -\frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{25(7+5x)^{5/2}} + \frac{17906\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{2085525(7+5x)^{3/2}} + 1426$$

Mathematica [A] time = 2.46, size = 251, normalized size = 0.76

$$-\frac{2\sqrt{2x-5}\,\sqrt{4x+1}\left(-236555\sqrt{682}\,(3x-2)\sqrt{\frac{8x^2-18x-5}{(2-3x)^2}}\,(5x+7)^3\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{2x-5}{3x-2}}\right),\frac{39}{62}\right)+713174\sqrt{}\right.}{1159843}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^(7/2),x]
```

```
[Out] (-2*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(31*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(137502130
+ 880765228*x + 1137407943*x^2 - 729949210*x^3 + 50105384*x^4) + 713174*Sqr
t[682]*(-2 + 3*x)*(7 + 5*x)^3*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*Ellipti
```

```
cE[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] - 236555*Sqrt[68
2]*(-2 + 3*x)*(7 + 5*x)^3*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticF[A
rcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62]))/(11598438735*Sqrt[
2 - 3*x]*(7 + 5*x)^(5/2)*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x + 8*x^2))
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{5\,x+7}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}}{625\,x^4+3500\,x^3+7350\,x^2+6860\,x+2401},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(7/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(625*x^4
+ 3500*x^3 + 7350*x^2 + 6860*x + 2401), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^(7/2), x)
```

maple [B] time = 0.05, size = 973, normalized size = 2.95

$$-\frac{2\left(285269600\sqrt{11}\,\sqrt{\frac{5x+7}{4x+1}}\,\sqrt{3}\,\sqrt{13}\,\sqrt{\frac{2x-5}{4x+1}}\,\sqrt{\frac{3x-2}{4x+1}}\,x^4\,\text{EllipticE}\left(\frac{\sqrt{31}\,\sqrt{11}\,\sqrt{\frac{5x+7}{4x+1}}}{31},\frac{\sqrt{31}\,\sqrt{78}}{39}\right)+17811200\sqrt{11}\,\sqrt{\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x+2)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)/(5*x+7)^(7/2),x)
```

```
[Out] -2/11598438735*(17811200*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*
((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticF(1/31*31^(1/2)*11^
(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))*x^4+285269600*11^(1/2
)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)
/(4*x+1))^(1/2)*EllipticE(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/
39*31^(1/2)*78^(1/2))*x^4+58776960*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)
*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticF(1/31*31
^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))*x^3+9413896
80*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2
)*((3*x-2)/(4*x+1))^(1/2)*EllipticE(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1)
)^(1/2),1/39*31^(1/2)*78^(1/2))*x^3+60958832*11^(1/2)*((5*x+7)/(4*x+1))^(1/
2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x^2*Ell
ipticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2
))+976335206*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*
x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x^2*EllipticE(1/31*31^(1/2)*11^(1/2)*((
5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+20571936*11^(1/2)*((5*x+7)/(4
*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/
2)*x*EllipticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)
*78^(1/2))+329486388*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*
x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x*EllipticE(1/31*31^(1/2)*11^(1
```

```
/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+2181872*11^(1/2)*((5*x+
7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1)
)^(1/2)*EllipticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1
/2)*78^(1/2))+34945526*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((
2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticE(1/31*31^(1/2)*11^(1
/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-3514495404*x^4+19294337
060*x^3-26198770563*x^2-3855274122*x+9191461480)*(4*x+1)^(1/2)*(2*x-5)^(1/2
)*(-3*x+2)^(1/2)/(120*x^4-182*x^3-385*x^2+197*x+70)/(5*x+7)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(7/2),x, algor
ithm="maxima")
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}{(5\,x+7)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^(7/2),x)
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**(7/2),x)
[Out] Timed out
```

3.84 $\int \frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{(7+5x)^{9/2}}\,dx$

Optimal. Leaf size=370

$$-\frac{1212290288\sqrt{\frac{11}{23}}\sqrt{5x+7}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{1867348636335\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}-\frac{65687975672\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{2257624501329015\sqrt{2x-5}}+328$$

```
[Out] -2/35*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(7/2)+2558/695175*
(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2)+23758016/579921936
75*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2)+32843987836/451
524900265803*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2)-65687
975672/2257624501329015*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^
(1/2)-1212290288/42949018635705*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x
)/(2-3*x))^(1/2)*EllipticF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)
/(2-3*x))^(1/2),1/23*I*897^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((
7+5*x)/(5-2*x))^(1/2)+32843987836/2257624501329015*EllipticE(1/23*897^(1/2)
*(1+4*x)^(1/2)/(-5+2*x)^(1/2),1/39*I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7
+5*x)/(5-2*x))^(1/2)/((2-3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A] time = 0.51, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used = {160, 1604, 1599, 1602, 12, 170, 418, 176, 424}

$$-\frac{65687975672\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{2257624501329015\sqrt{2x-5}}+\frac{32843987836\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{451524900265803\sqrt{5x+7}}+\frac{23758016\sqrt{2-3x}\sqrt{2x-}}{57992193675(5x-}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^(9/2),x]
```

```
[Out] (-2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(35*(7 + 5*x)^(7/2)) + (255
8*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(695175*(7 + 5*x)^(5/2)) + (2
3758016*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(57992193675*(7 + 5*x)^
(3/2)) + (32843987836*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(45152490
0265803*Sqrt[7 + 5*x]) - (65687975672*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 +
5*x])/(2257624501329015*Sqrt[-5 + 2*x]) + (32843987836*Sqrt[11/39]*Sqrt[2 -
 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqrt[1 + 4*x]
)/Sqrt[-5 + 2*x]], -23/39])/(57887807726385*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[
7 + 5*x]) - (1212290288*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 +
 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(1867348636335*Sqrt[-5 + 2*x]*Sqrt
[(7 + 5*x)/(5 - 2*x)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 160

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[((a + b*x)^(m + 1)*Sqrt[c
+ d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*(m + 1)), x] - Dist[1/(2*b*(m + 1)),
 Int[((a + b*x)^(m + 1)*Simp[d*e*g + c*f*g + c*e*h + 2*(d*f*g + d*e*h + c*f
*h)*x + 3*d*f*h*x^2, x])/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1599

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
(A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]
)/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), x] - Dist[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sq
rt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d*
f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*e
*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
- b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rule 1602

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[(C*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*f*h*Sqrt[c + d*
x]), x] + (Dist[1/(2*b*d*f*h), Int[(1*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f
*h) + (2*b*B*d*f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x])/(Sqrt[
a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(C*(d*e
- c*f)*(d*g - c*h))/(2*b*d*f*h), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rule 1604

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt
[e + f*x]*Sqrt[g + h*x])/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), x]
- Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m
 + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m +
 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
+ c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2
*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^
2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
+ c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{(7+5x)^{9/2}}\,dx = -\frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{35(7+5x)^{7/2}} + \frac{1}{35}\int \frac{-21+140x-72x^2}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+$$

$$= -\frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{35(7+5x)^{7/2}} + \frac{2558\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{695175(7+5x)^{5/2}} + \frac{\int}{}$$

$$= -\frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{35(7+5x)^{7/2}} + \frac{2558\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{695175(7+5x)^{5/2}} + \frac{23}{}$$

$$= -\frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{35(7+5x)^{7/2}} + \frac{2558\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{695175(7+5x)^{5/2}} + \frac{23}{}$$

$$= -\frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{35(7+5x)^{7/2}} + \frac{2558\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{695175(7+5x)^{5/2}} + \frac{23}{}$$

$$= -\frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{35(7+5x)^{7/2}} + \frac{2558\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{695175(7+5x)^{5/2}} + \frac{23}{}$$

$$= -\frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{35(7+5x)^{7/2}} + \frac{2558\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{695175(7+5x)^{5/2}} + \frac{23}{}$$

$$= -\frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{35(7+5x)^{7/2}} + \frac{2558\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{695175(7+5x)^{5/2}} + \frac{23}{}$$

Mathematica [A] time = 2.81, size = 259, normalized size = 0.70

$$\frac{2\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{5x+7}\left(\frac{242\left(19017205\sqrt{682}\,(3x-2)\sqrt{\frac{8x^2-18x-5}{(2-3x)^2}}\ \text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\,\sqrt{\frac{2x-5}{3x-2}}\right),\frac{39}{62}\right)+203578437\sqrt{\frac{5x+7}{3x-2}}\left(8x^2-18x-\right.\right.}{\sqrt{\frac{5x+7}{3x-2}}\left(8x^2-18x-5\right)}\right.}{2257624}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(7 + 5*x)^(9/2),x]
```

```
[Out] (2*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]*(-(((-2 + 3*x)*(1539551542327
0 + 113490310442229*x + 54668919175710*x^2 + 10263746198750*x^3))/(7 + 5*x)
^4) + (242*(203578437*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x + 8*x^2) - 6785
9479*Sqrt[682]*(-2 + 3*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticE[A
rcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + 19017205*Sqrt[682]
*(-2 + 3*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticF[ArcSin[Sqrt[31/
39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62]))/(Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5
- 18*x + 8*x^2))))/(2257624501329015*Sqrt[2 - 3*x])
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{5\,x+7}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}}{3125\,x^5+21875\,x^4+61250\,x^3+85750\,x^2+60025\,x+16807},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(9/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(3125*x^5
 + 21875*x^4 + 61250*x^3 + 85750*x^2 + 60025*x + 16807), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(9/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^(9/2), x)
```

maple [B] time = 0.05, size = 1160, normalized size = 3.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x+2)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)/(5*x+7)^(9/2),x)
```

```
[Out] -2/2257624501329015*(4737011092000*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)
*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticF(1/31*31
^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))*x^5+3284398
7836000*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))
^(1/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticE(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4
*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))*x^5+22263952132400*11^(1/2)*((5*x+7)/(
4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1
/2)*EllipticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*
78^(1/2))*x^4+154366742829200*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(
1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticE(1/31*31^(1/2
)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))*x^4+380974117074
10*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2
)*((3*x-2)/(4*x+1))^(1/2)*EllipticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1)
)^(1/2),1/39*31^(1/2)*78^(1/2))*x^3+264147772171030*11^(1/2)*((5*x+7)/(4*x+
1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*
EllipticE(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(
1/2))*x^3+28168636458578*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*
((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x^2*EllipticF(1/31*31^(1/2)
```

```
*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+195306773666774*1
1^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((
3*x-2)/(4*x+1))^(1/2)*x^2*EllipticE(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1)
)^(1/2),1/39*31^(1/2)*78^(1/2))+8240030794534*11^(1/2)*((5*x+7)/(4*x+1))^(1
/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x*Elli
pticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2)
)+57132116840722*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)
/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x*EllipticE(1/31*31^(1/2)*11^(1/2)*
((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+812397402278*11^(1/2)*((5*x
+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1
))^(1/2)*EllipticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(
1/2)*78^(1/2))+5632743913874*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1
/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticE(1/31*31^(1/2)
*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+5810951702460*x^5
-173342585590346*x^4+2153615020704860*x^3-4639703191080657*x^2+513664406072
72*x+1423213141652020)*(4*x+1)^(1/2)*(2*x-5)^(1/2)*(-3*x+2)^(1/2)/(120*x^4-
182*x^3-385*x^2+197*x+70)/(5*x+7)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(9/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(5*x + 7)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}{(5\,x+7)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^(9/2),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2))/(5*x + 7)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(-5+2*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**(9/2),x)
```

```
[Out] Timed out
```

3.85 $\int \frac{\sqrt{2-3x}\,\sqrt{1+4x}\,(7+5x)^{5/2}}{\sqrt{-5+2x}}\,dx$

Optimal. Leaf size=429

$$\frac{861015607\sqrt{\frac{11}{23}}\,\sqrt{5x+7}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\,\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{331776\sqrt{2x-5}\,\sqrt{\frac{5x+7}{5-2x}}}+\frac{1}{8}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(5x+7)^{5/2}+\frac{1445}{576}\sqrt{2-3}$$

```
[Out] 1445/576*(7+5*x)^(3/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)+1/8*(7+5*
x)^(5/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)+331574321009/711659520*
(2-3*x)*EllipticPi(1/23*253^(1/2)*(7+5*x)^(1/2)/(2-3*x)^(1/2),-69/55,1/39*I
*897^(1/2))*((5-2*x)/(2-3*x))^(1/2)*((-1-4*x)/(2-3*x))^(1/2)*429^(1/2)/(-5+
2*x)^(1/2)/(1+4*x)^(1/2)+2466927/4096*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(
1/2)/(-5+2*x)^(1/2)+1561915/27648*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2
)*(7+5*x)^(1/2)+861015607/7630848*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4
*x)/(2-3*x))^(1/2)*EllipticF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*
x)/(2-3*x))^(1/2),1/23*I*897^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/
((7+5*x)/(5-2*x))^(1/2)-2466927/8192*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)
/(-5+2*x)^(1/2),1/39*I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))
^(1/2)/((2-3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A] time = 0.53, antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.270, Rules used = {162, 1600, 1602, 1598, 170, 418, 165, 537, 176, 424}

$$\frac{1}{8}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(5x+7)^{5/2}+\frac{1445}{576}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(5x+7)^{3/2}+\frac{1561915\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x}}{27648}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2))/Sqrt[-5 + 2*x],x]
```

```
[Out] (2466927*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(4096*Sqrt[-5 + 2*x]) +
 (1561915*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/27648 +
 (1445*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2))/576 + (S
qrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2))/8 - (2466927*Sqr
t[429]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23
]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(8192*Sqrt[(2 - 3*x)/(5 - 2*x)]*
Sqrt[7 + 5*x]) + (861015607*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt
[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(331776*Sqrt[-5 + 2*x]*Sqrt[(7
 + 5*x)/(5 - 2*x)]) + (331574321009*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqr
t[-((1 + 4*x)/(2 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5
*x])/Sqrt[2 - 3*x]], -23/39])/(1658880*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*
x])
```

Rule 162

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*
(x_)])/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(2*(a + b*x)^m*Sqrt[c +
d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(d*(2*m + 3)), x] - Dist[1/(d*(2*m + 3)),
 Int[((a + b*x)^(m - 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[2
*b*c*e*g*m + a*(c*(f*g + e*h) - 2*d*e*g*(m + 1)) - (b*(2*d*e*g - c*(f*g + e
*h)*(2*m + 1)) - a*(2*c*f*h - d*(2*m + 1)*(f*g + e*h)))*x - (2*a*d*f*h*m +
b*(d*(f*g + e*h) - 2*c*f*h*(m + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, m}, x] && IntegerQ[2*m] && GtQ[m, 0]
```

Rule 165

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*(a + b*x)*Sqrt[((b*g -
 a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*
g - e*h)*(a + b*x))])/(Sqrt[c + d*x]*Sqrt[e + f*x]), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + ((b*c - a*d)*x^2)/(d*g - c*h)]*Sqrt[1 + ((b*e - a*f)*x^2)/(f*g
- e*h)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 1598

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 1600

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[(2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/
(d*f*h*(2*m + 3)), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1602

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[(C*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*f*h*Sqrt[c + d*
x]), x] + (Dist[1/(2*b*d*f*h), Int[(1*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f
*h) + (2*b*B*d*f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x])/(Sqrt[
a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(C*(d*e
- c*f)*(d*g - c*h))/(2*b*d*f*h), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\,\sqrt{1+4x}\,(7+5x)^{5/2}}{\sqrt{-5+2x}}\,dx = \frac{1}{8}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^{5/2} - \frac{1}{16}\int \frac{(7+5x)^{3/2}\left(-621-370x+\right.}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1}}$$

$$= \frac{1445}{576}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^{3/2} + \frac{1}{8}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}$$

$$= \frac{1561915\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{27648} + \frac{1445}{576}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1}$$

$$= \frac{2466927\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{4096\sqrt{-5+2x}} + \frac{1561915\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{27648}$$

$$= \frac{2466927\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{4096\sqrt{-5+2x}} + \frac{1561915\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{27648}$$

$$= \frac{2466927\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{4096\sqrt{-5+2x}} + \frac{1561915\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{27648}$$

$$= \frac{2466927\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{4096\sqrt{-5+2x}} + \frac{1561915\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{27648}$$

Mathematica [A] time = 3.85, size = 345, normalized size = 0.80

$$-\frac{\sqrt{2x-5}\sqrt{4x+1}\left(10666876180\sqrt{682}\sqrt{\frac{8x^2-18x-5}{(2-3x)^2}}\left(15x^2+11x-14\right)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{2x-5}{3x-2}}\right),\frac{39}{62}\right)-1\ldots\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2))/Sqrt[-5 + 2*x],x]
```

```
[Out] -1/41140224*(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(-12388907394*Sqrt[682]*Sqrt[(-5
- 18*x + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticE[ArcSin[Sqrt[31
/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + 10666876180*Sqrt[682]*Sqrt[(-5
- 18*x + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticF[ArcSin[Sqrt[31
/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + Sqrt[(7 + 5*x)/(-2 + 3*x)]*(186
*(-5752341805 - 26349657233*x - 12645389558*x^2 + 3088122056*x^3 + 10048195
20*x^4 + 439372800*x^5 + 82944000*x^6) + 10695945839*Sqrt[682]*(2 - 3*x)^2*
Sqrt[(1 + 4*x)/(-2 + 3*x)]*Sqrt[(-35 - 11*x + 10*x^2)/(2 - 3*x)^2]*Elliptic
Pi[117/62, ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62])))/(Sqrt
[2 - 3*x]*Sqrt[7 + 5*x]*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x + 8*x^2))
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(25\,x^2+70\,x+49\right)\sqrt{5\,x+7}\,\sqrt{4\,x+1}\,\sqrt{-3\,x+2}}{\sqrt{2\,x-5}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(5/2)*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((25*x^2 + 70*x + 49)*sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sq
rt(2*x - 5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5\,x+7)^{\frac{5}{2}}\sqrt{4\,x+1}\,\sqrt{-3\,x+2}}{\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(5/2)*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((5*x + 7)^(5/2)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

maple [B] time = 0.04, size = 890, normalized size = 2.07

$$-\frac{\sqrt{5x+7}\,\sqrt{-3x+2}\,\sqrt{4x+1}\,\sqrt{2x-5}\left(-355829760000x^6-1884909312000x^5-4310675740800x^4-13248\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x+7)^(5/2)*(-3*x+2)^(1/2)*(4*x+1)^(1/2)/(2*x-5)^(1/2),x)
```

```
[Out] -1/948879360*(5*x+7)^(1/2)*(-3*x+2)^(1/2)*(4*x+1)^(1/2)*(2*x-5)^(1/2)*(4241
70712240*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1)
)^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x^2*EllipticF(1/31*31^(1/2)*11^(1/2)*((5*x+
```

```
7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-3936108068752*11^(1/2)*((5*x+7)/(
4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1
/2)*x^2*EllipticPi(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),124/55,1/
39*31^(1/2)*78^(1/2))-4571906470560*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2
)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x^2*EllipticE(1/
31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+212085
356120*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^
(1/2)*((3*x-2)/(4*x+1))^(1/2)*x*EllipticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(
4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-1968054034376*11^(1/2)*((5*x+7)/(4*x+
1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*
x*EllipticPi(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),124/55,1/39*31^
(1/2)*78^(1/2))-2285953235280*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(
1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x*EllipticE(1/31*31^(1
/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+26510669515*11
^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3
*x-2)/(4*x+1))^(1/2)*EllipticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/
2),1/39*31^(1/2)*78^(1/2))-246006754297*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^
(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticPi(1
/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*78^(1/2)
)-285744154410*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(
4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticE(1/31*31^(1/2)*11^(1/2)*((5*
x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-355829760000*x^6-1884909312000*
x^5-4310675740800*x^4-13248043620240*x^3+85680578188920*x^2+78464986845960*
x-85333953104400)/(120*x^4-182*x^3-385*x^2+197*x+70)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x+7)^{\frac{5}{2}}\sqrt{4x+1}\sqrt{-3x+2}}{\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(5/2)*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((5*x + 7)^(5/2)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2-3x}\,\sqrt{4x+1}\,(5x+7)^{5/2}}{\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(5*x + 7)^(5/2))/(2*x - 5)^(1/2),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(5*x + 7)^(5/2))/(2*x - 5)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**(5/2)*(2-3*x)**(1/2)*(1+4*x)**(1/2)/(-5+2*x)**(1/2),x)
```

```
[Out] Timed out
```

3.86 $\int \frac{\sqrt{2-3x}\,\sqrt{1+4x}\,(7+5x)^{3/2}}{\sqrt{-5+2x}}\,dx$

Optimal. Leaf size=391

$$\frac{2824441\sqrt{\frac{11}{23}}\,\sqrt{5x+7}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\,\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{17280\sqrt{2x-5}\,\sqrt{\frac{5x+7}{5-2x}}}+\frac{1}{6}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(5x+7)^{3/2}+\frac{977}{288}\sqrt{2-3x}$$

```
[Out] 1/6*(7+5*x)^(3/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)+963142751/3706
5600*(2-3*x)*EllipticPi(1/23*253^(1/2)*(7+5*x)^(1/2)/(2-3*x)^(1/2),-69/55,1
/39*I*897^(1/2))*((5-2*x)/(2-3*x))^(1/2)*((-1-4*x)/(2-3*x))^(1/2)*429^(1/2)
/(-5+2*x)^(1/2)/(1+4*x)^(1/2)+66377/1920*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x
)^(1/2)/(-5+2*x)^(1/2)+977/288*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)*(
7+5*x)^(1/2)+2824441/397440*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)/(2
-3*x))^(1/2)*EllipticF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/(2-
3*x))^(1/2),1/23*I*897^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7+5*
x)/(5-2*x))^(1/2)-66377/3840*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x
)^(1/2),1/39*I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/2)/(
(2-3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.270, Rules used = {162, 1600, 1602, 1598, 170, 418, 165, 537, 176, 424}

$$\frac{1}{6}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(5x+7)^{3/2}+\frac{977}{288}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{5x+7}+\frac{66377\sqrt{2-3x}\,\sqrt{4x+1}\,\sqrt{5x+}}{1920\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2))/Sqrt[-5 + 2*x],x]
```

```
[Out] (66377*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(1920*Sqrt[-5 + 2*x]) + (
977*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/288 + (Sqrt[2
 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2))/6 - (66377*Sqrt[143/3
]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqr
t[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(1280*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[
7 + 5*x]) + (2824441*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*
x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(17280*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/
(5 - 2*x)]) + (963142751*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((1 + 4*
x)/(2 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2
 - 3*x]], -23/39])/(86400*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Rule 162

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*
(x_)])/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(2*(a + b*x)^m*Sqrt[c +
d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(d*(2*m + 3)), x] - Dist[1/(d*(2*m + 3)),
 Int[((a + b*x)^(m - 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[2
*b*c*e*g*m + a*(c*(f*g + e*h) - 2*d*e*g*(m + 1)) - (b*(2*d*e*g - c*(f*g + e
*h)*(2*m + 1)) - a*(2*c*f*h - d*(2*m + 1)*(f*g + e*h)))*x - (2*a*d*f*h*m +
b*(d*(f*g + e*h) - 2*c*f*h*(m + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, m}, x] && IntegerQ[2*m] && GtQ[m, 0]
```

Rule 165

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*(a + b*x)*Sqrt[((b*g -
```

```
 a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*
g - e*h)*(a + b*x))])/(Sqrt[c + d*x]*Sqrt[e + f*x]), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + ((b*c - a*d)*x^2)/(d*g - c*h)]*Sqrt[1 + ((b*e - a*f)*x^2)/(f*g
- e*h)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 1598

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 1600

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
```

```
ymbol] :> Simp[(2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/
(d*f*h*(2*m + 3)), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1602

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[(C*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*f*h*Sqrt[c + d*
x]), x] + (Dist[1/(2*b*d*f*h), Int[(1*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f
*h) + (2*b*B*d*f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x])/(Sqrt[
a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(C*(d*e
- c*f)*(d*g - c*h))/(2*b*d*f*h), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\,\sqrt{1+4x}\,(7+5x)^{3/2}}{\sqrt{-5+2x}}\,dx = \frac{1}{6}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^{3/2} - \frac{1}{12}\int \frac{\sqrt{7+5x}\,(-465+20x}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{}}$$

$$= \frac{977}{288}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x} + \frac{1}{6}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}$$

$$= \frac{66377\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{1920\sqrt{-5+2x}} + \frac{977}{288}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+}$$

$$= \frac{66377\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{1920\sqrt{-5+2x}} + \frac{977}{288}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+}$$

$$= \frac{66377\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{1920\sqrt{-5+2x}} + \frac{977}{288}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+}$$

$$= \frac{66377\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{1920\sqrt{-5+2x}} + \frac{977}{288}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+}$$

Mathematica [A] time = 3.77, size = 340, normalized size = 0.87

$$-\frac{\sqrt{2x-5}\,\sqrt{4x+1}\left(31389484\sqrt{682}\sqrt{\frac{8x^2-18x-5}{(2-3x)^2}}\left(15x^2+11x-14\right)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{2x-5}{3x-2}}\right),\frac{39}{62}\right)-3703}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2))/Sqrt[-5 + 2*x],x]
```

```
[Out] -1/2142720*(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(-37038366*Sqrt[682]*Sqrt[(-5 - 18
*x + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticE[ArcSin[Sqrt[31/39]
*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + 31389484*Sqrt[682]*Sqrt[(-5 - 18*x
+ 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticF[ArcSin[Sqrt[31/39]*Sq
rt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + Sqrt[(7 + 5*x)/(-2 + 3*x)]*(186*(-1723
2355 - 79187903*x - 38640362*x^2 + 10641080*x^3 + 4555200*x^4 + 1152000*x^5
) + 31069121*Sqrt[682]*(2 - 3*x)^2*Sqrt[(1 + 4*x)/(-2 + 3*x)]*Sqrt[(-35 - 1
1*x + 10*x^2)/(2 - 3*x)^2]*EllipticPi[117/62, ArcSin[Sqrt[31/39]*Sqrt[(-5 +
 2*x)/(-2 + 3*x)]], 39/62])))/(Sqrt[2 - 3*x]*Sqrt[7 + 5*x]*Sqrt[(7 + 5*x)/(
-2 + 3*x)]*(-5 - 18*x + 8*x^2))
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(5\,x+7)^{\frac{3}{2}}\sqrt{4\,x+1}\,\sqrt{-3\,x+2}}{\sqrt{2\,x-5}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(3/2)*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5\,x+7)^{\frac{3}{2}}\sqrt{4\,x+1}\,\sqrt{-3\,x+2}}{\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(3/2)*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

maple [B] time = 0.02, size = 885, normalized size = 2.26

$$-\frac{\sqrt{5x+7}\,\sqrt{-3x+2}\,\sqrt{4x+1}\,\sqrt{2x-5}\,\left(-4942080000x^{5}-19541808000x^{4}-45650233200x^{3}-13668351840\sqrt{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x+7)^(3/2)*(-3*x+2)^(1/2)*(4*x+1)^(1/2)/(2*x-5)^(1/2),x)
```

```
[Out] -1/49420800*(5*x+7)^(1/2)*(-3*x+2)^(1/2)*(4*x+1)^(1/2)*(2*x-5)^(1/2)*(12407
32240*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(
1/2)*((3*x-2)/(4*x+1))^(1/2)*x^2*EllipticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/
(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-11433436528*11^(1/2)*((5*x+7)/(4*x+1
))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x
^2*EllipticPi(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),124/55,1/39*31
^(1/2)*78^(1/2))-13668351840*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1
/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x^2*EllipticE(1/31*31^(
1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+620366120*11^
(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*
x-2)/(4*x+1))^(1/2)*x*EllipticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1
/2),1/39*31^(1/2)*78^(1/2))-5716718264*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(
1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x*EllipticPi(
```

```
1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*78^(1/2
))-6834175920*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4
*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x*EllipticE(1/31*31^(1/2)*11^(1/2)*((5
*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+77545765*11^(1/2)*((5*x+7)/(4*
x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2
)*EllipticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78
^(1/2))-714589783*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5
)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticPi(1/31*31^(1/2)*11^(1/2)*
((5*x+7)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*78^(1/2))-854271990*11^(1/2)*(
(5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4
*x+1))^(1/2)*EllipticE(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*
31^(1/2)*78^(1/2))-4942080000*x^5-19541808000*x^4-45650233200*x^3+259737071
880*x^2+236349193080*x-254967913200)/(120*x^4-182*x^3-385*x^2+197*x+70)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x+7)^{\frac{3}{2}}\sqrt{4x+1}\sqrt{-3x+2}}{\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(3/2)*(2-3*x)^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2-3x}\,\sqrt{4x+1}\,(5x+7)^{3/2}}{\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(5*x + 7)^(3/2))/(2*x - 5)^(1/2),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(5*x + 7)^(3/2))/(2*x - 5)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**(3/2)*(2-3*x)**(1/2)*(1+4*x)**(1/2)/(-5+2*x)**(1/2),x)
```

```
[Out] Timed out
```

3.87 $\int \frac{\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{\sqrt{-5+2x}}\,dx$

Optimal. Leaf size=351

$$\frac{8959\sqrt{\frac{11}{23}}\,\sqrt{5x+7}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\,\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{720\sqrt{2x-5}\,\sqrt{\frac{5x+7}{5-2x}}}+\frac{1}{4}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{5x+7}+\frac{509\sqrt{2-3x}\,\sqrt{4x+}}{240\sqrt{2x-}}$$

```
[Out] 2198489/1544400*(2-3*x)*EllipticPi(1/23*253^(1/2)*(7+5*x)^(1/2)/(2-3*x)^(1/
2),-69/55,1/39*I*897^(1/2))*((5-2*x)/(2-3*x))^(1/2)*((-1-4*x)/(2-3*x))^(1/2
)*429^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)+509/240*(2-3*x)^(1/2)*(1+4*x)^(1/2
)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)+1/4*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/
2)*(7+5*x)^(1/2)+8959/16560*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)/(2
-3*x))^(1/2)*EllipticF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/(2-
3*x))^(1/2),1/23*I*897^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7+5*
x)/(5-2*x))^(1/2)-509/480*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(
1/2),1/39*I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/2)/((2-
3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {161, 1602, 1598, 170, 418, 165, 537, 176, 424}

$$\frac{1}{4}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{5x+7}+\frac{509\sqrt{2-3x}\,\sqrt{4x+1}\,\sqrt{5x+7}}{240\sqrt{2x-5}}+\frac{8959\sqrt{\frac{11}{23}}\,\sqrt{5x+7}\,F\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\,\sqrt{2-3x}}\right)|-\right.}{720\sqrt{2x-5}\,\sqrt{\frac{5x+7}{5-2x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/Sqrt[-5 + 2*x],x]
```

```
[Out] (509*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(240*Sqrt[-5 + 2*x]) + (Sqr
t[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/4 - (509*Sqrt[143/3]
*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqrt
[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(160*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7
+ 5*x]) + (8959*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(S
qrt[2]*Sqrt[2 - 3*x])], -39/23])/(720*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 - 2*
x)]) + (2198489*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((1 + 4*x)/(2 - 3
*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]],
 -23/39])/(3600*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Rule 161

```
Int[((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)], x_Symbol] :> Simp[(2*(a + b*x)^(m + 1)*Sqrt[
c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*(2*m + 5)), x] + Dist[1/(b*(2*m +
5)), Int[((a + b*x)^m*Simp[3*b*c*e*g - a*(d*e*g + c*f*g + c*e*h) + 2*(b*(d*
e*g + c*f*g + c*e*h) - a*(d*f*g + d*e*h + c*f*h))*x - (3*a*d*f*h - b*(d*f*g
 + d*e*h + c*f*h))*x^2, x])/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IntegerQ[2*m] &&  !LtQ[m,
-1]
```

Rule 165

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*(a + b*x)*Sqrt[((b*g -
 a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*
```

```
g - e*h)*(a + b*x))])/(Sqrt[c + d*x]*Sqrt[e + f*x]), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + ((b*c - a*d)*x^2)/(d*g - c*h)]*Sqrt[1 + ((b*e - a*f)*x^2)/(f*g
- e*h)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 1598

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 1602

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[(C*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*f*h*Sqrt[c + d*
```

```
x]), x] + (Dist[1/(2*b*d*f*h), Int[(1*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f
*h) + (2*b*B*d*f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x])/(Sqrt[
a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(C*(d*e
- c*f)*(d*g - c*h))/(2*b*d*f*h), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{\sqrt{-5+2x}}\,dx = \frac{1}{4}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} + \frac{1}{8}\int \frac{309-410x-1018x^2}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7}$$

$$= \frac{509\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{240\sqrt{-5+2x}} + \frac{1}{4}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} - \frac{\int}{}$$

$$= \frac{509\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{240\sqrt{-5+2x}} + \frac{1}{4}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} + \frac{70}{}$$

$$= \frac{509\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{240\sqrt{-5+2x}} + \frac{1}{4}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} - \frac{50}{}$$

$$= \frac{509\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{240\sqrt{-5+2x}} + \frac{1}{4}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} - \frac{50}{}$$

Mathematica [A] time = 4.21, size = 347, normalized size = 0.99

$$\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}\left(66960(2-3x) - \frac{3\left(76756\sqrt{682}\sqrt{\frac{8x^2-18x-5}{(2-3x)^2}}\left(15x^2+11x-14\right)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{2x-5}{3x-2}}\right),\frac{39}{62}\right)+94674\sqrt{68}\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/Sqrt[-5 + 2*x],x]
```

```
[Out] (Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]*(66960*(2 - 3*x) - (3*(94674*Sq
rt[682]*(2 - 3*x)*(7 + 5*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticE
[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + 76756*Sqrt[682]*
Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticF[ArcSi
n[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + Sqrt[(7 + 5*x)/(-2 + 3
*x)]*(284022*(-35 - 151*x - 34*x^2 + 40*x^3) + 70919*Sqrt[682]*(2 - 3*x)^2*
Sqrt[(1 + 4*x)/(-2 + 3*x)]*Sqrt[(-35 - 11*x + 10*x^2)/(2 - 3*x)^2]*Elliptic
Pi[117/62, ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62])))/((2 -
 3*x)*((7 + 5*x)/(-2 + 3*x))^(3/2)*(5 + 18*x - 8*x^2))))/(267840*Sqrt[2 - 3
*x])
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{5\,x+7}\,\sqrt{4\,x+1}\,\sqrt{-3\,x+2}}{\sqrt{2\,x-5}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{5\,x+7}\,\sqrt{4\,x+1}\,\sqrt{-3\,x+2}}{\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

maple [B] time = 0.02, size = 880, normalized size = 2.51

$$-\frac{\sqrt{-3x+2}\,\sqrt{4x+1}\,\sqrt{5x+7}\,\sqrt{2x-5}\,\left(-61776000x^4-168339600x^3-34937760\sqrt{11}\,\sqrt{\frac{5x+7}{4x+1}}\,\sqrt{3}\,\sqrt{13}\,\sqrt{\frac{2x-}{4x-}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x+2)^(1/2)*(4*x+1)^(1/2)*(5*x+7)^(1/2)/(2*x-5)^(1/2),x)
```

```
[Out] -1/2059200*(-3*x+2)^(1/2)*(4*x+1)^(1/2)*(5*x+7)^(1/2)*(2*x-5)^(1/2)*(362296
0*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)
*((3*x-2)/(4*x+1))^(1/2)*x^2*EllipticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x
+1))^(1/2),1/39*31^(1/2)*78^(1/2))-26098192*11^(1/2)*((5*x+7)/(4*x+1))^(1/2
)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x^2*Elli
pticPi(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*
78^(1/2))-34937760*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-
5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x^2*EllipticE(1/31*31^(1/2)*11^(1
/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+1811480*11^(1/2)*((5*x+
7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1)
)^(1/2)*x*EllipticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^
(1/2)*78^(1/2))-13049096*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*
((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x*EllipticPi(1/31*31^(1/2)*
11^(1/2)*((5*x+7)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*78^(1/2))-17468880*11
^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3
*x-2)/(4*x+1))^(1/2)*x*EllipticE(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(
1/2),1/39*31^(1/2)*78^(1/2))+226435*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2
)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticF(1/31*3
1^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-1631137*11
^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3
*x-2)/(4*x+1))^(1/2)*EllipticPi(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1
/2),124/55,1/39*31^(1/2)*78^(1/2))-2183610*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)
*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticE
(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-168
339600*x^3-61776000*x^4+661123320*x^2+623542920*x-647446800)/(120*x^4-182*x
^3-385*x^2+197*x+70)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{5\,x+7}\,\sqrt{4\,x+1}\,\sqrt{-3\,x+2}}{\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(-3*x + 2)/sqrt(2*x - 5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{5\,x+7}}{\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(5*x + 7)^(1/2))/(2*x - 5)^(1/2),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(5*x + 7)^(1/2))/(2*x - 5)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2-3x}\,\sqrt{4x+1}\,\sqrt{5x+7}}{\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)*(7+5*x)**(1/2)/(-5+2*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)*sqrt(4*x + 1)*sqrt(5*x + 7)/sqrt(2*x - 5), x)
```

3.88 $\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}\sqrt{7+5x}}\,dx$

Optimal. Leaf size=365

$$\frac{7\sqrt{\frac{11}{23}}\sqrt{5x+7}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{10\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}+\frac{41\sqrt{\frac{11}{62}}\sqrt{2-3x}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{\frac{22}{23}}\sqrt{5x+7}}{\sqrt{2x-5}}\right),\frac{39}{62}\right)}{20\sqrt{-\frac{2-3x}{4x+1}}\sqrt{4x+1}}+\frac{\sqrt{2-}}{}$$

```
[Out] 41/1240*(1/(529+506*(7+5*x)/(-5+2*x)))^(1/2)*(529+506*(7+5*x)/(-5+2*x))^(1/
2)*EllipticF(506^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/(529+506*(7+5*x)/(-5+2*
x))^(1/2),1/62*2418^(1/2))*682^(1/2)*(2-3*x)^(1/2)/((-2+3*x)/(1+4*x))^(1/2)
/(1+4*x)^(1/2)+943/68200*(1/(529+506*(7+5*x)/(-5+2*x)))^(1/2)*(529+506*(7+5
*x)/(-5+2*x))^(1/2)*EllipticPi(506^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/(529+
506*(7+5*x)/(-5+2*x))^(1/2),78/55,1/62*2418^(1/2))*(2-3*x)^(1/2)*682^(1/2)/
((-2+3*x)/(1+4*x))^(1/2)/(1+4*x)^(1/2)+1/5*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5
*x)^(1/2)/(-5+2*x)^(1/2)+7/230*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)
/(2-3*x))^(1/2)*EllipticF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/
(2-3*x))^(1/2),1/23*I*897^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7
+5*x)/(5-2*x))^(1/2)-1/10*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(
1/2),1/39*I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/2)/((2-
3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.216$, Rules used = {173, 176, 424, 170, 418, 165, 536, 539}

$$\frac{\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{5\sqrt{2x-5}}+\frac{7\sqrt{\frac{11}{23}}\sqrt{5x+7}\,F\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{10\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}+\frac{41\sqrt{\frac{11}{62}}\sqrt{2-3x}\,F\left(\tan^{-1}\left(\frac{\sqrt{\frac{22}{23}}\sqrt{5x+7}}{\sqrt{2x-5}}\right)\right.}{20\sqrt{-\frac{2-3x}{4x+1}}\sqrt{4x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*Sqrt[7 + 5*x]),x]
```

```
[Out] (Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(5*Sqrt[-5 + 2*x]) - (Sqrt[429]
*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqrt
[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(10*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7 +
 5*x]) + (7*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[
2]*Sqrt[2 - 3*x])], -39/23])/(10*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 - 2*x)])
+ (41*Sqrt[11/62]*Sqrt[2 - 3*x]*EllipticF[ArcTan[(Sqrt[22/23]*Sqrt[7 + 5*x]
)/Sqrt[-5 + 2*x]], 39/62])/(20*Sqrt[-((2 - 3*x)/(1 + 4*x))]*Sqrt[1 + 4*x])
+ (943*Sqrt[2 - 3*x]*EllipticPi[78/55, ArcTan[(Sqrt[22/23]*Sqrt[7 + 5*x])/S
qrt[-5 + 2*x]], 39/62])/(100*Sqrt[682]*Sqrt[-((2 - 3*x)/(1 + 4*x))]*Sqrt[1
+ 4*x])
```

Rule 165

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*(a + b*x)*Sqrt[((b*g -
 a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*
g - e*h)*(a + b*x))])/(Sqrt[c + d*x]*Sqrt[e + f*x]), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + ((b*c - a*d)*x^2)/(d*g - c*h)]*Sqrt[1 + ((b*e - a*f)*x^2)/(f*g
- e*h)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 173

```
Int[(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(Sqrt[a + b*x]*Sqrt[c +
 d*x]*Sqrt[g + h*x])/(h*Sqrt[e + f*x]), x] + (-Dist[((d*e - c*f)*(f*g - e*h
))/(2*f*h), Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*(e + f*x)^(3/2)*Sqrt[g + h*x])
, x], x] + Dist[((d*e - c*f)*(b*f*g + b*e*h - 2*a*f*h))/(2*f^2*h), Int[1/(S
qrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(a*d
*f*h - b*(d*f*g + d*e*h - c*f*h))/(2*f^2*h), Int[Sqrt[e + f*x]/(Sqrt[a + b*
x]*Sqrt[c + d*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h},
 x]
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 536

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> -Dist[f/(b*e - a*f), Int[1/(Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] + Dist[b/(b*e - a*f), Int[Sqrt[e + f*x^2]/((a + b*x^2)*Sqrt[
c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[d/c, 0] && GtQ[f
/e, 0] &&  !SimplerSqrtQ[d/c, f/e]
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}\sqrt{7+5x}}\,dx = \frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{5\sqrt{-5+2x}} - \frac{41}{20}\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}\,dx + \frac{77}{20}\int \frac{}{\sqrt{2-3}}$$

$$= \frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{5\sqrt{-5+2x}} - \frac{\left(1599\sqrt{-\frac{2-3x}{-5+2x}}(-5+2x)\sqrt{\frac{1+4x}{-5+2x}}\right)\text{Subst}\left(\int \frac{}{(5-2x^2)}\right.}{10\sqrt{713}\sqrt{2-3x}\sqrt{1+4}}$$

$$= \frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{5\sqrt{-5+2x}} - \frac{\sqrt{429}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)\middle|-\frac{23}{39}\right)}{10\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} +$$

$$= \frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{5\sqrt{-5+2x}} - \frac{\sqrt{429}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)\middle|-\frac{23}{39}\right)}{10\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} +$$

Mathematica [A] time = 1.29, size = 318, normalized size = 0.87

$$\frac{\sqrt{2-3x}\left(1984\sqrt{682}\sqrt{\frac{5-2x}{5x+7}}\sqrt{\frac{4x+1}{5x+7}}\left(15x^2+11x-14\right)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{155-62x}{55x+77}}\right),\frac{23}{62}\right)-3410\sqrt{682}\sqrt{\frac{5-2x}{5x+7}}\sqrt{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*Sqrt[7 + 5*x]),x]
```

```
[Out] (Sqrt[2 - 3*x]*(-3410*Sqrt[682]*Sqrt[(5 - 2*x)/(7 + 5*x)]*Sqrt[(1 + 4*x)/(7
 + 5*x)]*(-14 + 11*x + 15*x^2)*EllipticE[ArcSin[Sqrt[(155 - 62*x)/(77 + 55*
x)]], 23/62] + 1984*Sqrt[682]*Sqrt[(5 - 2*x)/(7 + 5*x)]*Sqrt[(1 + 4*x)/(7 +
 5*x)]*(-14 + 11*x + 15*x^2)*EllipticF[ArcSin[Sqrt[(155 - 62*x)/(77 + 55*x)
]], 23/62] + Sqrt[(-2 + 3*x)/(7 + 5*x)]*(17050*(10 + 21*x - 70*x^2 + 24*x^3
) - 1599*Sqrt[682]*Sqrt[(1 + 4*x)/(7 + 5*x)]*(7 + 5*x)^2*Sqrt[(-10 + 19*x -
 6*x^2)/(7 + 5*x)^2]*EllipticPi[-55/62, ArcSin[Sqrt[(155 - 62*x)/(77 + 55*x
)]], 23/62])))/(34100*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*((-2 + 3*x)/(7 + 5*x))^(
3/2)*(7 + 5*x)^(3/2))
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{5x+7}\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{10x^2-11x-35},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2)/(-5+2*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(10*x^2 -
 11*x - 35), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{4x+1}\sqrt{-3x+2}}{\sqrt{5x+7}\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2)/(-5+2*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/(sqrt(5*x + 7)*sqrt(2*x - 5)), x)
```

maple [A] time = 0.03, size = 875, normalized size = 2.40

$$-\frac{\sqrt{-3x+2}\,\sqrt{4x+1}\,\sqrt{5x+7}\,\sqrt{2x-5}\,\left(-514800x^3-68640\sqrt{11}\,\sqrt{\frac{5x+7}{4x+1}}\,\sqrt{3}\,\sqrt{13}\,\sqrt{\frac{2x-5}{4x+1}}\,\sqrt{\frac{3x-2}{4x+1}}\,x^2\,\mathrm{EllipticE}\left(\frac{\sqrt{}}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x+2)^(1/2)*(4*x+1)^(1/2)/(5*x+7)^(1/2)/(2*x-5)^(1/2),x)
```

```
[Out] -1/42900*(-3*x+2)^(1/2)*(4*x+1)^(1/2)*(5*x+7)^(1/2)*(2*x-5)^(1/2)*(20240*11
^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3
*x-2)/(4*x+1))^(1/2)*x^2*EllipticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))
^(1/2),1/39*31^(1/2)*78^(1/2))-15088*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/
2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x^2*EllipticPi(
1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*78^(1/2
))-68640*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1)
)^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x^2*EllipticE(1/31*31^(1/2)*11^(1/2)*((5*x+
7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+10120*11^(1/2)*((5*x+7)/(4*x+1))^
(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x*El
lipticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/
2))-7544*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1)
)^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x*EllipticPi(1/31*31^(1/2)*11^(1/2)*((5*x+7
)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*78^(1/2))-34320*11^(1/2)*((5*x+7)/(4*
x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2
)*x*EllipticE(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*
78^(1/2))+1265*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(
4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticF(1/31*31^(1/2)*11^(1/2)*((5*
x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-943*11^(1/2)*((5*x+7)/(4*x+1))^
(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*Elli
pticPi(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*
78^(1/2))-4290*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(
4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticE(1/31*31^(1/2)*11^(1/2)*((5*
x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-514800*x^3+909480*x^2+1424280*x
-1201200)/(120*x^4-182*x^3-385*x^2+197*x+70)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{4\,x+1}\,\sqrt{-3\,x+2}}{\sqrt{5\,x+7}\,\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2)/(-5+2*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/(sqrt(5*x + 7)*sqrt(2*x - 5)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}}{\sqrt{2\,x-5}\,\sqrt{5\,x+7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^(1/2)),x)
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2-3x}\,\sqrt{4x+1}}{\sqrt{2x-5}\,\sqrt{5x+7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**(1/2)/(-5+2*x)**(1/2),x)
[Out] Integral(sqrt(2 - 3*x)*sqrt(4*x + 1)/(sqrt(2*x - 5)*sqrt(5*x + 7)), x)
```

3.89 $\int \frac{\sqrt{2-3x}\,\sqrt{1+4x}}{\sqrt{-5+2x}\,(7+5x)^{3/2}}\,dx$

Optimal. Leaf size=279

$$-\frac{4\sqrt{2-3x}\,\sqrt{4x+1}\,\sqrt{5x+7}}{195\sqrt{2x-5}}+\frac{2\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{39\sqrt{5x+7}}+\frac{2\sqrt{\frac{11}{39}}\,\sqrt{2-3x}\,\sqrt{\frac{5x+7}{5-2x}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\,\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{5\sqrt{\frac{2-3x}{5-2x}}\,\sqrt{5x+7}}-$$

```
[Out] -69/8525*(1+4*x)*EllipticPi(1/39*858^(1/2)*(7+5*x)^(1/2)/(1+4*x)^(1/2),78/5
5,1/62*2418^(1/2))*682^(1/2)*((-2+3*x)/(1+4*x))^(1/2)*((-5+2*x)/(1+4*x))^(1
/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)+2/39*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^
(1/2)/(7+5*x)^(1/2)-4/195*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x
)^(1/2)+2/195*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),1/39*I*
897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/2)/((2-3*x)/(5-2*x)
)^(1/2)/(7+5*x)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {164, 1586, 1595, 165, 537, 176, 424}

$$-\frac{4\sqrt{2-3x}\,\sqrt{4x+1}\,\sqrt{5x+7}}{195\sqrt{2x-5}}+\frac{2\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{39\sqrt{5x+7}}+\frac{2\sqrt{\frac{11}{39}}\,\sqrt{2-3x}\,\sqrt{\frac{5x+7}{5-2x}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\,\sqrt{4x+1}}{\sqrt{2x-5}}\right)|-\frac{23}{39}\right)}{5\sqrt{\frac{2-3x}{5-2x}}\,\sqrt{5x+7}}-$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^(3/2)),x]
```

```
[Out] (2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(39*Sqrt[7 + 5*x]) - (4*Sqrt
[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(195*Sqrt[-5 + 2*x]) + (2*Sqrt[11/39
]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqr
t[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(5*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7 +
 5*x]) - (69*Sqrt[2/341]*Sqrt[-((2 - 3*x)/(1 + 4*x))]*Sqrt[-((5 - 2*x)/(1 +
 4*x))]*(1 + 4*x)*EllipticPi[78/55, ArcSin[(Sqrt[22/39]*Sqrt[7 + 5*x])/Sqrt
[1 + 4*x]], 39/62])/(25*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x])
```

Rule 164

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*
(x_)])/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[((a + b*x)^(m + 1)*Sqrt[
c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((m + 1)*(b*c - a*d)), x] - Dist[1/(2
*(m + 1)*(b*c - a*d)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*
Sqrt[g + h*x]))*Simp[c*(f*g + e*h) + d*e*g*(2*m + 3) + 2*(c*f*h + d*(m + 2)
*(f*g + e*h))*x + d*f*h*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 165

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*(a + b*x)*Sqrt[((b*g -
 a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*
g - e*h)*(a + b*x))])/(Sqrt[c + d*x]*Sqrt[e + f*x]), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + ((b*c - a*d)*x^2)/(d*g - c*h)]*Sqrt[1 + ((b*e - a*f)*x^2)/(f*g
- e*h)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1595

```
Int[(Sqrt[(a_.) + (b_.)*(x_)]*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x_
)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(b
*B*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(d*f*h*Sqrt[a + b*x]), x] + (
-Dist[(B*(b*g - a*h))/(2*f*h), Int[Sqrt[e + f*x]/(Sqrt[a + b*x]*Sqrt[c + d*
x]*Sqrt[g + h*x]), x], x] + Dist[(B*(b*e - a*f)*(b*g - a*h))/(2*d*f*h), Int
[Sqrt[c + d*x]/((a + b*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x]) /; Fr
eeQ[{a, b, c, d, e, f, g, h, A, B}, x] && EqQ[2*A*d*f - B*(d*e + c*f), 0]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\,\sqrt{1+4x}}{\sqrt{-5+2x}\,(7+5x)^{3/2}}\,dx = \frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{39\sqrt{7+5x}} - \frac{1}{39}\int \frac{-25+130x-48x^2}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}}\,dx$$

$$= \frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{39\sqrt{7+5x}} - \frac{1}{39}\int \frac{(5-24x)\sqrt{-5+2x}}{\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}\,dx$$

$$= \frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{39\sqrt{7+5x}} - \frac{4\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{195\sqrt{-5+2x}} - \frac{3}{5}\int \frac{\sqrt{1+ \cdots}}{\sqrt{2-3x}\,\sqrt{-5 \cdots}}$$

$$= \frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{39\sqrt{7+5x}} - \frac{4\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{195\sqrt{-5+2x}} - \frac{\left(46\sqrt{\frac{3}{403}}\sqrt{-\frac{2-3x}{1+4x}}\,\cdots\right.}{\cdots}$$

$$= \frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{39\sqrt{7+5x}} - \frac{4\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{195\sqrt{-5+2x}} + \frac{2\sqrt{\frac{11}{39}}\,\sqrt{2-3x}\,\sqrt{\frac{7+\cdots}{5-\cdots}}}{5\sqrt{\cdots}}$$

Mathematica [A] time = 2.47, size = 326, normalized size = 1.17

$$\frac{\sqrt{2x-5}\,\sqrt{4x+1}\left(23\sqrt{682}\sqrt{\frac{8x^2-18x-5}{(2-3x)^2}}\left(15x^2+11x-14\right)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{2x-5}{3x-2}}\right),\frac{39}{62}\right)-62\sqrt{682}\sqrt{\frac{8x^2-18\cdots}{(2-3x\cdots}}\cdots\right.}{6\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^(3/2)),x]
```

```
[Out] (Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(-62*Sqrt[682]*Sqrt[(-5 - 18*x + 8*x^2)/(2 -
3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticE[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/
(-2 + 3*x)]], 39/62] + 23*Sqrt[682]*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*(
-14 + 11*x + 15*x^2)*EllipticF[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x
)]], 39/62] - 2*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-961*(-5 - 18*x + 8*x^2) + 39*S
qrt[682]*(2 - 3*x)^2*Sqrt[(1 + 4*x)/(-2 + 3*x)]*Sqrt[(-35 - 11*x + 10*x^2)/
(2 - 3*x)^2]*EllipticPi[117/62, ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*
x)]], 39/62])))/(6045*Sqrt[2 - 3*x]*Sqrt[7 + 5*x]*Sqrt[(7 + 5*x)/(-2 + 3*x)
]*(-5 - 18*x + 8*x^2))
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{5\,x+7}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}}{50\,x^3+15\,x^2-252\,x-245},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2)/(-5+2*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(50*x^3 +
 15*x^2 - 252*x - 245), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{4\,x+1}\,\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{3}{2}}\,\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2)/(-5+2*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^(3/2)*sqrt(2*x - 5)), x)
```

maple [B] time = 0.03, size = 870, normalized size = 3.12

$$2\sqrt{-3x+2}\,\sqrt{4x+1}\,\sqrt{5x+7}\,\sqrt{2x-5}\,\left(-880\sqrt{11}\,\sqrt{\frac{5x+7}{4x+1}}\,\sqrt{3}\,\sqrt{13}\,\sqrt{\frac{2x-5}{4x+1}}\,\sqrt{\frac{3x-2}{4x+1}}\,x^2\,\mathrm{EllipticE}\left(\frac{\sqrt{31}\,\sqrt{11}\,\sqrt{\frac{5x+7}{4x+1}}}{31}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x+2)^(1/2)*(4*x+1)^(1/2)/(5*x+7)^(3/2)/(2*x-5)^(1/2),x)
```

```
[Out] 2/10725*(-3*x+2)^(1/2)*(4*x+1)^(1/2)*(5*x+7)^(1/2)*(2*x-5)^(1/2)*(1104*11^(
1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x
-2)/(4*x+1))^(1/2)*x^2*EllipticPi(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^
(1/2),124/55,1/39*31^(1/2)*78^(1/2))-880*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3
^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x^2*Ellipti
cE(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+8
80*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2
)*((3*x-2)/(4*x+1))^(1/2)*x^2*EllipticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*
x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+552*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^
(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x*EllipticPi
(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*78^(1/
2))-440*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))
^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x*EllipticE(1/31*31^(1/2)*11^(1/2)*((5*x+7)/
(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+440*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)
*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x*Ellipti
cF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+6
9*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)
*((3*x-2)/(4*x+1))^(1/2)*EllipticPi(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1)
)^(1/2),124/55,1/39*31^(1/2)*78^(1/2))-55*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*
3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticE(
1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+55*1
1^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((
3*x-2)/(4*x+1))^(1/2)*EllipticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1
/2),1/39*31^(1/2)*78^(1/2))-7590*x^2+24035*x-12650)/(120*x^4-182*x^3-385*x^
2+197*x+70)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{4\,x+1}\,\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{3}{2}}\,\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2)/(-5+2*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^(3/2)*sqrt(2*x - 5)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}}{\sqrt{2\,x-5}\,{\left(5\,x+7\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^(3/2)),x)
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**(3/2)/(-5+2*x)**(1/2),x)
[Out] Timed out
```

3.90 $\int \frac{\sqrt{2-3x}\,\sqrt{1+4x}}{\sqrt{-5+2x}\,(7+5x)^{5/2}}\,dx$

Optimal. Leaf size=290

$$\frac{44\sqrt{\frac{11}{23}}\,\sqrt{5x+7}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\,\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{2691\sqrt{2x-5}\,\sqrt{\frac{5x+7}{5-2x}}}+\frac{3740\sqrt{2-3x}\,\sqrt{4x+1}\,\sqrt{5x+7}}{3253419\sqrt{2x-5}}-\frac{9350\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{}}{3253419\sqrt{5x+7}}$$

```
[Out] 2/117*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2)-9350/3253419
*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2)+3740/3253419*(2-3
*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)+44/61893*(1/(4+2*(1+4*
x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)/(2-3*x))^(1/2)*EllipticF((1+4*x)^(1/2)*2^(1
/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/(2-3*x))^(1/2),1/23*I*897^(1/2))*253^(1/2)*(
7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7+5*x)/(5-2*x))^(1/2)-1870/3253419*EllipticE(
1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),1/39*I*897^(1/2))*429^(1/2)*(2-
3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/2)/((2-3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {164, 1599, 1602, 12, 170, 418, 176, 424}

$$\frac{3740\sqrt{2-3x}\,\sqrt{4x+1}\,\sqrt{5x+7}}{3253419\sqrt{2x-5}}-\frac{9350\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{3253419\sqrt{5x+7}}+\frac{2\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{117(5x+7)^{3/2}}+\frac{44\sqrt{\frac{11}{23}}\,\sqrt{5x+}}{26}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^(5/2)),x]
```

```
[Out] (2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(117*(7 + 5*x)^(3/2)) - (935
0*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(3253419*Sqrt[7 + 5*x]) + (37
40*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(3253419*Sqrt[-5 + 2*x]) - (1
870*Sqrt[11/39]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(S
qrt[39/23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(83421*Sqrt[(2 - 3*x)/(
5 - 2*x)]*Sqrt[7 + 5*x]) + (44*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[S
qrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(2691*Sqrt[-5 + 2*x]*Sqrt[(
7 + 5*x)/(5 - 2*x)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 164

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*
(x_)])/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[((a + b*x)^(m + 1)*Sqrt[
c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((m + 1)*(b*c - a*d)), x] - Dist[1/(2
*(m + 1)*(b*c - a*d)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*
Sqrt[g + h*x]))*Simp[c*(f*g + e*h) + d*e*g*(2*m + 3) + 2*(c*f*h + d*(m + 2)
*(f*g + e*h))*x + d*f*h*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
```

```
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1599

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
(A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]
)/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), x] - Dist[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sq
rt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d*
f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*e
*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
- b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rule 1602

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[(C*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*f*h*Sqrt[c + d*
x]), x] + (Dist[1/(2*b*d*f*h), Int[(1*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f
*h) + (2*b*B*d*f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x])/(Sqrt[
a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(C*(d*e
- c*f)*(d*g - c*h))/(2*b*d*f*h), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}(7+5x)^{5/2}}\,dx = \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{117(7+5x)^{3/2}} - \frac{1}{117}\int \frac{-33+110x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,d$$

$$= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{117(7+5x)^{3/2}} - \frac{9350\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{3253419\sqrt{7+5x}} - \frac{\int \frac{-66308-1}{\sqrt{2-3x}\sqrt{-5}}}{3}$$

$$= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{117(7+5x)^{3/2}} - \frac{9350\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{3253419\sqrt{7+5x}} + \frac{3740\sqrt{2-}}{3253}$$

$$= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{117(7+5x)^{3/2}} - \frac{9350\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{3253419\sqrt{7+5x}} + \frac{3740\sqrt{2-}}{3253}$$

$$= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{117(7+5x)^{3/2}} - \frac{9350\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{3253419\sqrt{7+5x}} + \frac{3740\sqrt{2-}}{3253}$$

$$= \frac{2\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{117(7+5x)^{3/2}} - \frac{9350\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{3253419\sqrt{7+5x}} + \frac{3740\sqrt{2-}}{3253}$$

Mathematica [A] time = 1.95, size = 246, normalized size = 0.85

$$-\frac{2\sqrt{2x-5}\sqrt{4x+1}\left(506\sqrt{682}(3x-2)\sqrt{\frac{8x^2-18x-5}{(2-3x)^2}}(5x+7)^2\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{2x-5}{3x-2}}\right),\frac{39}{62}\right)-935\sqrt{682}(\right.}{3253419\sqrt{2-3x}(5x+7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^(5/2)),x]
```

```
[Out] (-2*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(31*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-23755 - 1
22348*x - 94580*x^2 + 58928*x^3) - 935*Sqrt[682]*(-2 + 3*x)*(7 + 5*x)^2*Sqr
t[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticE[ArcSin[Sqrt[31/39]*Sqrt[(-5 +
2*x)/(-2 + 3*x)]], 39/62] + 506*Sqrt[682]*(-2 + 3*x)*(7 + 5*x)^2*Sqrt[(-5 -
 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticF[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-
2 + 3*x)]], 39/62]))/(3253419*Sqrt[2 - 3*x]*(7 + 5*x)^(3/2)*Sqrt[(7 + 5*x)/
(-2 + 3*x)]*(-5 - 18*x + 8*x^2))
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{5\,x+7}\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{250\,x^4+425\,x^3-1155\,x^2-2989\,x-1715},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2)/(-5+2*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(250*x^4
+ 425*x^3 - 1155*x^2 - 2989*x - 1715), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{4\,x+1}\,\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{5}{2}}\,\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2)/(-5+2*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^(5/2)*sqrt(2*x - 5)), x)
```

maple [B] time = 0.03, size = 786, normalized size = 2.71

$$-\frac{2\left(-74800\sqrt{11}\,\sqrt{\frac{5x+7}{4x+1}}\,\sqrt{3}\,\sqrt{13}\,\sqrt{\frac{2x-5}{4x+1}}\,\sqrt{\frac{3x-2}{4x+1}}\,x^3\,\mathrm{EllipticE}\left(\frac{\sqrt{31}\,\sqrt{11}\,\sqrt{\frac{5x+7}{4x+1}}}{31},\frac{\sqrt{31}\,\sqrt{78}}{39}\right)+20240\sqrt{11}\,\sqrt{\frac{5x+7}{4x+1}}\,\sqrt{3}\,\sqrt{\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x+2)^(1/2)*(4*x+1)^(1/2)/(5*x+7)^(5/2)/(2*x-5)^(1/2),x)
```

```
[Out] -2/3253419*(20240*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5
)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticF(1/31*31^(1/2)*11^(1/2)*(
(5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))*x^3-74800*11^(1/2)*((5*x+7)/
(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(
1/2)*EllipticE(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)
*78^(1/2))*x^3+38456*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*
x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x^2*EllipticF(1/31*31^(1/2)*11^
(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-142120*11^(1/2)*((5*x
+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1
))^(1/2)*x^2*EllipticE(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*
31^(1/2)*78^(1/2))+15433*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*
((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x*EllipticF(1/31*31^(1/2)*1
1^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-57035*11^(1/2)*((5*
x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+
1))^(1/2)*x*EllipticE(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*3
1^(1/2)*78^(1/2))+1771*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((
2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticF(1/31*31^(1/2)*11^(1
/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-6545*11^(1/2)*((5*x+7)/
(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(
1/2)*EllipticE(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)
*78^(1/2))-1312518*x^3+3086255*x^2+1200968*x-1783420)*(2*x-5)^(1/2)*(4*x+1)
^(1/2)*(-3*x+2)^(1/2)/(120*x^4-182*x^3-385*x^2+197*x+70)/(5*x+7)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{4\,x+1}\,\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{5}{2}}\,\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2)/(-5+2*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^(5/2)*sqrt(2*x - 5)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2-3\,x}\,\sqrt{4\,x+1}}{\sqrt{2\,x-5}\,{(5\,x+7)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^(5/2)),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**(5/2)/(-5+2*x)**(1/2),x)
```

```
[Out] Timed out
```

3.91 $\int \frac{\sqrt{2-3x}\,\sqrt{1+4x}}{\sqrt{-5+2x}\,(7+5x)^{7/2}}\,dx$

Optimal. Leaf size=330

$$\frac{111628\sqrt{\frac{11}{23}}\,\sqrt{5x+7}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\,\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{74828637\sqrt{2x-5}\,\sqrt{\frac{5x+7}{5-2x}}}+\frac{8185936\sqrt{2-3x}\,\sqrt{4x+1}\,\sqrt{5x+7}}{90467822133\sqrt{2x-5}}-\frac{20464840\sqrt{2-3x}}{904678221}$$

```
[Out] 2/195*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2)-3646/1626709
5*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2)-20464840/9046782
2133*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2)+8185936/90467
822133*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)+111628/1721
058651*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)/(2-3*x))^(1/2)*Elliptic
F((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/(2-3*x))^(1/2),1/23*I*89
7^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7+5*x)/(5-2*x))^(1/2)-409
2968/90467822133*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),1/39
*I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/2)/((2-3*x)/(5-2
*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used = {164, 1604, 1599, 1602, 12, 170, 418, 176, 424}

$$\frac{8185936\sqrt{2-3x}\,\sqrt{4x+1}\,\sqrt{5x+7}}{90467822133\sqrt{2x-5}}-\frac{20464840\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{90467822133\sqrt{5x+7}}-\frac{3646\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{16267095(5x+7)^{3/2}}+\frac{2\sqrt{2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^(7/2)),x]
```

```
[Out] (2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(195*(7 + 5*x)^(5/2)) - (364
6*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(16267095*(7 + 5*x)^(3/2)) -
(20464840*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(90467822133*Sqrt[7 +
 5*x]) + (8185936*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(90467822133*S
qrt[-5 + 2*x]) - (4092968*Sqrt[11/39]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x
)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(
2319687747*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7 + 5*x]) + (111628*Sqrt[11/23]*S
qrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/2
3])/(74828637*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 - 2*x)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 164

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*
(x_)])/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[((a + b*x)^(m + 1)*Sqrt[
c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((m + 1)*(b*c - a*d)), x] - Dist[1/(2
*(m + 1)*(b*c - a*d)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*
Sqrt[g + h*x]))*Simp[c*(f*g + e*h) + d*e*g*(2*m + 3) + 2*(c*f*h + d*(m + 2)
*(f*g + e*h))*x + d*f*h*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1599

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
(A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]
)/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), x] - Dist[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sq
rt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d*
f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*e
*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
- b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rule 1602

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[(C*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*f*h*Sqrt[c + d*
x]), x] + (Dist[1/(2*b*d*f*h), Int[(1*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f
*h) + (2*b*B*d*f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x])/(Sqrt[
a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(C*(d*e
- c*f)*(d*g - c*h))/(2*b*d*f*h), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rule 1604

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt
[e + f*x]*Sqrt[g + h*x])/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), x]
- Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m
 + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m +
 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
+ c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2
*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^
2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
+ c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\,\sqrt{1+4x}}{\sqrt{-5+2x}\,(7+5x)^{7/2}}\,dx = \frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{195(7+5x)^{5/2}} - \frac{1}{195}\int \frac{-41+90x+48x^2}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^{5/2}}\,dx$$

$$= \frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{195(7+5x)^{5/2}} - \frac{3646\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{16267095(7+5x)^{3/2}} - \frac{\int \frac{-489390\ldots}{\sqrt{2-3x}\,\sqrt{-5+2x}\ldots}}{162\ldots}$$

$$= \frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{195(7+5x)^{5/2}} - \frac{3646\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{16267095(7+5x)^{3/2}} - \frac{20464840\sqrt{2-\ldots}}{904678\ldots}$$

$$= \frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{195(7+5x)^{5/2}} - \frac{3646\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{16267095(7+5x)^{3/2}} - \frac{20464840\sqrt{2-\ldots}}{904678\ldots}$$

$$= \frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{195(7+5x)^{5/2}} - \frac{3646\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{16267095(7+5x)^{3/2}} - \frac{20464840\sqrt{2-\ldots}}{904678\ldots}$$

$$= \frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{195(7+5x)^{5/2}} - \frac{3646\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{16267095(7+5x)^{3/2}} - \frac{20464840\sqrt{2-\ldots}}{904678\ldots}$$

$$= \frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{195(7+5x)^{5/2}} - \frac{3646\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{16267095(7+5x)^{3/2}} - \frac{20464840\sqrt{2-\ldots}}{904678\ldots}$$

Mathematica [A] time = 1.87, size = 251, normalized size = 0.76

$$-\frac{2\sqrt{2x-5}\,\sqrt{4x+1}\left(958111\sqrt{682}\,(3x-2)\sqrt{\frac{8x^2-18x-5}{(2-3x)^2}}\,(5x+7)^3\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{2x-5}{3x-2}}\right),\frac{39}{62}\right)-2046484\sqrt{\ldots}\right.}{904678\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^(7/2)),x]
```

```
[Out] (-2*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(31*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-374624540
 - 2271416114*x - 2953846743*x^2 + 643813106*x^3 + 370051256*x^4) - 2046484
*Sqrt[682]*(-2 + 3*x)*(7 + 5*x)^3*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*Ell
```

```
ipticE[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + 958111*Sqr
t[682]*(-2 + 3*x)*(7 + 5*x)^3*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*Ellipti
cF[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62]))/(90467822133*S
qrt[2 - 3*x]*(7 + 5*x)^(5/2)*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x + 8*x^2)
)
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{5\,x+7}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}}{1250\,x^5+3875\,x^4-2800\,x^3-23030\,x^2-29498\,x-12005}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(7/2)/(-5+2*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(1250*x^5
 + 3875*x^4 - 2800*x^3 - 23030*x^2 - 29498*x - 12005), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{4\,x+1}\,\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{7}{2}}\,\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(7/2)/(-5+2*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^(7/2)*sqrt(2*x - 5)), x)
```

maple [B] time = 0.03, size = 973, normalized size = 2.95

$$\frac{2\left(818593600\sqrt{11}\,\sqrt{\frac{5x+7}{4x+1}}\,\sqrt{3}\,\sqrt{13}\,\sqrt{\frac{2x-5}{4x+1}}\,\sqrt{\frac{3x-2}{4x+1}}\,x^4\,\text{EllipticE}\left(\frac{\sqrt{31}\,\sqrt{11}\,\sqrt{\frac{5x+7}{4x+1}}}{31}, \frac{\sqrt{31}\,\sqrt{78}}{39}\right)-126500000\sqrt{11}\,\sqrt{\frac{5}{4}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x+2)^(1/2)*(4*x+1)^(1/2)/(5*x+7)^(7/2)/(2*x-5)^(1/2),x)
```

```
[Out] 2/90467822133*(818593600*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*
((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticE(1/31*31^(1/2)*11^
(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))*x^4-126500000*11^(1/2
)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)
/(4*x+1))^(1/2)*EllipticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/
39*31^(1/2)*78^(1/2))*x^4+2701358880*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/
2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticE(1/31*
31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))*x^3-41745
0000*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1
/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+
1))^(1/2),1/39*31^(1/2)*78^(1/2))*x^3+2801636596*11^(1/2)*((5*x+7)/(4*x+1))
^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x^2
*EllipticE(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^
(1/2))-432946250*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)
/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x^2*EllipticF(1/31*31^(1/2)*11^(1/2
)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+945475608*11^(1/2)*((5*x+
7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1)
)^(1/2)*x*EllipticE(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^
(1/2)*78^(1/2))-146107500*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)
```

```
*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x*EllipticF(1/31*31^(1/2)*
11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+100277716*11^(1/2)
*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/
(4*x+1))^(1/2)*EllipticE(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/3
9*31^(1/2)*78^(1/2))-15496250*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(
1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticF(1/31*31^(1/2
)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+5843757936*x^4+1
0390893586*x^3-65568669813*x^2-3127552098*x+26993559920)*(2*x-5)^(1/2)*(4*x
+1)^(1/2)*(-3*x+2)^(1/2)/(120*x^4-182*x^3-385*x^2+197*x+70)/(5*x+7)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{4x+1}\,\sqrt{-3x+2}}{(5x+7)^{\frac{7}{2}}\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(7/2)/(-5+2*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^(7/2)*sqrt(2*x - 5)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2-3x}\,\sqrt{4x+1}}{\sqrt{2x-5}\,(5x+7)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^(7/2)),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^(7/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**(7/2)/(-5+2*x)**(1/2),x)
```

```
[Out] Timed out
```

3.92 $\int \frac{\sqrt{2-3x}\,\sqrt{1+4x}}{\sqrt{-5+2x}\,(7+5x)^{9/2}}\,dx$

Optimal. Leaf size=370

$$\frac{258506776\sqrt{\frac{11}{23}}\,\sqrt{5x+7}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\,\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{1618368818157\sqrt{2x-5}\,\sqrt{\frac{5x+7}{5-2x}}}+\frac{16377776536\sqrt{2-3x}\,\sqrt{4x+1}\,\sqrt{5x+7}}{1956607901151813\sqrt{2x-5}}-\frac{40944}{1}$$

```
[Out] 2/273*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(7/2)+98/1807455*(
2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(5/2)-3217468/50259901185
*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2)-40944441340/19566
07901151813*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2)+163777
76536/1956607901151813*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(
1/2)+258506776/37222482817611*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)/
(2-3*x))^(1/2)*EllipticF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/(
2-3*x))^(1/2),1/23*I*897^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7+
5*x)/(5-2*x))^(1/2)-8188888268/1956607901151813*EllipticE(1/23*897^(1/2)*(1
+4*x)^(1/2)/(-5+2*x)^(1/2),1/39*I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*
x)/(5-2*x))^(1/2)/((2-3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A] time = 0.52, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {164, 1604, 1599, 1602, 12, 170, 418, 176, 424}

$$\frac{16377776536\sqrt{2-3x}\,\sqrt{4x+1}\,\sqrt{5x+7}}{1956607901151813\sqrt{2x-5}}-\frac{40944441340\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}{1956607901151813\sqrt{5x+7}}-\frac{3217468\sqrt{2-3x}\,\sqrt{2x-5}}{50259901185(5x+7}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^(9/2)),x]
```

```
[Out] (2*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(273*(7 + 5*x)^(7/2)) + (98*
Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(1807455*(7 + 5*x)^(5/2)) - (32
17468*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(50259901185*(7 + 5*x)^(3
/2)) - (40944441340*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(1956607901
151813*Sqrt[7 + 5*x]) + (16377776536*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5
*x])/(1956607901151813*Sqrt[-5 + 2*x]) - (8188888268*Sqrt[11/39]*Sqrt[2 - 3
*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqrt[1 + 4*x])/
Sqrt[-5 + 2*x]], -23/39])/(50169433362867*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7
+ 5*x]) + (258506776*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*
x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(1618368818157*Sqrt[-5 + 2*x]*Sqrt[(7
 + 5*x)/(5 - 2*x)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 164

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*
(x_)])/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[((a + b*x)^(m + 1)*Sqrt[
c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((m + 1)*(b*c - a*d)), x] - Dist[1/(2
*(m + 1)*(b*c - a*d)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*
Sqrt[g + h*x]))*Simp[c*(f*g + e*h) + d*e*g*(2*m + 3) + 2*(c*f*h + d*(m + 2)
*(f*g + e*h))*x + d*f*h*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
```

```
f, g, h, m}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1599

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
(A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]
)/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), x] - Dist[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sq
rt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d*
f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*e
*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
- b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rule 1602

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[(C*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*f*h*Sqrt[c + d*
x]), x] + (Dist[1/(2*b*d*f*h), Int[(1*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f
*h) + (2*b*B*d*f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x])/(Sqrt[
a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(C*(d*e
- c*f)*(d*g - c*h))/(2*b*d*f*h), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
```

```
x]
```

Rule 1604

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt
[e + f*x]*Sqrt[g + h*x])/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), x]
- Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m
 + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m +
 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
+ c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2
*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^
2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
+ c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\sqrt{1+4x}}{\sqrt{-5+2x}\,(7+5x)^{9/2}}\,dx = \frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{273(7+5x)^{7/2}} - \frac{1}{273}\int \frac{-49+70x+96x^2}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^{7/2}}\,d$$

$$= \frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{273(7+5x)^{7/2}} + \frac{98\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{1807455(7+5x)^{5/2}} - \frac{\int \frac{-958104+228}{\sqrt{2-3x}\,\sqrt{-5+2}}}{379}$$

$$= \frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{273(7+5x)^{7/2}} + \frac{98\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{1807455(7+5x)^{5/2}} - \frac{3217468\sqrt{2-}}{502599}$$

$$= \frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{273(7+5x)^{7/2}} + \frac{98\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{1807455(7+5x)^{5/2}} - \frac{3217468\sqrt{2-}}{502599}$$

$$= \frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{273(7+5x)^{7/2}} + \frac{98\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{1807455(7+5x)^{5/2}} - \frac{3217468\sqrt{2-}}{502599}$$

$$= \frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{273(7+5x)^{7/2}} + \frac{98\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{1807455(7+5x)^{5/2}} - \frac{3217468\sqrt{2-}}{502599}$$

$$= \frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{273(7+5x)^{7/2}} + \frac{98\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{1807455(7+5x)^{5/2}} - \frac{3217468\sqrt{2-}}{502599}$$

$$= \frac{2\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{273(7+5x)^{7/2}} + \frac{98\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}{1807455(7+5x)^{5/2}} - \frac{3217468\sqrt{2-}}{502599}$$

Mathematica [A] time = 2.45, size = 258, normalized size = 0.70

$$\frac{2\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{5x+7}\left(\frac{(3x-2)\left(2559027583750x^3+12313608173580x^2+19165803061167x+2552362046246\right)}{(5x+7)^4} - \frac{22\left(71545594\sqrt{682}\right.}{}\right.}{19566079}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[1 + 4*x])/(Sqrt[-5 + 2*x]*(7 + 5*x)^(9/2)),x]
```

```
[Out] (2*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]*(((-2 + 3*x)*(2552362046246 +
 19165803061167*x + 12313608173580*x^2 + 2559027583750*x^3))/(7 + 5*x)^4 -
(22*(558333291*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x + 8*x^2) - 186111097*S
qrt[682]*(-2 + 3*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticE[ArcSin[
Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + 71545594*Sqrt[682]*(-2 +
 3*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticF[ArcSin[Sqrt[31/39]*Sq
rt[(-5 + 2*x)/(-2 + 3*x)]], 39/62]))/(Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x
 + 8*x^2))))/(1956607901151813*Sqrt[2 - 3*x])
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{5\,x+7}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}}{6250\,x^{6}+28125\,x^{5}+13125\,x^{4}-134750\,x^{3}-308700\,x^{2}-266511\,x-84035},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(9/2)/(-5+2*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(6250*x^6
 + 28125*x^5 + 13125*x^4 - 134750*x^3 - 308700*x^2 - 266511*x - 84035), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{4\,x+1}\,\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{9}{2}}\,\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(9/2)/(-5+2*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^(9/2)*sqrt(2*x - 5)), x)
```

maple [B] time = 0.04, size = 1160, normalized size = 3.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x+2)^(1/2)*(4*x+1)^(1/2)/(5*x+7)^(9/2)/(2*x-5)^(1/2),x)
```

```
[Out] 2/1956607901151813*(8188888268000*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*
13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticE(1/31*31^
(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))*x^5-17517821
2000*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1
/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+
1))^(1/2),1/39*31^(1/2)*78^(1/2))*x^5+38487774859600*11^(1/2)*((5*x+7)/(4*x
+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)
*EllipticE(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^
(1/2))*x^4-823337596400*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*(
(2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticF(1/31*31^(1/2)*11^(
1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))*x^4+65859133895390*11^
(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*
x-2)/(4*x+1))^(1/2)*EllipticE(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2
),1/39*31^(1/2)*78^(1/2))*x^3-1408870770010*11^(1/2)*((5*x+7)/(4*x+1))^(1/2
)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*Elliptic
F(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))*x^
```

```
3+48695224085662*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)
/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x^2*EllipticE(1/31*31^(1/2)*11^(1/2
)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-1041697237658*11^(1/2)*((
5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*
x+1))^(1/2)*x^2*EllipticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/
39*31^(1/2)*78^(1/2))+14244571142186*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/
2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x*EllipticE(1/3
1*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-3047224
99774*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(
1/2)*((3*x-2)/(4*x+1))^(1/2)*x*EllipticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4
*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+1404394337962*11^(1/2)*((5*x+7)/(4*x+1
))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*E
llipticE(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1
/2))-30043063358*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)
/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticF(1/31*31^(1/2)*11^(1/2)*((
5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+33052545587580*x^5+8373262836
7442*x^4-43651554581534*x^3-1041927172311711*x^2+131120048990980*x+36770679
4166900)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(-3*x+2)^(1/2)/(120*x^4-182*x^3-385*x^
2+197*x+70)/(5*x+7)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{4x+1}\,\sqrt{-3x+2}}{(5x+7)^{\frac{9}{2}}\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(9/2)/(-5+2*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(4*x + 1)*sqrt(-3*x + 2)/((5*x + 7)^(9/2)*sqrt(2*x - 5)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2-3x}\,\sqrt{4x+1}}{\sqrt{2x-5}\,(5x+7)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^(9/2)),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(4*x + 1)^(1/2))/((2*x - 5)^(1/2)*(5*x + 7)^(9/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)*(1+4*x)**(1/2)/(7+5*x)**(9/2)/(-5+2*x)**(1/2),x)
```

```
[Out] Timed out
```

3.93 $\int \frac{\sqrt{2-3x}\,(7+5x)^{5/2}}{\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx$

Optimal. Leaf size=391

$$\frac{5241511\sqrt{\frac{11}{23}}\,\sqrt{5x+7}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\,\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{13824\sqrt{2x-5}\,\sqrt{\frac{5x+7}{5-2x}}}+\frac{5}{24}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(5x+7)^{3/2}+\frac{6955\sqrt{2-3x}}{}$$

```
[Out] 5/24*(7+5*x)^(3/2)*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)+295576909/593
0496*(2-3*x)*EllipticPi(1/23*253^(1/2)*(7+5*x)^(1/2)/(2-3*x)^(1/2),-69/55,1
/39*I*897^(1/2))*((5-2*x)/(2-3*x))^(1/2)*((-1-4*x)/(2-3*x))^(1/2)*429^(1/2)
/(-5+2*x)^(1/2)/(1+4*x)^(1/2)+102487/1536*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*
x)^(1/2)/(-5+2*x)^(1/2)+6955/1152*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2
)*(7+5*x)^(1/2)+5241511/317952*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)
/(2-3*x))^(1/2)*EllipticF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/
(2-3*x))^(1/2),1/23*I*897^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7
+5*x)/(5-2*x))^(1/2)-102487/3072*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5
+2*x)^(1/2),1/39*I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/
2)/((2-3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.270$, Rules used = {174, 1600, 1602, 1598, 170, 418, 165, 537, 176, 424}

$$\frac{5}{24}\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(5x+7)^{3/2}+\frac{6955\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{5x+7}}{1152}+\frac{102487\sqrt{2-3x}\,\sqrt{4x+1}\,\sqrt{5x}}{1536\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*(7 + 5*x)^(5/2))/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (102487*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(1536*Sqrt[-5 + 2*x]) +
(6955*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/1152 + (5*S
qrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2))/24 - (102487*Sqr
t[143/3]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/
23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(1024*Sqrt[(2 - 3*x)/(5 - 2*x)
]*Sqrt[7 + 5*x]) + (5241511*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt
[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(13824*Sqrt[-5 + 2*x]*Sqrt[(7
+ 5*x)/(5 - 2*x)]) + (295576909*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-(
(1 + 4*x)/(2 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])
/Sqrt[2 - 3*x]], -23/39])/(13824*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Rule 165

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*(a + b*x)*Sqrt[((b*g -
 a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*
g - e*h)*(a + b*x))])/(Sqrt[c + d*x]*Sqrt[e + f*x]), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + ((b*c - a*d)*x^2)/(d*g - c*h)]*Sqrt[1 + ((b*e - a*f)*x^2)/(f*g
- e*h)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
```

```
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 174

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.
)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(2*b*(a + b*x)^(m - 1)
*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(f*h*(2*m + 1)), x] - Dist[1/(f
*h*(2*m + 1)), Int[((a + b*x)^(m - 2)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]))*Simp[a*b*(d*e*g + c*(f*g + e*h)) + 2*b^2*c*e*g*(m - 1) - a^2*c*f*h*
(2*m + 1) + (b^2*(2*m - 1)*(d*e*g + c*(f*g + e*h)) - a^2*d*f*h*(2*m + 1) +
2*a*b*(d*f*g + d*e*h - 2*c*f*h*m))*x - b*(a*d*f*h*(4*m - 1) + b*(c*f*h - 2*
d*(f*g + e*h)*m))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x]
&& IntegerQ[2*m] && GtQ[m, 1]
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 1598

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 1600

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[(2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/
(d*f*h*(2*m + 3)), x] + Dist[1/(d*f*h*(2*m + 3)), Int[((a + b*x)^(m - 1)/(S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*
(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*(
2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b*
B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*
m] && GtQ[m, 0]
```

Rule 1602

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[(C*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*f*h*Sqrt[c + d*
x]), x] + (Dist[1/(2*b*d*f*h), Int[(1*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f
*h) + (2*b*B*d*f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x])/(Sqrt[
a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(C*(d*e
- c*f)*(d*g - c*h))/(2*b*d*f*h), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\,(7+5x)^{5/2}}{\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx = \frac{5}{24}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^{3/2} - \frac{1}{48}\int \frac{\sqrt{7+5x}\,\left(-6189+3136x+13910\right.}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}$$

$$= \frac{6955\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{1152} + \frac{5}{24}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^{3}$$

$$= \frac{102487\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{1536\sqrt{-5+2x}} + \frac{6955\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{1152} + \frac{5}{24}$$

$$= \frac{102487\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{1536\sqrt{-5+2x}} + \frac{6955\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{1152} + \frac{5}{24}$$

$$= \frac{102487\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{1536\sqrt{-5+2x}} + \frac{6955\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{1152} + \frac{5}{24}$$

$$= \frac{102487\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{1536\sqrt{-5+2x}} + \frac{6955\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{1152} + \frac{5}{24}$$

Mathematica [A] time = 2.62, size = 340, normalized size = 0.87

$$-\frac{\sqrt{2x-5}\,\sqrt{4x+1}\,\left(46704724\sqrt{682}\,\sqrt{\frac{8x^2-18x-5}{(2-3x)^2}}\,\left(15x^2+11x-14\right)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\,\sqrt{\frac{2x-5}{3x-2}}\right),\frac{39}{62}\right)-571877\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[2 - 3*x]*(7 + 5*x)^(5/2))/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] -1/1714176*(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(-57187746*Sqrt[682]*Sqrt[(-5 - 18
*x + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticE[ArcSin[Sqrt[31/39]
*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + 46704724*Sqrt[682]*Sqrt[(-5 - 18*x
+ 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*EllipticF[ArcSin[Sqrt[31/39]*Sq
rt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + Sqrt[(7 + 5*x)/(-2 + 3*x)]*(186*(-2744
7805 - 124999073*x - 56065622*x^2 + 20626760*x^3 + 6542400*x^4 + 1152000*x^
5) + 47673695*Sqrt[682]*(2 - 3*x)^2*Sqrt[(1 + 4*x)/(-2 + 3*x)]*Sqrt[(-35 -
11*x + 10*x^2)/(2 - 3*x)^2]*EllipticPi[117/62, ArcSin[Sqrt[31/39]*Sqrt[(-5
+ 2*x)/(-2 + 3*x)]], 39/62])))/(Sqrt[2 - 3*x]*Sqrt[7 + 5*x]*Sqrt[(7 + 5*x)/
(-2 + 3*x)]*(-5 - 18*x + 8*x^2))
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(25\,x^2+70\,x+49\right)\sqrt{5\,x+7}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}}{8\,x^2-18\,x-5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(5/2)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((25*x^2 + 70*x + 49)*sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqr
t(-3*x + 2)/(8*x^2 - 18*x - 5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5\,x+7)^{\frac{5}{2}}\sqrt{-3\,x+2}}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(5/2)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((5*x + 7)^(5/2)*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

maple [B] time = 0.04, size = 885, normalized size = 2.26

$$-\frac{\sqrt{5x+7}\,\sqrt{-3x+2}\,\sqrt{2x-5}\,\sqrt{4x+1}\,\left(-988416000x^5-5613379200x^4-17697760080x^3-4220824608\sqrt{1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x+7)^(5/2)*(-3*x+2)^(1/2)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] -1/7907328*(5*x+7)^(1/2)*(-3*x+2)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(193959
920*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/
2)*((3*x-2)/(4*x+1))^(1/2)*x^2*EllipticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4
*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-3508783952*11^(1/2)*((5*x+7)/(4*x+1))^
(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x^2*
EllipticPi(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),124/55,1/39*31^(1
/2)*78^(1/2))-4220824608*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*
((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x^2*EllipticE(1/31*31^(1/2)
*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+96979960*11^(1/2)
*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/
```

```
(4*x+1))^(1/2)*x*EllipticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1
/39*31^(1/2)*78^(1/2))-1754391976*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*
13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x*EllipticPi(1/31*
31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*78^(1/2))-21
10412304*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1)
)^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x*EllipticE(1/31*31^(1/2)*11^(1/2)*((5*x+7)
/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+12122495*11^(1/2)*((5*x+7)/(4*x+1))
^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*Ell
ipticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2
))-219298997*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*
x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticPi(1/31*31^(1/2)*11^(1/2)*((5*x
+7)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*78^(1/2))-263801538*11^(1/2)*((5*x+
7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1)
)^(1/2)*EllipticE(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1
/2)*78^(1/2))-988416000*x^5-5613379200*x^4-17697760080*x^3+77122472856*x^2+
75329218536*x-78013375440)/(120*x^4-182*x^3-385*x^2+197*x+70)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x+7)^{\frac{5}{2}}\sqrt{-3x+2}}{\sqrt{4x+1}\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(5/2)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="maxima")
[Out] integrate((5*x + 7)^(5/2)*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2-3x}\,(5x+7)^{5/2}}{\sqrt{4x+1}\,\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(5*x + 7)^(5/2))/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
[Out] int(((2 - 3*x)^(1/2)*(5*x + 7)^(5/2))/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**(5/2)*(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
[Out] Timed out
```

3.94 $\int \frac{\sqrt{2-3x}\,(7+5x)^{3/2}}{\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx$

Optimal. Leaf size=351

$$\frac{17515\sqrt{\frac{11}{23}}\sqrt{5x+7}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{576\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}+\frac{5}{16}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}+\frac{785\sqrt{2-3x}\,\sqrt{\ldots}}{192\sqrt{\ldots}}$$

```
[Out] 3730013/1235520*(2-3*x)*EllipticPi(1/23*253^(1/2)*(7+5*x)^(1/2)/(2-3*x)^(1/
2),-69/55,1/39*I*897^(1/2))*((5-2*x)/(2-3*x))^(1/2)*((-1-4*x)/(2-3*x))^(1/2
)*429^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)+785/192*(2-3*x)^(1/2)*(1+4*x)^(1/2
)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)+5/16*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1
/2)*(7+5*x)^(1/2)+17515/13248*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)/
(2-3*x))^(1/2)*EllipticF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/(
2-3*x))^(1/2),1/23*I*897^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7+
5*x)/(5-2*x))^(1/2)-785/384*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x)
^(1/2),1/39*I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/2)/((
2-3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used = {174, 1602, 1598, 170, 418, 165, 537, 176, 424}

$$\frac{5}{16}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}+\frac{785\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{192\sqrt{2x-5}}+\frac{17515\sqrt{\frac{11}{23}}\sqrt{5x+7}\,F\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\ldots\right.\right.}{576\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*(7 + 5*x)^(3/2))/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (785*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(192*Sqrt[-5 + 2*x]) + (5*S
qrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/16 - (785*Sqrt[143
/3]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*S
qrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(128*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt
[7 + 5*x]) + (17515*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x
]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(576*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5
- 2*x)]) + (3730013*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((1 + 4*x)/(2
 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*
x]], -23/39])/(2880*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Rule 165

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*(a + b*x)*Sqrt[((b*g -
 a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*
g - e*h)*(a + b*x))])/(Sqrt[c + d*x]*Sqrt[e + f*x]), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + ((b*c - a*d)*x^2)/(d*g - c*h)]*Sqrt[1 + ((b*e - a*f)*x^2)/(f*g
- e*h)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
```

```
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 174

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.
)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(2*b*(a + b*x)^(m - 1)
*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(f*h*(2*m + 1)), x] - Dist[1/(f
*h*(2*m + 1)), Int[((a + b*x)^(m - 2)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]))*Simp[a*b*(d*e*g + c*(f*g + e*h)) + 2*b^2*c*e*g*(m - 1) - a^2*c*f*h*
(2*m + 1) + (b^2*(2*m - 1)*(d*e*g + c*(f*g + e*h)) - a^2*d*f*h*(2*m + 1) +
2*a*b*(d*f*g + d*e*h - 2*c*f*h*m))*x - b*(a*d*f*h*(4*m - 1) + b*(c*f*h - 2*
d*(f*g + e*h)*m))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x]
&& IntegerQ[2*m] && GtQ[m, 1]
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 1598

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 1602

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
```

```
l] :> Simp[(C*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*f*h*Sqrt[c + d*
x]), x] + (Dist[1/(2*b*d*f*h), Int[(1*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f
*h) + (2*b*B*d*f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x])/(Sqrt[
a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(C*(d*e
- c*f)*(d*g - c*h))/(2*b*d*f*h), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\,(7+5x)^{3/2}}{\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx = \frac{5}{16}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x} - \frac{1}{32}\int \frac{-4121+4074x+7850x^2}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+\cdots}}$$

$$= \frac{785\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{192\sqrt{-5+2x}} + \frac{5}{16}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x} + \frac{\int \frac{\cdots}{\sqrt{2-\cdots}}}{\cdots}$$

$$= \frac{785\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{192\sqrt{-5+2x}} + \frac{5}{16}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x} + \frac{12032\cdots}{\cdots}$$

$$= \frac{785\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{192\sqrt{-5+2x}} + \frac{5}{16}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x} - \frac{785\sqrt{\cdots}}{\cdots}$$

$$= \frac{785\sqrt{2-3x}\,\sqrt{1+4x}\,\sqrt{7+5x}}{192\sqrt{-5+2x}} + \frac{5}{16}\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x} - \frac{785\sqrt{\cdots}}{\cdots}$$

Mathematica [A] time = 3.34, size = 349, normalized size = 0.99

$$\frac{\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{5x+7}\left(\frac{(2-3x)\left(\frac{998820\sqrt{682}\,(5x+7)\sqrt{\frac{8x^2-18x-5}{(2-3x)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{2x-5}{3x-2}}\right),\frac{39}{62}\right)}{(2-3x)^2} - \frac{1314090\sqrt{682}\,(5x+7)\sqrt{\frac{8x^2-18\cdots}{(2-3\cdots}}\cdots}{(2\cdots}\cdots\right)}{\cdots}\cdots\right)}{6\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[2 - 3*x]*(7 + 5*x)^(3/2))/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]*(200880 + ((2 - 3
*x)*((-1314090*Sqrt[682]*(7 + 5*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*El
lipticE[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62])/(2 - 3*x)^
2 + (998820*Sqrt[682]*(7 + 5*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*Ellip
ticF[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62])/(2 - 3*x)^2 +
 Sqrt[(7 + 5*x)/(-2 + 3*x)]*((3942270*(-35 - 151*x - 34*x^2 + 40*x^3))/(-2
+ 3*x)^3 + (1082907*Sqrt[682]*((1 + 4*x)/(-2 + 3*x))^(3/2)*Sqrt[(-35 - 11*x
 + 10*x^2)/(2 - 3*x)^2]*EllipticPi[117/62, ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*
x)/(-2 + 3*x)]], 39/62])/(1 + 4*x))))/(((7 + 5*x)/(-2 + 3*x))^(3/2)*(5 + 18
*x - 8*x^2))))/642816
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(5\,x+7)^{\frac{3}{2}}\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}}{8\,x^2-18\,x-5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(3/2)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(8*x^2
- 18*x - 5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5\,x+7)^{\frac{3}{2}}\sqrt{-3\,x+2}}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(3/2)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((5*x + 7)^(3/2)*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

maple [B] time = 0.03, size = 880, normalized size = 2.51

$$\sqrt{5x+7}\,\sqrt{-3x+2}\,\sqrt{2x-5}\,\sqrt{4x+1}\,\left(61776000x^4+310424400x^3+53882400\sqrt{11}\,\sqrt{\frac{5x+7}{4x+1}}\,\sqrt{3}\,\sqrt{13}\,\sqrt{\frac{2x-5}{4x+1}}\,\sqrt{\frac{3}{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x+7)^(3/2)*(-3*x+2)^(1/2)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] 1/1647360*(5*x+7)^(1/2)*(-3*x+2)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(1963280
*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*
((3*x-2)/(4*x+1))^(1/2)*x^2*EllipticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+
1))^(1/2),1/39*31^(1/2)*78^(1/2))+44278864*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)
*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x^2*Ellip
ticPi(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*7
8^(1/2))+53882400*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5
)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x^2*EllipticE(1/31*31^(1/2)*11^(1/
2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+981640*11^(1/2)*((5*x+7)
/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^
(1/2)*x*EllipticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1
/2)*78^(1/2))+22139432*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((
2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x*EllipticPi(1/31*31^(1/2)*11
^(1/2)*((5*x+7)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*78^(1/2))+26941200*11^(
1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x
-2)/(4*x+1))^(1/2)*x*EllipticE(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/
2),1/39*31^(1/2)*78^(1/2))+122705*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*
13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticF(1/31*31^
(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+2767429*11^(
1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x
-2)/(4*x+1))^(1/2)*EllipticPi(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2
),124/55,1/39*31^(1/2)*78^(1/2))+3367650*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3
^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticE(1
/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+31042
```

```
4400*x^3+61776000*x^4-912139800*x^2-1016644200*x+978978000)/(120*x^4-182*x^
3-385*x^2+197*x+70)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x+7)^{\frac{3}{2}}\sqrt{-3x+2}}{\sqrt{4x+1}\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(3/2)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((5*x + 7)^(3/2)*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2-3x}\,(5x+7)^{3/2}}{\sqrt{4x+1}\,\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(5*x + 7)^(3/2))/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
```

```
[Out] int(((2 - 3*x)^(1/2)*(5*x + 7)^(3/2))/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**(3/2)*(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Timed out
```

3.95 $\int \frac{\sqrt{2-3x}\,\sqrt{7+5x}}{\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx$

Optimal. Leaf size=365

$$-\frac{39\sqrt{\frac{11}{23}}\sqrt{5x+7}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{8\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}+\frac{179\sqrt{\frac{11}{62}}\sqrt{2-3x}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{\frac{22}{23}}\sqrt{5x+7}}{\sqrt{2x-5}}\right),\frac{39}{62}\right)}{16\sqrt{-\frac{2-3x}{4x+1}}\sqrt{4x+1}}+\frac{\sqrt{2-}}{}$$

```
[Out] 179/992*(1/(529+506*(7+5*x)/(-5+2*x)))^(1/2)*(529+506*(7+5*x)/(-5+2*x))^(1/
2)*EllipticF(506^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/(529+506*(7+5*x)/(-5+2*
x))^(1/2),1/62*2418^(1/2))*682^(1/2)*(2-3*x)^(1/2)/((-2+3*x)/(1+4*x))^(1/2)
/(1+4*x)^(1/2)+4117/54560*(1/(529+506*(7+5*x)/(-5+2*x)))^(1/2)*(529+506*(7+
5*x)/(-5+2*x))^(1/2)*EllipticPi(506^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/(529
+506*(7+5*x)/(-5+2*x))^(1/2),78/55,1/62*2418^(1/2))*(2-3*x)^(1/2)*682^(1/2)
/((-2+3*x)/(1+4*x))^(1/2)/(1+4*x)^(1/2)+1/4*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+
5*x)^(1/2)/(-5+2*x)^(1/2)-39/184*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*
x)/(2-3*x))^(1/2)*EllipticF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x
)/(2-3*x))^(1/2),1/23*I*897^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/(
(7+5*x)/(5-2*x))^(1/2)-1/8*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^
(1/2),1/39*I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/2)/((2
-3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {173, 176, 424, 170, 418, 165, 536, 539}

$$\frac{\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{4\sqrt{2x-5}}-\frac{39\sqrt{\frac{11}{23}}\sqrt{5x+7}\,F\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{8\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}+\frac{179\sqrt{\frac{11}{62}}\sqrt{2-3x}\,F\left(\tan^{-1}\left(\frac{\sqrt{\frac{22}{23}}\sqrt{5x+7}}{\sqrt{2x-5}}\right)|\right.}{16\sqrt{-\frac{2-3x}{4x+1}}\sqrt{4x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - 3*x]*Sqrt[7 + 5*x])/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(4*Sqrt[-5 + 2*x]) - (Sqrt[429]
*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/23]*Sqrt
[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(8*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7 +
5*x]) - (39*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[
2]*Sqrt[2 - 3*x])], -39/23])/(8*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]) +
 (179*Sqrt[11/62]*Sqrt[2 - 3*x]*EllipticF[ArcTan[(Sqrt[22/23]*Sqrt[7 + 5*x]
)/Sqrt[-5 + 2*x]], 39/62])/(16*Sqrt[-((2 - 3*x)/(1 + 4*x))]*Sqrt[1 + 4*x])
+ (4117*Sqrt[2 - 3*x]*EllipticPi[78/55, ArcTan[(Sqrt[22/23]*Sqrt[7 + 5*x])/
Sqrt[-5 + 2*x]], 39/62])/(80*Sqrt[682]*Sqrt[-((2 - 3*x)/(1 + 4*x))]*Sqrt[1
+ 4*x])
```

Rule 165

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*(a + b*x)*Sqrt[((b*g -
 a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*
g - e*h)*(a + b*x))])/(Sqrt[c + d*x]*Sqrt[e + f*x]), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + ((b*c - a*d)*x^2)/(d*g - c*h)]*Sqrt[1 + ((b*e - a*f)*x^2)/(f*g
- e*h)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 173

```
Int[(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(Sqrt[a + b*x]*Sqrt[c +
 d*x]*Sqrt[g + h*x])/(h*Sqrt[e + f*x]), x] + (-Dist[((d*e - c*f)*(f*g - e*h
))/(2*f*h), Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*(e + f*x)^(3/2)*Sqrt[g + h*x])
, x], x] + Dist[((d*e - c*f)*(b*f*g + b*e*h - 2*a*f*h))/(2*f^2*h), Int[1/(S
qrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(a*d
*f*h - b*(d*f*g + d*e*h - c*f*h))/(2*f^2*h), Int[Sqrt[e + f*x]/(Sqrt[a + b*
x]*Sqrt[c + d*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h},
 x]
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 536

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> -Dist[f/(b*e - a*f), Int[1/(Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] + Dist[b/(b*e - a*f), Int[Sqrt[e + f*x^2]/((a + b*x^2)*Sqrt[
c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[d/c, 0] && GtQ[f
/e, 0] &&  !SimplerSqrtQ[d/c, f/e]
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}\sqrt{7+5x}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx = \frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{4\sqrt{-5+2x}} - \frac{179}{16}\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}\,dx - \frac{429}{16}\int \frac{}{\sqrt{2-3x}}$$

$$= \frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{4\sqrt{-5+2x}} - \frac{\left(6981\sqrt{-\frac{2-3x}{-5+2x}}(-5+2x)\sqrt{\frac{1+4x}{-5+2x}}\right)\text{Subst}\left(\int \frac{}{\left(5-2x^2\right)\sqrt{1+}}\right.}{8\sqrt{713}\sqrt{2-3x}\sqrt{1+4x}}$$

$$= \frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{4\sqrt{-5+2x}} - \frac{\sqrt{429}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{8\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} - \frac{39}{}$$

$$= \frac{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{4\sqrt{-5+2x}} - \frac{\sqrt{429}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right)|-\frac{23}{39}\right)}{8\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}} - \frac{39}{}$$

Mathematica [A] time = 1.47, size = 347, normalized size = 0.95

$$-\frac{-1265\sqrt{341}\sqrt{\frac{3x-2}{4x+1}}\sqrt{\frac{5x+7}{4x+1}}\left(8x^2-18x-5\right)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{22}{39}}\sqrt{\frac{5x+7}{4x+1}}\right),\frac{39}{62}\right)+6820\sqrt{341}\sqrt{\frac{3x-2}{4x+1}}\sqrt{\frac{5x+7}{4x+1}}\left(8x^2\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[2 - 3*x]*Sqrt[7 + 5*x])/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] -1/27280*(6820*Sqrt[341]*Sqrt[(-2 + 3*x)/(1 + 4*x)]*Sqrt[(7 + 5*x)/(1 + 4*x
)]*(-5 - 18*x + 8*x^2)*EllipticE[ArcSin[Sqrt[22/39]*Sqrt[(7 + 5*x)/(1 + 4*x
)]], 39/62] - 1265*Sqrt[341]*Sqrt[(-2 + 3*x)/(1 + 4*x)]*Sqrt[(7 + 5*x)/(1 +
 4*x)]*(-5 - 18*x + 8*x^2)*EllipticF[ArcSin[Sqrt[22/39]*Sqrt[(7 + 5*x)/(1 +
 4*x)]], 39/62] + Sqrt[(-5 + 2*x)/(1 + 4*x)]*(13640*Sqrt[2]*(70 - 83*x - 53
*x^2 + 30*x^3) + 4117*Sqrt[341]*Sqrt[(-2 + 3*x)/(1 + 4*x)]*(1 + 4*x)^2*Sqrt
[(-35 - 11*x + 10*x^2)/(1 + 4*x)^2]*EllipticPi[78/55, ArcSin[Sqrt[22/39]*Sq
rt[(7 + 5*x)/(1 + 4*x)]], 39/62]))/(Sqrt[2 - 3*x]*Sqrt[-10 + 4*x]*Sqrt[(-5
+ 2*x)/(1 + 4*x)]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{5\,x+7}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}}{8\,x^2-18\,x-5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(1/2)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(8*x^2 -
18*x - 5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{5\,x+7}\,\sqrt{-3\,x+2}}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(1/2)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(5*x + 7)*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

maple [A] time = 0.03, size = 875, normalized size = 2.40

$$\frac{\sqrt{5x+7}\;\sqrt{-3x+2}\;\sqrt{2x-5}\;\sqrt{4x+1}\left(514800x^3+68640\sqrt{11}\;\sqrt{\frac{5x+7}{4x+1}}\;\sqrt{3}\;\sqrt{13}\;\sqrt{\frac{2x-5}{4x+1}}\;\sqrt{\frac{3x-2}{4x+1}}\;x^2\,\mathrm{EllipticE}\left(\frac{\sqrt{}}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x+7)^(1/2)*(-3*x+2)^(1/2)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] 1/34320*(5*x+7)^(1/2)*(-3*x+2)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(20240*11^
(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*
x-2)/(4*x+1))^(1/2)*x^2*EllipticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^
(1/2),1/39*31^(1/2)*78^(1/2))+65872*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2
)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x^2*EllipticPi(1
/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*78^(1/2)
)+68640*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))
^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x^2*EllipticE(1/31*31^(1/2)*11^(1/2)*((5*x+7
)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+10120*11^(1/2)*((5*x+7)/(4*x+1))^(
1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x*Ell
ipticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2
))+32936*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1)
)^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x*EllipticPi(1/31*31^(1/2)*11^(1/2)*((5*x+7
)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*78^(1/2))+34320*11^(1/2)*((5*x+7)/(4*
x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2
)*x*EllipticE(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*
78^(1/2))+1265*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(
4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticF(1/31*31^(1/2)*11^(1/2)*((5*
x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+4117*11^(1/2)*((5*x+7)/(4*x+1))
^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*Ell
ipticPi(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)
*78^(1/2))+4290*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/
(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticE(1/31*31^(1/2)*11^(1/2)*((5
*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+514800*x^3-909480*x^2-1424280*
x+1201200)/(120*x^4-182*x^3-385*x^2+197*x+70)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{5\,x+7}\,\sqrt{-3\,x+2}}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(1/2)*(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(5*x + 7)*sqrt(-3*x + 2)/(sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2-3\,x}\,\sqrt{5\,x+7}}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2 - 3*x)^(1/2)*(5*x + 7)^(1/2))/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
[Out] int(((2 - 3*x)^(1/2)*(5*x + 7)^(1/2))/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2-3x}\,\sqrt{5x+7}}{\sqrt{2x-5}\,\sqrt{4x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**(1/2)*(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
[Out] Integral(sqrt(2 - 3*x)*sqrt(5*x + 7)/(sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

3.96 $\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}}\,dx$

Optimal. Leaf size=101

$$\frac{62(2-3x)\sqrt{\frac{5-2x}{2-3x}}\,\sqrt{-\frac{4x+1}{2-3x}}\,\Pi\left(-\frac{69}{55};\sin^{-1}\left(\frac{\sqrt{\frac{11}{23}}\,\sqrt{5x+7}}{\sqrt{2-3x}}\right)|-\frac{23}{39}\right)}{5\sqrt{429}\,\sqrt{2x-5}\,\sqrt{4x+1}}$$

```
[Out] 62/2145*(2-3*x)*EllipticPi(1/23*253^(1/2)*(7+5*x)^(1/2)/(2-3*x)^(1/2),-69/5
5,1/39*I*897^(1/2))*((5-2*x)/(2-3*x))^(1/2)*((-1-4*x)/(2-3*x))^(1/2)*429^(1
/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {165, 537}

$$\frac{62(2-3x)\sqrt{\frac{5-2x}{2-3x}}\,\sqrt{-\frac{4x+1}{2-3x}}\,\Pi\left(-\frac{69}{55};\sin^{-1}\left(\frac{\sqrt{\frac{11}{23}}\,\sqrt{5x+7}}{\sqrt{2-3x}}\right)|-\frac{23}{39}\right)}{5\sqrt{429}\,\sqrt{2x-5}\,\sqrt{4x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]),x]
```

```
[Out] (62*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((1 + 4*x)/(2 - 3*x))]*Ellipt
icPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2 - 3*x]], -23/39])/(5
*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Rule 165

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*(a + b*x)*Sqrt[((b*g -
 a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*
g - e*h)*(a + b*x))])/(Sqrt[c + d*x]*Sqrt[e + f*x]), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + ((b*c - a*d)*x^2)/(d*g - c*h)]*Sqrt[1 + ((b*e - a*f)*x^2)/(f*g
- e*h)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}}\,dx = \frac{\left(62(2-3x)\sqrt{-\frac{-5+2x}{2-3x}}\,\sqrt{-\frac{1+4x}{2-3x}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{11x^2}{23}}\,\sqrt{1+\frac{11x^2}{39}}\,(5+3x^2)}\,dx, x, \frac{\sqrt{7}}{\sqrt{2}}\right.}{\sqrt{897}\,\sqrt{-5+2x}\,\sqrt{1+4x}}$$

$$= \frac{62(2-3x)\sqrt{\frac{5-2x}{2-3x}}\,\sqrt{-\frac{1+4x}{2-3x}}\,\Pi\left(-\frac{69}{55};\sin^{-1}\left(\frac{\sqrt{\frac{11}{23}}\,\sqrt{7+5x}}{\sqrt{2-3x}}\right)|-\frac{23}{39}\right)}{5\sqrt{429}\,\sqrt{-5+2x}\,\sqrt{1+4x}}$$

Mathematica [A] time = 0.59, size = 170, normalized size = 1.68

$$\frac{\sqrt{\frac{4x+1}{5x+7}}(5x+7)^{3/2}\left(117\sqrt{\frac{-6x^2+19x-10}{(5x+7)^2}}\Pi\left(-\frac{55}{62};\sin^{-1}\left(\sqrt{\frac{155-62x}{55x+77}}\right)|\frac{23}{62}\right)-62\sqrt{\frac{5-2x}{5x+7}}\sqrt{\frac{3x-2}{5x+7}}\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{155-62x}{55x+77}}\right.\right.\right.}{5\sqrt{682}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]),x]
```

```
[Out] (Sqrt[(1 + 4*x)/(7 + 5*x)]*(7 + 5*x)^(3/2)*(-62*Sqrt[(5 - 2*x)/(7 + 5*x)]*S
qrt[(-2 + 3*x)/(7 + 5*x)]*EllipticF[ArcSin[Sqrt[(155 - 62*x)/(77 + 55*x)]],
 23/62] + 117*Sqrt[(-10 + 19*x - 6*x^2)/(7 + 5*x)^2]*EllipticPi[-55/62, Arc
Sin[Sqrt[(155 - 62*x)/(77 + 55*x)]], 23/62]))/(5*Sqrt[682]*Sqrt[2 - 3*x]*Sq
rt[-5 + 2*x]*Sqrt[1 + 4*x])
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{5x+7}\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{40x^3-34x^2-151x-35},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(40*x^3 -
 34*x^2 - 151*x - 35), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-3x+2}}{\sqrt{5x+7}\sqrt{4x+1}\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(-3*x + 2)/(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

maple [B] time = 0.03, size = 172, normalized size = 1.70

$$\frac{\left(55\,\text{EllipticF}\left(\frac{\sqrt{31}\sqrt{11}\sqrt{\frac{5x+7}{4x+1}}}{31},\frac{\sqrt{31}\sqrt{78}}{39}\right)+69\,\text{EllipticPi}\left(\frac{\sqrt{31}\sqrt{11}\sqrt{\frac{5x+7}{4x+1}}}{31},\frac{124}{55},\frac{\sqrt{31}\sqrt{78}}{39}\right)\right)\sqrt{\frac{3x-2}{4x+1}}\sqrt{\frac{2x-5}{4x+1}}\sqrt{13}\sqrt{3}\sqrt{\cdots}}{128700x^3-227370x^2-356070x+300300}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x+2)^(1/2)/(5*x+7)^(1/2)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] 1/4290*(55*EllipticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31
^(1/2)*78^(1/2))+69*EllipticPi(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/
2),124/55,1/39*31^(1/2)*78^(1/2)))*((3*x-2)/(4*x+1))^(1/2)*((2*x-5)/(4*x+1)
)^(1/2)*13^(1/2)*3^(1/2)*((5*x+7)/(4*x+1))^(1/2)*11^(1/2)*(4*x+1)^(3/2)*(2*
x-5)^(1/2)*(5*x+7)^(1/2)*(-3*x+2)^(1/2)/(30*x^3-53*x^2-83*x+70)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-3x+2}}{\sqrt{5x+7}\sqrt{4x+1}\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(-3*x + 2)/(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2-3\,x}}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{5\,x+7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(1/2)),x)
```

```
[Out] int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2-3x}}{\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{5x+7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)/(7+5*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral(sqrt(2 - 3*x)/(sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)), x)
```

3.97 $\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^{3/2}}\,dx$

Optimal. Leaf size=60

$$\frac{2\sqrt{\frac{11}{39}}\,\sqrt{5-2x}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{22}}\,\sqrt{4x+1}}{\sqrt{5x+7}}\right)|\frac{62}{39}\right)}{23\sqrt{2x-5}}$$

```
[Out] 2/897*EllipticE(1/22*858^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2),1/39*2418^(1/2))
*429^(1/2)*(5-2*x)^(1/2)/(-5+2*x)^(1/2)
```

Rubi [B] time = 0.13, antiderivative size = 195, normalized size of antiderivative = 3.25, number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {176, 422, 418, 492, 411}

$$-\frac{62\sqrt{2x-5}\,\sqrt{4x+1}}{897\sqrt{2-3x}\,\sqrt{5x+7}}-\frac{\sqrt{\frac{22}{31}}\,\sqrt{4x+1}\,F\left(\tan^{-1}\left(\frac{\sqrt{\frac{31}{11}}\,\sqrt{2x-5}}{\sqrt{5x+7}}\right)|\frac{39}{62}\right)}{39\sqrt{2-3x}\,\sqrt{-\frac{4x+1}{2-3x}}}+\frac{2\sqrt{682}\,\sqrt{4x+1}\,E\left(\tan^{-1}\left(\frac{\sqrt{\frac{31}{11}}\,\sqrt{2x-5}}{\sqrt{5x+7}}\right)|\frac{39}{62}\right)}{897\sqrt{2-3x}\,\sqrt{-\frac{4x+1}{2-3x}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2)),x]
```

```
[Out] (-62*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(897*Sqrt[2 - 3*x]*Sqrt[7 + 5*x]) + (2*S
qrt[682]*Sqrt[1 + 4*x]*EllipticE[ArcTan[(Sqrt[31/11]*Sqrt[-5 + 2*x])/Sqrt[7
 + 5*x]], 39/62])/(897*Sqrt[2 - 3*x]*Sqrt[-((1 + 4*x)/(2 - 3*x))]) - (Sqrt[
22/31]*Sqrt[1 + 4*x]*EllipticF[ArcTan[(Sqrt[31/11]*Sqrt[-5 + 2*x])/Sqrt[7 +
 5*x]], 39/62])/(39*Sqrt[2 - 3*x]*Sqrt[-((1 + 4*x)/(2 - 3*x))])
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
```

```
&& PosQ[b/a]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx = \frac{\left(\sqrt{2}\sqrt{2-3x}\sqrt{\frac{1+4x}{7+5x}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{31x^2}{11}}}{\sqrt{1+\frac{23x^2}{22}}}\,dx, x, \frac{\sqrt{-5+2x}}{\sqrt{7+5x}}\right)}{39\sqrt{1+4x}\sqrt{-\frac{2-3x}{7+5x}}}$$

$$= \frac{\left(\sqrt{2}\sqrt{2-3x}\sqrt{\frac{1+4x}{7+5x}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{23x^2}{22}}\sqrt{1+\frac{31x^2}{11}}}\,dx, x, \frac{\sqrt{-5+2x}}{\sqrt{7+5x}}\right)}{39\sqrt{1+4x}\sqrt{-\frac{2-3x}{7+5x}}} + \frac{(31\sqrt{}}{}$$

$$= -\frac{62\sqrt{-5+2x}\sqrt{1+4x}}{897\sqrt{2-3x}\sqrt{7+5x}} - \frac{\sqrt{\frac{22}{31}}\sqrt{1+4x}\,F\left(\tan^{-1}\left(\frac{\sqrt{\frac{31}{11}}\sqrt{-5+2x}}{\sqrt{7+5x}}\right)|\frac{39}{62}\right)}{39\sqrt{2-3x}\sqrt{-\frac{1+4x}{2-3x}}} - \cdots$$

$$= -\frac{62\sqrt{-5+2x}\sqrt{1+4x}}{897\sqrt{2-3x}\sqrt{7+5x}} + \frac{2\sqrt{682}\sqrt{1+4x}\,E\left(\tan^{-1}\left(\frac{\sqrt{\frac{31}{11}}\sqrt{-5+2x}}{\sqrt{7+5x}}\right)|\frac{39}{62}\right)}{897\sqrt{2-3x}\sqrt{-\frac{1+4x}{2-3x}}} - \cdots$$

Mathematica [B] time = 1.86, size = 237, normalized size = 3.95

$$\frac{\sqrt{2x-5}\sqrt{4x+1}\left(-23\sqrt{682}\sqrt{\frac{8x^2-18x-5}{(2-3x)^2}}\left(15x^2+11x-14\right)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{2x-5}{3x-2}}\right),\frac{39}{62}\right)-1922\sqrt{\frac{5x+7}{3x-2}}\cdots\right.}{27807\sqrt{2-3x}\sqrt{5x+7}\sqrt{\frac{5x+7}{3x-2}}\left(8x^2\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2)),x]
```

```
[Out] (Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(-1922*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x
+ 8*x^2) + 62*Sqrt[682]*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x +
 15*x^2)*EllipticE[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62]
- 23*Sqrt[682]*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2)*
EllipticF[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62]))/(27807*
Sqrt[2 - 3*x]*Sqrt[7 + 5*x]*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x + 8*x^2))
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{5\,x+7}\sqrt{4\,x+1}\sqrt{2\,x-5}\sqrt{-3\,x+2}}{200\,x^4+110\,x^3-993\,x^2-1232\,x-245}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^(3/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(200*x^4
+ 110*x^3 - 993*x^2 - 1232*x - 245), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{3}{2}}\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^(3/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(-3*x + 2)/((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

maple [B] time = 0.03, size = 330, normalized size = 5.50

$$\frac{2\sqrt{-3x+2}\,\sqrt{5x+7}\,\sqrt{2x-5}\,\sqrt{4x+1}\left(16\sqrt{11}\,\sqrt{\frac{5x+7}{4x+1}}\,\sqrt{3}\,\sqrt{13}\,\sqrt{\frac{2x-5}{4x+1}}\,\sqrt{\frac{3x-2}{4x+1}}\,x^2\,\mathrm{EllipticE}\left(\frac{\sqrt{31}\,\sqrt{11}\,\sqrt{\frac{5x+7}{4x+1}}}{31},\frac{\sqrt{31}}{39}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x+2)^(1/2)/(5*x+7)^(3/2)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] 2/897*(-3*x+2)^(1/2)*(5*x+7)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(16*11^(1/2)
*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/
(4*x+1))^(1/2)*x^2*EllipticE(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)
,1/39*31^(1/2)*78^(1/2))+8*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2
)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x*EllipticE(1/31*31^(1/2)
*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+11^(1/2)*((5*x+7)
/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^
(1/2)*EllipticE(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2
)*78^(1/2))+138*x^2-437*x+230)/(120*x^4-182*x^3-385*x^2+197*x+70)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-3\,x+2}}{(5\,x+7)^{\frac{3}{2}}\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^(3/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(-3*x + 2)/((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{2-3\,x}}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,{(5\,x+7)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(3/2)),x)
```

```
[Out] int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(3/2)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)/(7+5*x)**(3/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
[Out] Timed out
```

3.98 $\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\,\sqrt{1+4x}\,(7+5x)^{5/2}}\,dx$

Optimal. Leaf size=290

$$\frac{716\sqrt{\frac{11}{23}}\sqrt{5x+7}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{61893\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}+\frac{39332\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{74828637\sqrt{2x-5}}-\frac{98330\sqrt{2-3x}\sqrt{2x-5}}{74828637\sqrt{5x+}}$$

```
[Out] -10/2691*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2)-98330/748
28637*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2)+39332/748286
37*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)+716/1423539*(1/
(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)/(2-3*x))^(1/2)*EllipticF((1+4*x)^
(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/(2-3*x))^(1/2),1/23*I*897^(1/2))*2
53^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7+5*x)/(5-2*x))^(1/2)-19666/7482863
7*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),1/39*I*897^(1/2))*4
29^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/2)/((2-3*x)/(5-2*x))^(1/2)/(7+5
*x)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {177, 1599, 1602, 12, 170, 418, 176, 424}

$$\frac{39332\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{74828637\sqrt{2x-5}}-\frac{98330\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{74828637\sqrt{5x+7}}-\frac{10\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{2691(5x+7)^{3/2}}+\frac{716\sqrt{\frac{11}{23}}\sqrt{5x}}{61}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2)),x]
```

```
[Out] (-10*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(2691*(7 + 5*x)^(3/2)) - (
98330*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(74828637*Sqrt[7 + 5*x])
+ (39332*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(74828637*Sqrt[-5 + 2*x
]) - (19666*Sqrt[11/39]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[A
rcSin[(Sqrt[39/23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(1918683*Sqrt[(
2 - 3*x)/(5 - 2*x)]*Sqrt[7 + 5*x]) + (716*Sqrt[11/23]*Sqrt[7 + 5*x]*Ellipti
cF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(61893*Sqrt[-5 +
 2*x]*Sqrt[(7 + 5*x)/(5 - 2*x)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
```

```
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 177

```
Int[(((a_.) + (b_.)*(x_))^(m_)*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.
)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*S
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((m + 1)*(b*e - a*f)*(b*g - a*h))
, x] + Dist[1/(2*(m + 1)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(
Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[2*a*c*f*h*(m + 1) - b*(d*e
*g + c*(2*m + 3)*(f*g + e*h)) + 2*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e
*h + c*f*h))*x - b*d*f*h*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, m}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1599

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
(A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]
)/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), x] - Dist[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sq
rt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d*
f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*e
*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
- b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rule 1602

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[(C*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*f*h*Sqrt[c + d*
x]), x] + (Dist[1/(2*b*d*f*h), Int[(1*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f
*h) + (2*b*B*d*f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x])/(Sqrt[
a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(C*(d*e
- c*f)*(d*g - c*h))/(2*b*d*f*h), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$\int \frac{\sqrt{2-3x}}{\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}\,dx = -\frac{10\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2691(7+5x)^{3/2}} - \frac{\int \frac{-771+854x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx}{2691}$$

$$= -\frac{10\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2691(7+5x)^{3/2}} - \frac{98330\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{74828637\sqrt{7+5x}} - \frac{\int}{}$$

$$= -\frac{10\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2691(7+5x)^{3/2}} - \frac{98330\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{74828637\sqrt{7+5x}} + \frac{39}{}$$

$$= -\frac{10\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2691(7+5x)^{3/2}} - \frac{98330\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{74828637\sqrt{7+5x}} + \frac{39}{}$$

$$= -\frac{10\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2691(7+5x)^{3/2}} - \frac{98330\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{74828637\sqrt{7+5x}} + \frac{39}{}$$

$$= -\frac{10\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{2691(7+5x)^{3/2}} - \frac{98330\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{74828637\sqrt{7+5x}} + \frac{39}{}$$

Mathematica [A] time = 1.92, size = 248, normalized size = 0.86

$$-\frac{2\sqrt{2x-5}\sqrt{4x+1}\left(31\left(92\sqrt{682}(3x-2)\sqrt{\frac{8x^2-18x-5}{(2-3x)^2}}(5x+7)^2\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{2x-5}{3x-2}}\right),\frac{39}{62}\right)+\sqrt{\frac{5x+7}{3x-2}}\left(28\right.\right.\right.}{74828637\sqrt{2-3x}(5x+}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - 3*x]/(Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2)),x]
```

```
[Out] (-2*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(-9833*Sqrt[682]*(-2 + 3*x)*(7 + 5*x)^2*Sq
rt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticE[ArcSin[Sqrt[31/39]*Sqrt[(-5 +
 2*x)/(-2 + 3*x)]], 39/62] + 31*(Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-389005 - 1578
968*x - 20372*x^2 + 285680*x^3) + 92*Sqrt[682]*(-2 + 3*x)*(7 + 5*x)^2*Sqrt[
(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticF[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*
x)/(-2 + 3*x)]], 39/62])))/(74828637*Sqrt[2 - 3*x]*(7 + 5*x)^(3/2)*Sqrt[(7
+ 5*x)/(-2 + 3*x)]*(-5 - 18*x + 8*x^2))
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{5x+7}\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{1000x^5+1950x^4-4195x^3-13111x^2-9849x-1715}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^(5/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(1000*x^5
 + 1950*x^4 - 4195*x^3 - 13111*x^2 - 9849*x - 1715), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-3x+2}}{(5x+7)^{\frac{5}{2}}\sqrt{4x+1}\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^(5/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(-3*x + 2)/((5*x + 7)^(5/2)*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

maple [B] time = 0.03, size = 786, normalized size = 2.71

$$2\left(786640\sqrt{11}\,\sqrt{\frac{5x+7}{4x+1}}\,\sqrt{3}\,\sqrt{13}\,\sqrt{\frac{2x-5}{4x+1}}\,\sqrt{\frac{3x-2}{4x+1}}\,x^3\,\mathrm{EllipticE}\left(\frac{\sqrt{31}\,\sqrt{11}\,\sqrt{\frac{5x+7}{4x+1}}}{31},\frac{\sqrt{31}\,\sqrt{78}}{39}\right)+101200\sqrt{11}\,\sqrt{\frac{5x+7}{4x+1}}\,\sqrt{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x+2)^(1/2)/(5*x+7)^(5/2)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] 2/74828637*(101200*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-
5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticF(1/31*31^(1/2)*11^(1/2)*
((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))*x^3+786640*11^(1/2)*((5*x+7
)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))
^(1/2)*EllipticE(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/
2)*78^(1/2))*x^3+192280*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*(
(2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x^2*EllipticF(1/31*31^(1/2)*
11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+1494616*11^(1/2)*(
(5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4
*x+1))^(1/2)*x^2*EllipticE(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1
/39*31^(1/2)*78^(1/2))+77165*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1
/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x*EllipticF(1/31*31^(1/
2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+599813*11^(1/2)
*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/
(4*x+1))^(1/2)*x*EllipticE(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1
/39*31^(1/2)*78^(1/2))+8855*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/
2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticF(1/31*31^(1/2)*
11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+68831*11^(1/2)*((5
*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x
+1))^(1/2)*EllipticE(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31
^(1/2)*78^(1/2))+3447930*x^3-2253977*x^2-21690932*x+14440780)*(4*x+1)^(1/2)
*(2*x-5)^(1/2)*(-3*x+2)^(1/2)/(120*x^4-182*x^3-385*x^2+197*x+70)/(5*x+7)^(1
/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-3x+2}}{(5x+7)^{\frac{5}{2}}\sqrt{4x+1}\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)^(1/2)/(7+5*x)^(5/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(-3*x + 2)/((5*x + 7)^(5/2)*sqrt(4*x + 1)*sqrt(2*x - 5)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2-3x}}{\sqrt{4x+1}\,\sqrt{2x-5}\,(5x+7)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(5/2)),x)
[Out] int((2 - 3*x)^(1/2)/((4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*x)**(1/2)/(7+5*x)**(5/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
[Out] Timed out
```

3.99 $\int \frac{\sqrt{a+bx}\,\sqrt{c+dx}}{\sqrt{e+fx}\,\sqrt{g+hx}}\,dx$

Optimal. Leaf size=721

$$\frac{\sqrt{g+hx}\,(de-cf)(-2afh+beh+bfg)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\ \text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{e+fx}\,\sqrt{bg-ah}}{\sqrt{a+bx}\,\sqrt{fg-eh}}\right),-\frac{(bc-ad)(fg-eh)}{(bg-ah)(de-cf)}\right)}{f^2h\sqrt{c+dx}\,\sqrt{bg-ah}\,\sqrt{fg-eh}\,\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}+\frac{(e+f}{}$$

```
[Out] (a*d*f*h-b*(-c*f*h+d*e*h+d*f*g))*(f*x+e)*EllipticPi((-a*f+b*e)^(1/2)*(h*x+g
)^(1/2)/(-a*h+b*g)^(1/2)/(f*x+e)^(1/2),f*(-a*h+b*g)/(-a*f+b*e)/h,((-c*f+d*e
)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))*(-a*h+b*g)^(1/2)*((-e*h+f*g)*(b*
x+a)/(-a*h+b*g)/(f*x+e))^(1/2)*((-e*h+f*g)*(d*x+c)/(-c*h+d*g)/(f*x+e))^(1/2
)/f^2/h^2/(-a*f+b*e)^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)+(b*x+a)^(1/2)*(d*x+c
)^(1/2)*(h*x+g)^(1/2)/h/(f*x+e)^(1/2)+(-c*f+d*e)*(-2*a*f*h+b*e*h+b*f*g)*Ell
ipticF((-a*h+b*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(b*x+a)^(1/2),(-(-a*
d+b*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*h+b*g))^(1/2))*((-a*f+b*e)*(d*x+c)/(-c*f+d
*e)/(b*x+a))^(1/2)*(h*x+g)^(1/2)/f^2/h/(-a*h+b*g)^(1/2)/(-e*h+f*g)^(1/2)/(d
*x+c)^(1/2)/(-(-a*f+b*e)*(h*x+g)/(-e*h+f*g)/(b*x+a))^(1/2)-EllipticE((-e*h+
f*g)^(1/2)*(d*x+c)^(1/2)/(-c*h+d*g)^(1/2)/(f*x+e)^(1/2),((-a*f+b*e)*(-c*h+d
*g)/(-a*d+b*c)/(-e*h+f*g))^(1/2))*(-c*h+d*g)^(1/2)*(-e*h+f*g)^(1/2)*(b*x+a)
^(1/2)*((-c*f+d*e)*(h*x+g)/(-c*h+d*g)/(f*x+e))^(1/2)/f/h/(-(-c*f+d*e)*(b*x+
a)/(-a*d+b*c)/(f*x+e))^(1/2)/(h*x+g)^(1/2)
```

Rubi [A] time = 0.67, antiderivative size = 721, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {173, 176, 424, 170, 419, 165, 537}

$$\frac{(e+fx)\sqrt{bg-ah}\,\sqrt{\frac{(a+bx)(fg-eh)}{(e+fx)(bg-ah)}}\,\sqrt{\frac{(c+dx)(fg-eh)}{(e+fx)(dg-ch)}}\,(adfh-b(-cfh+deh+dfg))\Pi\left(\frac{f(bg-ah)}{(be-af)h};\sin^{-1}\left(\frac{\sqrt{be-af}\,\sqrt{g+hx}}{\sqrt{bg-ah}\,\sqrt{e+fx}}\right)\right.}{f^2h^2\sqrt{a+bx}\,\sqrt{c+dx}\,\sqrt{be-af}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x]*Sqrt[c + d*x])/(Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[g + h*x])/(h*Sqrt[e + f*x]) - (Sqrt[d*g -
 c*h]*Sqrt[f*g - e*h]*Sqrt[a + b*x]*Sqrt[((d*e - c*f)*(g + h*x))/((d*g - c*
h)*(e + f*x))]*EllipticE[ArcSin[(Sqrt[f*g - e*h]*Sqrt[c + d*x])/(Sqrt[d*g -
 c*h]*Sqrt[e + f*x])], ((b*e - a*f)*(d*g - c*h))/((b*c - a*d)*(f*g - e*h))]
)/(f*h*Sqrt[-(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))]*Sqrt[g + h*
x]) + ((d*e - c*f)*(b*f*g + b*e*h - 2*a*f*h)*Sqrt[((b*e - a*f)*(c + d*x))/(
(d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g - a*h]*Sqr
t[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h))/(
(d*e - c*f)*(b*g - a*h)))])/(f^2*h*Sqrt[b*g - a*h]*Sqrt[f*g - e*h]*Sqrt[c +
 d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]) + (Sqrt[b*g
 - a*h]*(a*d*f*h - b*(d*f*g + d*e*h - c*f*h))*Sqrt[((f*g - e*h)*(a + b*x))/
((b*g - a*h)*(e + f*x))]*Sqrt[((f*g - e*h)*(c + d*x))/((d*g - c*h)*(e + f*x
))]*(e + f*x)*EllipticPi[(f*(b*g - a*h))/((b*e - a*f)*h), ArcSin[(Sqrt[b*e
- a*f]*Sqrt[g + h*x])/(Sqrt[b*g - a*h]*Sqrt[e + f*x])], ((d*e - c*f)*(b*g -
 a*h))/((b*e - a*f)*(d*g - c*h))])/(f^2*Sqrt[b*e - a*f]*h^2*Sqrt[a + b*x]*S
qrt[c + d*x])
```

Rule 165

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*(a + b*x)*Sqrt[((b*g -
 a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*
```

```
g - e*h)*(a + b*x))])/(Sqrt[c + d*x]*Sqrt[e + f*x]), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + ((b*c - a*d)*x^2)/(d*g - c*h)]*Sqrt[1 + ((b*e - a*f)*x^2)/(f*g
- e*h)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 173

```
Int[(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(Sqrt[a + b*x]*Sqrt[c +
 d*x]*Sqrt[g + h*x])/(h*Sqrt[e + f*x]), x] + (-Dist[((d*e - c*f)*(f*g - e*h
))/(2*f*h), Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*(e + f*x)^(3/2)*Sqrt[g + h*x])
, x], x] + Dist[((d*e - c*f)*(b*f*g + b*e*h - 2*a*f*h))/(2*f^2*h), Int[1/(S
qrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(a*d
*f*h - b*(d*f*g + d*e*h - c*f*h))/(2*f^2*h), Int[Sqrt[e + f*x]/(Sqrt[a + b*
x]*Sqrt[c + d*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h},
 x]
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}}{\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}}{h\sqrt{e+fx}} - \frac{((de-cf)(fg-eh))\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}(e+fx)^{3/2}\sqrt{g+hx}}\,dx}{2fh} + \frac{((de-$$

$$= \frac{\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}}{h\sqrt{e+fx}} + \frac{\left((adfh-b(dfg+deh-cfh))\sqrt{\frac{(fg-eh)(a+bx)}{(bg-ah)(e+fx)}}\sqrt{\frac{(fg-eh)(}{(dg-ch)(}}\right.}{f^2}$$

$$= \frac{\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}}{h\sqrt{e+fx}} - \frac{\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{\frac{(de-cf)(g+hx)}{(dg-ch)(e+fx)}}E\left(\sin^{-1}\left(\right.\right.}{fh\sqrt{-\frac{(de-cf)(a+bx)}{(bc-ad)(e+fx)}}\sqrt{g+hx}}$$

Mathematica [B] time = 15.05, size = 6667, normalized size = 9.25

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*x]*Sqrt[c + d*x])/(Sqrt[e + f*x]*Sqrt[g + h*x]),x]
[Out] Result too large to show
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algori
thm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx+a}\sqrt{dx+c}}{\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algori
thm="giac")
[Out] integrate(sqrt(b*x + a)*sqrt(d*x + c)/(sqrt(f*x + e)*sqrt(h*x + g)), x)
```

maple [B] time = 0.16, size = 18077, normalized size = 25.07

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx+a}\sqrt{dx+c}}{\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate(sqrt(b*x + a)*sqrt(d*x + c)/(sqrt(f*x + e)*sqrt(h*x + g)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+b\,x}\,\sqrt{c+d\,x}}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)^(1/2)*(c + d*x)^(1/2))/((e + f*x)^(1/2)*(g + h*x)^(1/2)),x)
```

```
[Out] int(((a + b*x)^(1/2)*(c + d*x)^(1/2))/((e + f*x)^(1/2)*(g + h*x)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}}{\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x)*sqrt(c + d*x)/(sqrt(e + f*x)*sqrt(g + h*x)), x)
```

3.100 $\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal. Leaf size=161

$$-\frac{2\sqrt{c+dx}\,E\left(\tan^{-1}\left(\frac{\sqrt{af-be}\sqrt{g+hx}}{\sqrt{bg-ah}\sqrt{e+fx}}\right)|\frac{(ad-bc)(fg-eh)}{(af-be)(dg-ch)}\right)}{\sqrt{a+bx}\sqrt{af-be}\sqrt{bg-ah}\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}}$$

```
[Out] -2*(1/(1+(a*f-b*e)*(h*x+g)/(-a*h+b*g)/(f*x+e)))^(1/2)*(1+(a*f-b*e)*(h*x+g)/
(-a*h+b*g)/(f*x+e))^(1/2)*EllipticE((a*f-b*e)^(1/2)*(h*x+g)^(1/2)/(-a*h+b*g
)^(1/2)/(f*x+e)^(1/2)/(1+(a*f-b*e)*(h*x+g)/(-a*h+b*g)/(f*x+e))^(1/2),((a*d-
b*c)*(-e*h+f*g)/(a*f-b*e)/(-c*h+d*g))^(1/2))*(d*x+c)^(1/2)/(a*f-b*e)^(1/2)/
(-a*h+b*g)^(1/2)/(b*x+a)^(1/2)/((-a*h+b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))^(1/2
)
```

Rubi [A] time = 0.09, antiderivative size = 208, normalized size of antiderivative = 1.29, number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {176, 424}

$$-\frac{2\sqrt{c+dx}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}\,E\left(\sin^{-1}\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right)|-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{\sqrt{g+hx}\,(be-af)\sqrt{bg-ah}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/((a + b*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (-2*Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*
h)*(a + b*x)))]*EllipticE[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*g
- e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*g - a*h
)))])/((b*e - a*f)*Sqrt[b*g - a*h]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f
)*(a + b*x))]*Sqrt[g + h*x])
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{\left(2\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{(bc-ad)x^2}{de-cf}}}{\sqrt{1-\frac{(bg-ah)x^2}{fg-eh}}}\,dx, x, \frac{\sqrt{e+fx}}{\sqrt{a+bx}}\right)}{(be-af)\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}}$$

$$= -\frac{2\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}\,E\left(\sin^{-1}\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right)|-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{(be-af)\sqrt{bg-ah}\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}}$$

Mathematica [A] time = 5.21, size = 206, normalized size = 1.28

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\,E\left(\sin^{-1}\left(\sqrt{\frac{(af-be)(g+hx)}{(fg-eh)(a+bx)}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{(a+bx)^{3/2}(eh-fg)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{-\frac{(e+fx)(g+hx)(be-af)(bg-ah)}{(a+bx)^2(fg-eh)^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/((a + b*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]*EllipticE[ArcSin[Sqrt[((-(b*e)
 + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], ((b*c - a*d)*(f*g - e*h))/((b
*e - a*f)*(d*g - c*h))])/((-(f*g) + e*h)*(a + b*x)^(3/2)*Sqrt[((b*g - a*h)*
(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[-(((b*e - a*f)*(b*g - a*h)*(e + f*
x)*(g + h*x))/((f*g - e*h)^2*(a + b*x)^2))])
```

fricas [F] time = 3.36, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}{b^2fhx^4+a^2eg+\left(b^2fg+\left(b^2e+2\,abf\right)h\right)x^3+\left(\left(b^2e+2\,abf\right)g+\left(2\,abe+a^2f\right)h\right)x^2+\left(a^2eh+\left(2\,abe+a^2f\right)g\right)x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algori
thm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)/(b^2*f*h*x
^4 + a^2*e*g + (b^2*f*g + (b^2*e + 2*a*b*f)*h)*x^3 + ((b^2*e + 2*a*b*f)*g +
 (2*a*b*e + a^2*f)*h)*x^2 + (a^2*e*h + (2*a*b*e + a^2*f)*g)*x), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algori
thm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.13, size = 4590, normalized size = 28.51

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

```
[Out] 2*(-2*EllipticE(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h
-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*x*a*c*e*f*h^2*((a*f-b*e)*(h*x+g)/(a*h-b*g
)/(f*x+e))^(1/2)*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)*((e*h-f*g)*(b*
x+a)/(a*h-b*g)/(f*x+e))^(1/2)-2*EllipticE(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x
+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*x*b*d*e*f*g^2*(
(a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*
x+e))^(1/2)*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)-EllipticF(((a*f-b*e
)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e)
)^(1/2))*x^2*a*d*f^2*g*h*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)*((e*h-
f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))
^(1/2)-EllipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*
h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*x^2*b*c*f^2*g*h*((a*f-b*e)*(h*x+g)/(a*h-
b*g)/(f*x+e))^(1/2)*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)*((e*h-f*g)*
(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)+EllipticE(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*
x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*x^2*a*d*f^2*g*
h*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)*((e*h-f*g)*(d*x+c)/(c*h-d*g)/
(f*x+e))^(1/2)*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)+EllipticE(((a*f-
b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b
*e))^(1/2))*x^2*b*c*f^2*g*h*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)*((e
*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+
e))^(1/2)+2*EllipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e
)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*x*a*c*e*f*h^2*((a*f-b*e)*(h*x+g)/(a
*h-b*g)/(f*x+e))^(1/2)*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)*((e*h-f*
g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)+2*EllipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g
)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*x*b*d*e*f
*g^2*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)*((e*h-f*g)*(d*x+c)/(c*h-d*
g)/(f*x+e))^(1/2)*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)+EllipticF(((a
*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*
f-b*e))^(1/2))*x^2*a*c*f^2*h^2*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)*
((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f
*x+e))^(1/2)+EllipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*
e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*x^2*b*d*f^2*g^2*((a*f-b*e)*(h*x+g)
/(a*h-b*g)/(f*x+e))^(1/2)*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)*((e*h
-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)-EllipticE(((a*f-b*e)*(h*x+g)/(a*h-b*
g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*x^2*a*c*
f^2*h^2*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)*((e*h-f*g)*(d*x+c)/(c*h
-d*g)/(f*x+e))^(1/2)*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)-EllipticE(
((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/
(a*f-b*e))^(1/2))*x^2*b*d*f^2*g^2*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/
2)*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)*((e*h-f*g)*(b*x+a)/(a*h-b*g)
/(f*x+e))^(1/2)-EllipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f
-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*a*d*e^2*g*h*((a*f-b*e)*(h*x+g)/
(a*h-b*g)/(f*x+e))^(1/2)*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)*((e*h-
f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)-EllipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g
)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*b*c*e^2*g
*h*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)*((e*h-f*g)*(d*x+c)/(c*h-d*g)
/(f*x+e))^(1/2)*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)+EllipticE(((a*f
-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-
b*e))^(1/2))*a*d*e^2*g*h*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)*((e*h-
f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))
^(1/2)+EllipticE(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*
h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*b*c*e^2*g*h*((a*f-b*e)*(h*x+g)/(a*h-b*g)
/(f*x+e))^(1/2)*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)*((e*h-f*g)*(b*x
+a)/(a*h-b*g)/(f*x+e))^(1/2)+a*c*e*f*g*h-x^2*b*d*e^2*h^2-a*c*f^2*g^2-2*Elli
pticF(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h
-d*g)/(a*f-b*e))^(1/2))*x*a*d*e*f*g*h*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))
^(1/2)*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)*((e*h-f*g)*(b*x+a)/(a*h-
b*g)/(f*x+e))^(1/2)-2*EllipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)
```

```
,((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*x*b*c*e*f*g*h*((a*f-b*e)*
(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2
)*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)+2*EllipticE(((a*f-b*e)*(h*x+g
)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))
*x*a*d*e*f*g*h*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)*((e*h-f*g)*(d*x+
c)/(c*h-d*g)/(f*x+e))^(1/2)*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)+2*E
llipticE(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(
c*h-d*g)/(a*f-b*e))^(1/2))*x*b*c*e*f*g*h*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+
e))^(1/2)*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)*((e*h-f*g)*(b*x+a)/(a
*h-b*g)/(f*x+e))^(1/2)+x^2*a*d*e*f*h^2+EllipticF(((a*f-b*e)*(h*x+g)/(a*h-b*
g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*b*d*e^2*
g^2*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)*((e*h-f*g)*(d*x+c)/(c*h-d*g
)/(f*x+e))^(1/2)*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)-EllipticE(((a*
f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f
-b*e))^(1/2))*a*c*e^2*h^2*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)*((e*h
-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e)
)^(1/2)-EllipticE(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a
*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*b*d*e^2*g^2*((a*f-b*e)*(h*x+g)/(a*h-b*g
)/(f*x+e))^(1/2)*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)*((e*h-f*g)*(b*
x+a)/(a*h-b*g)/(f*x+e))^(1/2)-x*a*d*f^2*g^2-x*b*c*e^2*h^2-b*c*e^2*g*h+b*c*e
*f*g^2+x*b*d*e*f*g^2+EllipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),
((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*a*c*e^2*h^2*((a*f-b*e)*(h*
x+g)/(a*h-b*g)/(f*x+e))^(1/2)*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)*(
(e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)-x^2*a*d*f^2*g*h+x*a*c*e*f*h^2-x*
a*c*f^2*g*h-x*b*d*e^2*g*h+x^2*b*d*e*f*g*h+x*a*d*e*f*g*h+x*b*c*e*f*g*h)/(h*x
+g)^(1/2)/(f*x+e)^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(e*h-f*g)/(a*h-b*g)/(a*
f-b*e)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx+c}}{(bx+a)^{\frac{3}{2}}\sqrt{fx+e}\,\sqrt{hx+g}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate(sqrt(d*x + c)/((b*x + a)^(3/2)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c+d\,x}}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,{\left(a+b\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(3/2)),x)
```

```
[Out] int((c + d*x)^(1/2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**(3/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] Timed out
```

3.101 $\int \frac{(7+5x)^{5/2}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$

Optimal. Leaf size=351

$$\frac{29047\sqrt{\frac{23}{11}}\sqrt{5x+7}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{576\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}-\frac{25}{48}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}-\frac{2135\sqrt{2-3x}}{192}$$

```
[Out] -3431855/247104*(2-3*x)*EllipticPi(1/23*253^(1/2)*(7+5*x)^(1/2)/(2-3*x)^(1/
2),-69/55,1/39*I*897^(1/2))*((5-2*x)/(2-3*x))^(1/2)*((-1-4*x)/(2-3*x))^(1/2
)*429^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2)-2135/192*(2-3*x)^(1/2)*(1+4*x)^(1/
2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)-25/48*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^
(1/2)*(7+5*x)^(1/2)+29047/6336*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)
/(2-3*x))^(1/2)*EllipticF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/
(2-3*x))^(1/2),1/23*I*897^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7
+5*x)/(5-2*x))^(1/2)+2135/384*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*
x)^(1/2),1/39*I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/2)/
((2-3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used = {167, 1602, 1598, 170, 418, 165, 537, 176, 424}

$$-\frac{25}{48}\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}-\frac{2135\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{192\sqrt{2x-5}}+\frac{29047\sqrt{\frac{23}{11}}\sqrt{5x+7}\,F\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2}}\right.\right.}{576\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x)^(5/2)/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (-2135*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(192*Sqrt[-5 + 2*x]) - (2
5*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/48 + (2135*Sqrt
[143/3]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/2
3]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(128*Sqrt[(2 - 3*x)/(5 - 2*x)]*
Sqrt[7 + 5*x]) + (29047*Sqrt[23/11]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 +
 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(576*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)
/(5 - 2*x)]) - (3431855*(2 - 3*x)*Sqrt[(5 - 2*x)/(2 - 3*x)]*Sqrt[-((1 + 4*x
)/(2 - 3*x))]*EllipticPi[-69/55, ArcSin[(Sqrt[11/23]*Sqrt[7 + 5*x])/Sqrt[2
- 3*x]], -23/39])/(576*Sqrt[429]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])
```

Rule 165

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*(a + b*x)*Sqrt[((b*g -
 a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*
g - e*h)*(a + b*x))])/(Sqrt[c + d*x]*Sqrt[e + f*x]), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + ((b*c - a*d)*x^2)/(d*g - c*h)]*Sqrt[1 + ((b*e - a*f)*x^2)/(f*g
- e*h)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 167

```
Int[((a_.) + (b_.)*(x_))^(m_)/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*
(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(2*b^2*(a + b*x)^(m - 2)
*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(d*f*h*(2*m - 1)), x] - Dist[1/
(d*f*h*(2*m - 1)), Int[((a + b*x)^(m - 3)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt
```

```
[g + h*x]))*Simp[a*b^2*(d*e*g + c*f*g + c*e*h) + 2*b^3*c*e*g*(m - 2) - a^3*
d*f*h*(2*m - 1) + b*(2*a*b*(d*f*g + d*e*h + c*f*h) + b^2*(2*m - 3)*(d*e*g +
 c*f*g + c*e*h) - 3*a^2*d*f*h*(2*m - 1))*x - 2*b^2*(m - 1)*(3*a*d*f*h - b*(
d*f*g + d*e*h + c*f*h))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x] && IntegerQ[2*m] && GeQ[m, 2]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 1598

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(A*b
 - a*B)/b, Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
 h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Rule 1602

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
```

```
l] :> Simp[(C*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*f*h*Sqrt[c + d*
x]), x] + (Dist[1/(2*b*d*f*h), Int[(1*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f
*h) + (2*b*B*d*f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x])/(Sqrt[
a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(C*(d*e
- c*f)*(d*g - c*h))/(2*b*d*f*h), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$\int \frac{(7+5x)^{5/2}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = -\frac{25}{48}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x} + \frac{1}{96}\int \frac{28003+89810x+}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1}}$$

$$= -\frac{2135\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{192\sqrt{-5+2x}} - \frac{25}{48}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+}$$

$$= -\frac{2135\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{192\sqrt{-5+2x}} - \frac{25}{48}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+}$$

$$= -\frac{2135\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{192\sqrt{-5+2x}} - \frac{25}{48}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+}$$

$$= -\frac{2135\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{192\sqrt{-5+2x}} - \frac{25}{48}\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+}$$

Mathematica [A] time = 2.61, size = 347, normalized size = 0.99

$$\frac{\sqrt{2x-5}\sqrt{4x+1}\sqrt{5x+7}\left(\frac{17113116\sqrt{682}(3x-2)(5x+7)\sqrt{\frac{8x^2-18x-5}{(2-3x)^2}}\ \text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{2x-5}{3x-2}}\right),\frac{39}{62}\right)-13104630\sqrt{682}(3x-2)(5x+7)}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(7 + 5*x)^(5/2)/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]*(1227600*(-2 + 3*x) + (-1310463
0*Sqrt[682]*(-2 + 3*x)*(7 + 5*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*Elli
pticE[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] + 17113116*Sq
rt[682]*(-2 + 3*x)*(7 + 5*x)*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*Elliptic
F[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] - 385*Sqrt[(7 + 5
*x)/(-2 + 3*x)]*(-102114*(-35 - 151*x - 34*x^2 + 40*x^3) - 47445*Sqrt[682]*
(2 - 3*x)^2*Sqrt[(1 + 4*x)/(-2 + 3*x)]*Sqrt[(-35 - 11*x + 10*x^2)/(2 - 3*x)
^2]*EllipticPi[117/62, ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/
62]))/((2 - 3*x)*((7 + 5*x)/(-2 + 3*x))^(3/2)*(5 + 18*x - 8*x^2))))/(235699
2*Sqrt[2 - 3*x])
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(25\,x^2+70\,x+49\right)\sqrt{5\,x+7}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}}{24\,x^3-70\,x^2+21\,x+10},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(5/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(25*x^2 + 70*x + 49)*sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sq
rt(-3*x + 2)/(24*x^3 - 70*x^2 + 21*x + 10), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x+7)^{\frac{5}{2}}}{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(5/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((5*x + 7)^(5/2)/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

maple [B] time = 0.04, size = 880, normalized size = 2.51

$$\sqrt{5x+7}\,\sqrt{-3x+2}\,\sqrt{2x-5}\,\sqrt{4x+1}\,\left(-20592000x^4-188588400x^3-29309280\sqrt{11}\,\sqrt{\frac{5x+7}{4x+1}}\,\sqrt{3}\,\sqrt{13}\,\sqrt{\frac{2x-5}{4x+1}}\,\sqrt{\quad}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x+7)^(5/2)/(-3*x+2)^(1/2)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] 1/329472*(5*x+7)^(1/2)*(-3*x+2)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(11088208
*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*
((3*x-2)/(4*x+1))^(1/2)*x^2*EllipticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+
1))^(1/2),1/39*31^(1/2)*78^(1/2))-40739440*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)
*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x^2*Ellip
ticPi(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*7
8^(1/2))-29309280*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5
)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x^2*EllipticE(1/31*31^(1/2)*11^(1/
2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+5544104*11^(1/2)*((5*x+7
)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))
^(1/2)*x*EllipticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(
1/2)*78^(1/2))-20369720*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*(
(2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x*EllipticPi(1/31*31^(1/2)*1
1^(1/2)*((5*x+7)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*78^(1/2))-14654640*11^
(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*
x-2)/(4*x+1))^(1/2)*x*EllipticE(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1
/2),1/39*31^(1/2)*78^(1/2))+693013*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)
*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticF(1/31*31
^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-2546215*11^
(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*
x-2)/(4*x+1))^(1/2)*EllipticPi(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/
2),124/55,1/39*31^(1/2)*78^(1/2))-1831830*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*
3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticE(
1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-1885
88400*x^3-20592000*x^4+454413960*x^2+574362360*x-524924400)/(120*x^4-182*x^
3-385*x^2+197*x+70)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x+7)^{\frac{5}{2}}}{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(5/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((5*x + 7)^(5/2)/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(5x+7)^{5/2}}{\sqrt{2-3x}\,\sqrt{4x+1}\,\sqrt{2x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + 7)^(5/2)/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
```

```
[Out] int((5*x + 7)^(5/2)/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**(5/2)/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Timed out
```

3.102 $\int \frac{(7+5x)^{3/2}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$

Optimal. Leaf size=469

$$\frac{65\sqrt{\frac{11}{23}}\sqrt{5x+7}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{8\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}-\frac{895\sqrt{\frac{11}{62}}\sqrt{2-3x}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{\frac{22}{23}}\sqrt{5x+7}}{\sqrt{2x-5}}\right),\frac{39}{62}\right)}{48\sqrt{-\frac{2-3x}{4x+1}}\sqrt{4x+1}}-\frac{5\sqrt{2-}}{}$$

```
[Out] -895/2976*(1/(529+506*(7+5*x)/(-5+2*x)))^(1/2)*(529+506*(7+5*x)/(-5+2*x))^(
1/2)*EllipticF(506^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/(529+506*(7+5*x)/(-5+
2*x))^(1/2),1/62*2418^(1/2))*682^(1/2)*(2-3*x)^(1/2)/((-2+3*x)/(1+4*x))^(1/
2)/(1+4*x)^(1/2)-4117/32736*(1/(529+506*(7+5*x)/(-5+2*x)))^(1/2)*(529+506*(
7+5*x)/(-5+2*x))^(1/2)*EllipticPi(506^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/(5
29+506*(7+5*x)/(-5+2*x))^(1/2),78/55,1/62*2418^(1/2))*(2-3*x)^(1/2)*682^(1/
2)/((-2+3*x)/(1+4*x))^(1/2)/(1+4*x)^(1/2)+23/132*(7+5*x)*EllipticPi(1/11*34
1^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2),55/124,1/62*2418^(1/2))*682^(1/2)*((2-3
*x)/(7+5*x))^(1/2)*((5-2*x)/(7+5*x))^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)-5/1
2*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)+65/184*(1/(4+2*(
1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)/(2-3*x))^(1/2)*EllipticF((1+4*x)^(1/2)*
2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/(2-3*x))^(1/2),1/23*I*897^(1/2))*253^(1/
2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7+5*x)/(5-2*x))^(1/2)+5/24*EllipticE(1/23
*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),1/39*I*897^(1/2))*429^(1/2)*(2-3*x)
^(1/2)*((7+5*x)/(5-2*x))^(1/2)/((2-3*x)/(5-2*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 469, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.270, Rules used = {166, 173, 176, 424, 170, 418, 165, 536, 539, 537}

$$-\frac{5\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{12\sqrt{2x-5}}+\frac{65\sqrt{\frac{11}{23}}\sqrt{5x+7}\,F\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{8\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}-\frac{895\sqrt{\frac{11}{62}}\sqrt{2-3x}\,F\left(\tan^{-1}\left(\frac{\sqrt{\frac{22}{23}}\sqrt{5x+}}{\sqrt{2x-5}}\right.\right.}{48\sqrt{-\frac{2-3x}{4x+1}}\sqrt{4x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x)^(3/2)/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (-5*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(12*Sqrt[-5 + 2*x]) + (5*Sqr
t[143/3]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*EllipticE[ArcSin[(Sqrt[39/
23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(8*Sqrt[(2 - 3*x)/(5 - 2*x)]*S
qrt[7 + 5*x]) + (65*Sqrt[11/23]*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x
]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(8*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 -
2*x)]) - (895*Sqrt[11/62]*Sqrt[2 - 3*x]*EllipticF[ArcTan[(Sqrt[22/23]*Sqrt[
7 + 5*x])/Sqrt[-5 + 2*x]], 39/62])/(48*Sqrt[-((2 - 3*x)/(1 + 4*x))]*Sqrt[1
+ 4*x]) + (23*Sqrt[31/22]*Sqrt[(2 - 3*x)/(7 + 5*x)]*Sqrt[(5 - 2*x)/(7 + 5*x
)]*(7 + 5*x)*EllipticPi[55/124, ArcSin[(Sqrt[31/11]*Sqrt[1 + 4*x])/Sqrt[7 +
 5*x]], 39/62])/(6*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]) - (4117*Sqrt[2 - 3*x]*Elli
pticPi[78/55, ArcTan[(Sqrt[22/23]*Sqrt[7 + 5*x])/Sqrt[-5 + 2*x]], 39/62])/(
48*Sqrt[682]*Sqrt[-((2 - 3*x)/(1 + 4*x))]*Sqrt[1 + 4*x])
```

Rule 165

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*(a + b*x)*Sqrt[((b*g -
 a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*
g - e*h)*(a + b*x))])/(Sqrt[c + d*x]*Sqrt[e + f*x]), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + ((b*c - a*d)*x^2)/(d*g - c*h)]*Sqrt[1 + ((b*e - a*f)*x^2)/(f*g
- e*h)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
```

```
f, g, h}, x]
```

Rule 166

```
Int[((a_.) + (b_.)*(x_))^(3/2)/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[b/d, Int[(Sqrt[a + b*x]
*Sqrt[c + d*x])/(Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] - Dist[(b*c - a*d)/d,
 Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] /; F
reeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 173

```
Int[(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(Sqrt[a + b*x]*Sqrt[c +
 d*x]*Sqrt[g + h*x])/(h*Sqrt[e + f*x]), x] + (-Dist[((d*e - c*f)*(f*g - e*h
))/(2*f*h), Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*(e + f*x)^(3/2)*Sqrt[g + h*x])
, x], x] + Dist[((d*e - c*f)*(b*f*g + b*e*h - 2*a*f*h))/(2*f^2*h), Int[1/(S
qrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(a*d
*f*h - b*(d*f*g + d*e*h - c*f*h))/(2*f^2*h), Int[Sqrt[e + f*x]/(Sqrt[a + b*
x]*Sqrt[c + d*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h},
 x]
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 536

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> -Dist[f/(b*e - a*f), Int[1/(Sqrt[c + d*x^2]*Sqrt[e + f
```

```
*x^2]), x], x] + Dist[b/(b*e - a*f), Int[Sqrt[e + f*x^2]/((a + b*x^2)*Sqrt[
c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[d/c, 0] && GtQ[f
/e, 0] &&  !SimplerSqrtQ[d/c, f/e]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 539

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[(c*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcT
an[Rt[d/c, 2]*x], 1 - (c*f)/(d*e)])/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c
*(e + f*x^2))/(e*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\int \frac{(7+5x)^{3/2}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx = -\left(\frac{5}{3}\int \frac{\sqrt{2-3x}\sqrt{7+5x}}{\sqrt{-5+2x}\sqrt{1+4x}}\,dx\right) + \frac{31}{3}\int \frac{\sqrt{7+5x}}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}\,dx$$

$$= -\frac{5\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{12\sqrt{-5+2x}} + \frac{895}{48}\int \frac{\sqrt{-5+2x}}{\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}\,dx + \frac{715}{16}$$

$$= -\frac{5\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{12\sqrt{-5+2x}} + \frac{23\sqrt{\frac{31}{22}}\sqrt{\frac{2-3x}{7+5x}}\sqrt{\frac{5-2x}{7+5x}}(7+5x)\Pi\left(\frac{55}{124};\sin^{-1}\right.}{6\sqrt{2-3x}\sqrt{-5+2x}}$$

$$= -\frac{5\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{12\sqrt{-5+2x}} + \frac{5\sqrt{\frac{143}{3}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right.\right.}{8\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}}$$

$$= -\frac{5\sqrt{2-3x}\sqrt{1+4x}\sqrt{7+5x}}{12\sqrt{-5+2x}} + \frac{5\sqrt{\frac{143}{3}}\sqrt{2-3x}\sqrt{\frac{7+5x}{5-2x}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{23}}\sqrt{1+4x}}{\sqrt{-5+2x}}\right.\right.}{8\sqrt{\frac{2-3x}{5-2x}}\sqrt{7+5x}}$$

Mathematica [A] time = 1.32, size = 347, normalized size = 0.74

$$\frac{\sqrt{2x-5}\left(-6969\sqrt{341}\sqrt{\frac{3x-2}{4x+1}}\sqrt{\frac{5x+7}{4x+1}}\left(8x^2-18x-5\right)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{22}{39}}\sqrt{\frac{5x+7}{4x+1}}\right),\frac{39}{62}\right)+6820\sqrt{341}\sqrt{\frac{3x-2}{4x+1}}\sqrt{\frac{5}{4}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(7 + 5*x)^(3/2)/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (Sqrt[-5 + 2*x]*(6820*Sqrt[341]*Sqrt[(-2 + 3*x)/(1 + 4*x)]*Sqrt[(7 + 5*x)/(
1 + 4*x)]*(-5 - 18*x + 8*x^2)*EllipticE[ArcSin[Sqrt[22/39]*Sqrt[(7 + 5*x)/(
1 + 4*x)]], 39/62] - 6969*Sqrt[341]*Sqrt[(-2 + 3*x)/(1 + 4*x)]*Sqrt[(7 + 5*
x)/(1 + 4*x)]*(-5 - 18*x + 8*x^2)*EllipticF[ArcSin[Sqrt[22/39]*Sqrt[(7 + 5*
x)/(1 + 4*x)]], 39/62] + Sqrt[(-5 + 2*x)/(1 + 4*x)]*(13640*Sqrt[2]*(70 - 83
*x - 53*x^2 + 30*x^3) + 9821*Sqrt[341]*Sqrt[(-2 + 3*x)/(1 + 4*x)]*(1 + 4*x)
^2*Sqrt[(-35 - 11*x + 10*x^2)/(1 + 4*x)^2]*EllipticPi[78/55, ArcSin[Sqrt[22
/39]*Sqrt[(7 + 5*x)/(1 + 4*x)]], 39/62])))/(16368*Sqrt[4 - 6*x]*((-5 + 2*x)
/(1 + 4*x))^(3/2)*(1 + 4*x)^(3/2)*Sqrt[7 + 5*x])
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{(5x+7)^{\frac{3}{2}}\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{24x^3-70x^2+21x+10}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(3/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-(5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(24*x^
3 - 70*x^2 + 21*x + 10), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x+7)^{\frac{3}{2}}}{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(3/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((5*x + 7)^(3/2)/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

maple [A] time = 0.03, size = 875, normalized size = 1.87

$$\sqrt{5x+7}\,\sqrt{-3x+2}\,\sqrt{2x-5}\,\sqrt{4x+1}\,\left(-514800x^3-68640\sqrt{11}\,\sqrt{\frac{5x+7}{4x+1}}\,\sqrt{3}\,\sqrt{13}\,\sqrt{\frac{2x-5}{4x+1}}\,\sqrt{\frac{3x-2}{4x+1}}\,x^2\,\text{EllipticE}\right($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x+7)^(3/2)/(-3*x+2)^(1/2)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] 1/20592*(5*x+7)^(1/2)*(-3*x+2)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(71024*11^
(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*
x-2)/(4*x+1))^(1/2)*x^2*EllipticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^
(1/2),1/39*31^(1/2)*78^(1/2))-157136*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/
2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x^2*EllipticPi(
1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*78^(1/2
))-68640*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1)
)^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x^2*EllipticE(1/31*31^(1/2)*11^(1/2)*((5*x+
7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+35512*11^(1/2)*((5*x+7)/(4*x+1))^
(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x*El
lipticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/
2))-78568*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1
))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x*EllipticPi(1/31*31^(1/2)*11^(1/2)*((5*x+
7)/(4*x+1))^(1/2),124/55,1/39*31^(1/2)*78^(1/2))-34320*11^(1/2)*((5*x+7)/(4
*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/
```

```
2)*x*EllipticE(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)
*78^(1/2))+4439*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/
(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticF(1/31*31^(1/2)*11^(1/2)*((5
*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-9821*11^(1/2)*((5*x+7)/(4*x+1)
)^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*El
lipticPi(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),124/55,1/39*31^(1/2
)*78^(1/2))-4290*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)
/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticE(1/31*31^(1/2)*11^(1/2)*((
5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))-514800*x^3+909480*x^2+1424280
*x-1201200)/(120*x^4-182*x^3-385*x^2+197*x+70)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5\,x+7)^{\frac{3}{2}}}{\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(3/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((5*x + 7)^(3/2)/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(5\,x+7)^{3/2}}{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + 7)^(3/2)/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
```

```
[Out] int((5*x + 7)^(3/2)/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**(3/2)/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Timed out
```

3.103 $\int \frac{\sqrt{7+5x}}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx$

Optimal. Leaf size=100

$$\frac{23\sqrt{\frac{2-3x}{5x+7}}\,\sqrt{\frac{5-2x}{5x+7}}\,(5x+7)\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{\sqrt{\frac{31}{11}}\,\sqrt{4x+1}}{\sqrt{5x+7}}\right)|\frac{39}{62}\right)}{2\sqrt{682}\,\sqrt{2-3x}\,\sqrt{2x-5}}$$

```
[Out] 23/1364*(7+5*x)*EllipticPi(1/11*341^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2),55/12
4,1/62*2418^(1/2))*682^(1/2)*((2-3*x)/(7+5*x))^(1/2)*((5-2*x)/(7+5*x))^(1/2
)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {165, 537}

$$\frac{23\sqrt{\frac{2-3x}{5x+7}}\,\sqrt{\frac{5-2x}{5x+7}}\,(5x+7)\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{\sqrt{\frac{31}{11}}\,\sqrt{4x+1}}{\sqrt{5x+7}}\right)|\frac{39}{62}\right)}{2\sqrt{682}\,\sqrt{2-3x}\,\sqrt{2x-5}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[7 + 5*x]/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (23*Sqrt[(2 - 3*x)/(7 + 5*x)]*Sqrt[(5 - 2*x)/(7 + 5*x)]*(7 + 5*x)*EllipticP
i[55/124, ArcSin[(Sqrt[31/11]*Sqrt[1 + 4*x])/Sqrt[7 + 5*x]], 39/62])/(2*Sqr
t[682]*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x])
```

Rule 165

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*(a + b*x)*Sqrt[((b*g -
 a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*
g - e*h)*(a + b*x))])/(Sqrt[c + d*x]*Sqrt[e + f*x]), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + ((b*c - a*d)*x^2)/(d*g - c*h)]*Sqrt[1 + ((b*e - a*f)*x^2)/(f*g
- e*h)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{7+5x}}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}}\,dx &= \frac{\left(23\sqrt{2}\,\sqrt{\frac{2-3x}{7+5x}}\,\sqrt{-\frac{-5+2x}{7+5x}}\,(7+5x)\right)\text{Subst}\left(\int \frac{1}{\left(4-5x^2\right)\sqrt{1-\frac{31x^2}{11}}\,\sqrt{1-\frac{39x^2}{22}}}\,dx, x, \ldots\right.}{11\sqrt{2-3x}\,\sqrt{-5+2x}}\\ &= \frac{23\sqrt{\frac{2-3x}{7+5x}}\,\sqrt{\frac{5-2x}{7+5x}}\,(7+5x)\Pi\left(\frac{55}{124};\sin^{-1}\left(\frac{\sqrt{\frac{31}{11}}\,\sqrt{1+4x}}{\sqrt{7+5x}}\right)|\frac{39}{62}\right)}{2\sqrt{682}\,\sqrt{2-3x}\,\sqrt{-5+2x}}\end{aligned}$$

Mathematica [A] time = 0.17, size = 95, normalized size = 0.95

$$-\frac{62\sqrt{4x+1}\sqrt{\frac{5-2x}{5x+7}}\Pi\left(-\frac{55}{69};\sin^{-1}\left(\frac{\sqrt{\frac{23}{11}}\sqrt{2-3x}}{\sqrt{5x+7}}\right)|-\frac{39}{23}\right)}{3\sqrt{253}\sqrt{2x-5}\sqrt{\frac{4x+1}{5x+7}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[7 + 5*x]/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]),x]
```

```
[Out] (-62*Sqrt[1 + 4*x]*Sqrt[(5 - 2*x)/(7 + 5*x)]*EllipticPi[-55/69, ArcSin[(Sqr
t[23/11]*Sqrt[2 - 3*x])/Sqrt[7 + 5*x]], -39/23])/(3*Sqrt[253]*Sqrt[-5 + 2*x
]*Sqrt[(1 + 4*x)/(7 + 5*x)])
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{5x+7}\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{24x^3-70x^2+21x+10}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(24*x^3
- 70*x^2 + 21*x + 10), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{5x+7}}{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(5*x + 7)/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

maple [B] time = 0.03, size = 170, normalized size = 1.70

$$\frac{23\left(\text{EllipticF}\left(\frac{\sqrt{31}\sqrt{11}\sqrt{\frac{5x+7}{4x+1}}}{31}, \frac{\sqrt{31}\sqrt{78}}{39}\right)-\text{EllipticPi}\left(\frac{\sqrt{31}\sqrt{11}\sqrt{\frac{5x+7}{4x+1}}}{31}, \frac{124}{55}, \frac{\sqrt{31}\sqrt{78}}{39}\right)\right)\sqrt{\frac{3x-2}{4x+1}}\sqrt{\frac{2x-5}{4x+1}}\sqrt{13}\sqrt{3}\sqrt{\frac{5x+7}{4x+1}}}{858\left(30x^3-53x^2-83x+70\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x+7)^(1/2)/(-3*x+2)^(1/2)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] 23/858*(EllipticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1
/2)*78^(1/2))-EllipticPi(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),124
/55,1/39*31^(1/2)*78^(1/2)))*((3*x-2)/(4*x+1))^(1/2)*((2*x-5)/(4*x+1))^(1/2
)*13^(1/2)*3^(1/2)*((5*x+7)/(4*x+1))^(1/2)*11^(1/2)*(4*x+1)^(3/2)*(2*x-5)^(
1/2)*(-3*x+2)^(1/2)*(5*x+7)^(1/2)/(30*x^3-53*x^2-83*x+70)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{5x+7}}{\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(5*x + 7)/(sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{5\,x+7}}{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + 7)^(1/2)/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)),x)
```

```
[Out] int((5*x + 7)^(1/2)/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{5x+7}}{\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7+5*x)**(1/2)/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral(sqrt(5*x + 7)/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)), x)
```

3.104 $\int \frac{1}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}}\,dx$

Optimal. Leaf size=71

$$\frac{2\sqrt{5x+7}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\,\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{\sqrt{253}\,\sqrt{2x-5}\,\sqrt{\frac{5x+7}{5-2x}}}$$

```
[Out] 2/253*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)/(2-3*x))^(1/2)*EllipticF
((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/(2-3*x))^(1/2),1/23*I*897
^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7+5*x)/(5-2*x))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {170, 418}

$$\frac{2\sqrt{5x+7}\,F\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\,\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{\sqrt{253}\,\sqrt{2x-5}\,\sqrt{\frac{5x+7}{5-2x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]),x]
```

```
[Out] (2*Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -
39/23])/(Sqrt[253]*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 - 2*x)])
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2-3x}\,\sqrt{-5+2x}\,\sqrt{1+4x}\,\sqrt{7+5x}}\,dx &= \frac{\left(\sqrt{\frac{2}{253}}\sqrt{-\frac{-5+2x}{2-3x}}\,\sqrt{7+5x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{2}}\,\sqrt{1+\frac{31x^2}{23}}}\,dx,x,\frac{\sqrt{1+4x}}{\sqrt{2-3x}}\right)}{\sqrt{-5+2x}\,\sqrt{\frac{7+5x}{2-3x}}}\\ &= \frac{2\sqrt{7+5x}\,F\left(\tan^{-1}\left(\frac{\sqrt{1+4x}}{\sqrt{2}\,\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{\sqrt{253}\,\sqrt{-5+2x}\,\sqrt{\frac{7+5x}{5-2x}}}\end{aligned}$$

Mathematica [A] time = 0.16, size = 90, normalized size = 1.27

$$-\frac{2\sqrt{4x+1}\sqrt{\frac{5-2x}{5x+7}}\ \text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{23}{11}}\sqrt{2-3x}}{\sqrt{5x+7}}\right),-\frac{39}{23}\right)}{\sqrt{253}\sqrt{2x-5}\sqrt{\frac{4x+1}{5x+7}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x]),x]
```

```
[Out] (-2*Sqrt[1 + 4*x]*Sqrt[(5 - 2*x)/(7 + 5*x)]*EllipticF[ArcSin[(Sqrt[23/11]*S
qrt[2 - 3*x])/Sqrt[7 + 5*x]], -39/23])/(Sqrt[253]*Sqrt[-5 + 2*x]*Sqrt[(1 +
4*x)/(7 + 5*x)])
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{5x+7}\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{120x^4-182x^3-385x^2+197x+70},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral(-sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(120*x^4
 - 182*x^3 - 385*x^2 + 197*x + 70), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{5x+7}\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate(1/(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

maple [A] time = 0.03, size = 134, normalized size = 1.89

$$\frac{2\sqrt{\frac{3x-2}{4x+1}}\sqrt{\frac{2x-5}{4x+1}}\sqrt{13}\sqrt{3}\sqrt{\frac{5x+7}{4x+1}}\sqrt{11}\,(4x+1)^{\frac{3}{2}}\sqrt{2x-5}\sqrt{-3x+2}\sqrt{5x+7}\ \text{EllipticF}\left(\frac{\sqrt{31}\sqrt{11}\sqrt{\frac{5x+7}{4x+1}}}{31},\frac{\sqrt{31}\sqrt{7}}{39}\right.}{429\left(30x^3-53x^2-83x+70\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x+7)^(1/2)/(-3*x+2)^(1/2)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] 2/429*EllipticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2
)*78^(1/2))*((3*x-2)/(4*x+1))^(1/2)*((2*x-5)/(4*x+1))^(1/2)*13^(1/2)*3^(1/2
)*((5*x+7)/(4*x+1))^(1/2)*11^(1/2)*(4*x+1)^(3/2)*(2*x-5)^(1/2)*(-3*x+2)^(1/
2)*(5*x+7)^(1/2)/(30*x^3-53*x^2-83*x+70)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{5x+7}\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)^(1/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(1/(sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{5\,x+7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(1/2)),x)
```

```
[Out] int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,\sqrt{5x+7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)**(1/2)/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)*sqrt(5*x + 7)), x)
```

3.105 $\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx$

Optimal. Leaf size=195

$$\frac{2\sqrt{\frac{3}{143}}(2-3x)\sqrt{\frac{5-2x}{2-3x}}\sqrt{-\frac{4x+1}{2-3x}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{11}{23}}\sqrt{5x+7}}{\sqrt{2-3x}}\right),-\frac{23}{39}\right)}{31\sqrt{2x-5}\sqrt{4x+1}}+\frac{10\sqrt{\frac{11}{39}}\sqrt{2-3x}\sqrt{\frac{5-2x}{5x+7}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{39}{22}}\sqrt{4x+}}{\sqrt{5x+7}}\right.\right.}{713\sqrt{2x-5}\sqrt{\frac{2-3x}{5x+7}}}$$

```
[Out] 2/4433*(2-3*x)*EllipticF(1/23*253^(1/2)*(7+5*x)^(1/2)/(2-3*x)^(1/2),1/39*I*
897^(1/2))*429^(1/2)*((5-2*x)/(2-3*x))^(1/2)*((-1-4*x)/(2-3*x))^(1/2)/(-5+2
*x)^(1/2)/(1+4*x)^(1/2)+10/27807*EllipticE(1/22*858^(1/2)*(1+4*x)^(1/2)/(7+
5*x)^(1/2),1/39*2418^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((5-2*x)/(7+5*x))^(1/2)
/(-5+2*x)^(1/2)/((2-3*x)/(7+5*x))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 270, normalized size of antiderivative = 1.38, number of steps used = 8, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {171, 170, 418, 176, 422, 492, 411}

$$-\frac{10\sqrt{2x-5}\sqrt{4x+1}}{897\sqrt{2-3x}\sqrt{5x+7}}+\frac{6\sqrt{5x+7}F\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{31\sqrt{253}\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}-\frac{5\sqrt{\frac{22}{31}}\sqrt{4x+1}F\left(\tan^{-1}\left(\frac{\sqrt{\frac{31}{11}}\sqrt{2x-5}}{\sqrt{5x+7}}\right)|\frac{39}{62}\right)}{1209\sqrt{2-3x}\sqrt{-\frac{4x+1}{2-3x}}}+10$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2)),x]
```

```
[Out] (-10*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(897*Sqrt[2 - 3*x]*Sqrt[7 + 5*x]) + (10*
Sqrt[22/31]*Sqrt[1 + 4*x]*EllipticE[ArcTan[(Sqrt[31/11]*Sqrt[-5 + 2*x])/Sqr
t[7 + 5*x]], 39/62])/(897*Sqrt[2 - 3*x]*Sqrt[-((1 + 4*x)/(2 - 3*x))]) + (6*
Sqrt[7 + 5*x]*EllipticF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/
23])/(31*Sqrt[253]*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]) - (5*Sqrt[22/3
1]*Sqrt[1 + 4*x]*EllipticF[ArcTan[(Sqrt[31/11]*Sqrt[-5 + 2*x])/Sqrt[7 + 5*x
]], 39/62])/(1209*Sqrt[2 - 3*x]*Sqrt[-((1 + 4*x)/(2 - 3*x))])
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 171

```
Int[1/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_
.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> -Dist[d/(b*c - a*d), Int[1
/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[b
/(b*c - a*d), Int[Sqrt[c + d*x]/((a + b*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x
]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
```

```
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,dx = \frac{3}{31}\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}\sqrt{7+5x}}\,dx + \frac{5}{31}\int \frac{}{\sqrt{-5+}}$$

$$= \frac{\left(5\sqrt{2}\sqrt{2-3x}\sqrt{\frac{1+4x}{7+5x}}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{31x^2}{11}}}{\sqrt{1+\frac{23x^2}{22}}}\,dx, x, \frac{\sqrt{-5+2x}}{\sqrt{7+5x}}\right)}{1209\sqrt{1+4x}\sqrt{-\frac{2-3x}{7+5x}}} + \frac{(3}{}$$

$$= \frac{6\sqrt{7+5x}\,F\left(\tan^{-1}\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right)}{31\sqrt{253}\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}} + \frac{\left(5\sqrt{2}\sqrt{2-3x}\sqrt{\frac{1+4x}{7+5x}}\right.}{1}$$

$$= -\frac{10\sqrt{-5+2x}\sqrt{1+4x}}{897\sqrt{2-3x}\sqrt{7+5x}} + \frac{6\sqrt{7+5x}\,F\left(\tan^{-1}\left(\frac{\sqrt{1+4x}}{\sqrt{2}\sqrt{2-3x}}\right)|-\frac{39}{23}\right.}{31\sqrt{253}\sqrt{-5+2x}\sqrt{\frac{7+5x}{5-2x}}}$$

$$= -\frac{10\sqrt{-5+2x}\sqrt{1+4x}}{897\sqrt{2-3x}\sqrt{7+5x}} + \frac{10\sqrt{\frac{22}{31}}\sqrt{1+4x}\,E\left(\tan^{-1}\left(\frac{\sqrt{\frac{31}{11}}\sqrt{-5+2}}{\sqrt{7+5x}}\right.\right.}{897\sqrt{2-3x}\sqrt{-\frac{1+4x}{2-3x}}}$$

Mathematica [A] time = 1.69, size = 237, normalized size = 1.22

$$-\frac{2\sqrt{2x-5}\sqrt{4x+1}\left(-23\sqrt{682}\sqrt{\frac{8x^2-18x-5}{(2-3x)^2}}\left(15x^2+11x-14\right)\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{2x-5}{3x-2}}\right),\frac{39}{62}\right)+1705\sqrt{\frac{5x}{3x}}\right.}{305877\sqrt{2-3x}\sqrt{5x+7}\sqrt{\frac{5x+7}{3x-2}}\,(}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(3/2)),x]
```

```
[Out] (-2*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(1705*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*
x + 8*x^2) - 55*Sqrt[682]*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x
 + 15*x^2)*EllipticE[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62
] - 23*Sqrt[682]*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*(-14 + 11*x + 15*x^2
)*EllipticF[ArcSin[Sqrt[31/39]*Sqrt[(-5 + 2*x)/(-2 + 3*x)]], 39/62]))/(3058
77*Sqrt[2 - 3*x]*Sqrt[7 + 5*x]*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x + 8*x^
2))
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{5x+7}\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{600x^5-70x^4-3199x^3-1710x^2+1729x+490}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)^(3/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral(-sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(600*x^5
 - 70*x^4 - 3199*x^3 - 1710*x^2 + 1729*x + 490), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(5\,x+7)^{\frac{3}{2}}\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)^(3/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate(1/((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x
)
```

maple [B] time = 0.03, size = 599, normalized size = 3.07

$$2\sqrt{5x+7}\,\sqrt{-3x+2}\,\sqrt{2x-5}\,\sqrt{4x+1}\left(880\sqrt{11}\,\sqrt{\frac{5x+7}{4x+1}}\,\sqrt{3}\,\sqrt{13}\,\sqrt{\frac{2x-5}{4x+1}}\,\sqrt{\frac{3x-2}{4x+1}}\,x^2\,\mathrm{EllipticE}\left(\frac{\sqrt{31}\,\sqrt{11}\,\sqrt{\frac{5x+7}{4x+1}}}{31}, \frac{\sqrt{3}\cdots}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x+7)^(3/2)/(-3*x+2)^(1/2)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] 2/305877*(5*x+7)^(1/2)*(-3*x+2)^(1/2)*(2*x-5)^(1/2)*(4*x+1)^(1/2)*(1104*11^
(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*
x-2)/(4*x+1))^(1/2)*x^2*EllipticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^
(1/2),1/39*31^(1/2)*78^(1/2))+880*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*
13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x^2*EllipticE(1/31
*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+552*11^(
1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x
-2)/(4*x+1))^(1/2)*x*EllipticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/
2),1/39*31^(1/2)*78^(1/2))+440*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^
(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x*EllipticE(1/31*31^(
1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+69*11^(1/2)*(
(5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4
*x+1))^(1/2)*EllipticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*
31^(1/2)*78^(1/2))+55*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2
*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticE(1/31*31^(1/2)*11^(1/
2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+7590*x^2-24035*x+12650)/
(120*x^4-182*x^3-385*x^2+197*x+70)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(5\,x+7)^{\frac{3}{2}}\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,\sqrt{-3\,x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)^(3/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(1/((5*x + 7)^(3/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{2-3\,x}\,\sqrt{4\,x+1}\,\sqrt{2\,x-5}\,{(5\,x+7)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(3/2)),x)
[Out] int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{2-3x}\,\sqrt{2x-5}\,\sqrt{4x+1}\,(5x+7)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)**(3/2)/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
[Out] Integral(1/(sqrt(2 - 3*x)*sqrt(2*x - 5)*sqrt(4*x + 1)*(5*x + 7)**(3/2)), x)
```

3.106 $\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}\,dx$

Optimal. Leaf size=288

$$\frac{103964\sqrt{5x+7}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{4x+1}}{\sqrt{2}\sqrt{2-3x}}\right),-\frac{39}{23}\right)}{1918683\sqrt{253}\sqrt{2x-5}\sqrt{\frac{5x+7}{5-2x}}}+\frac{358120\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{2319687747\sqrt{2x-5}}-\frac{895300\sqrt{2-3x}\sqrt{2x-5}}{2319687747\sqrt{5x-}}$$

```
[Out] -50/83421*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(3/2)-895300/2
319687747*(2-3*x)^(1/2)*(-5+2*x)^(1/2)*(1+4*x)^(1/2)/(7+5*x)^(1/2)+358120/2
319687747*(2-3*x)^(1/2)*(1+4*x)^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)+103964/4
85426799*(1/(4+2*(1+4*x)/(2-3*x)))^(1/2)*(4+2*(1+4*x)/(2-3*x))^(1/2)*Ellipt
icF((1+4*x)^(1/2)*2^(1/2)/(2-3*x)^(1/2)/(4+2*(1+4*x)/(2-3*x))^(1/2),1/23*I*
897^(1/2))*253^(1/2)*(7+5*x)^(1/2)/(-5+2*x)^(1/2)/((7+5*x)/(5-2*x))^(1/2)-1
79060/2319687747*EllipticE(1/23*897^(1/2)*(1+4*x)^(1/2)/(-5+2*x)^(1/2),1/39
*I*897^(1/2))*429^(1/2)*(2-3*x)^(1/2)*((7+5*x)/(5-2*x))^(1/2)/((2-3*x)/(5-2
*x))^(1/2)/(7+5*x)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {172, 1599, 1602, 12, 170, 418, 176, 424}

$$\frac{358120\sqrt{2-3x}\sqrt{4x+1}\sqrt{5x+7}}{2319687747\sqrt{2x-5}}-\frac{895300\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{2319687747\sqrt{5x+7}}-\frac{50\sqrt{2-3x}\sqrt{2x-5}\sqrt{4x+1}}{83421(5x+7)^{3/2}}+\frac{103964\sqrt{5\ldots}}{1918\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2)),x]
```

```
[Out] (-50*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(83421*(7 + 5*x)^(3/2)) -
(895300*Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x])/(2319687747*Sqrt[7 + 5*
x]) + (358120*Sqrt[2 - 3*x]*Sqrt[1 + 4*x]*Sqrt[7 + 5*x])/(2319687747*Sqrt[-
5 + 2*x]) - (179060*Sqrt[11/39]*Sqrt[2 - 3*x]*Sqrt[(7 + 5*x)/(5 - 2*x)]*Ell
ipticE[ArcSin[(Sqrt[39/23]*Sqrt[1 + 4*x])/Sqrt[-5 + 2*x]], -23/39])/(594791
73*Sqrt[(2 - 3*x)/(5 - 2*x)]*Sqrt[7 + 5*x]) + (103964*Sqrt[7 + 5*x]*Ellipti
cF[ArcTan[Sqrt[1 + 4*x]/(Sqrt[2]*Sqrt[2 - 3*x])], -39/23])/(1918683*Sqrt[25
3]*Sqrt[-5 + 2*x]*Sqrt[(7 + 5*x)/(5 - 2*x)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 172

```
Int[((a_.) + (b_.)*(x_))^(m_)/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*
(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(b^2*(a + b*x)^(m + 1)*S
```

```
qrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((m + 1)*(b*c - a*d)*(b*e - a*f)*
(b*g - a*h)), x] - Dist[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)),
Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[2*
a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(
d*e*g + c*f*g + c*e*h) - 2*b*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h +
c*f*h))*x + d*f*h*(2*m + 5)*b^2*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1599

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
(A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]
)/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)), x] - Dist[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h)), Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*Sq
rt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d*
f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*e
*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
- b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rule 1602

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[(C*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*f*h*Sqrt[c + d*
x]), x] + (Dist[1/(2*b*d*f*h), Int[(1*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f
*h) + (2*b*B*d*f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x])/(Sqrt[
a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(C*(d*e
- c*f)*(d*g - c*h))/(2*b*d*f*h), Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e
+ f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C},
x]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{5/2}}\,dx = -\frac{50\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{83421(7+5x)^{3/2}} + \frac{\int \frac{11928-4270x}{\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}(7+5x)^{3/2}}\,d}{83421}$$

$$= -\frac{50\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{83421(7+5x)^{3/2}} - \frac{895300\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1}}{2319687747\sqrt{7+5x}}$$

$$= -\frac{50\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{83421(7+5x)^{3/2}} - \frac{895300\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1}}{2319687747\sqrt{7+5x}}$$

$$= -\frac{50\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{83421(7+5x)^{3/2}} - \frac{895300\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1}}{2319687747\sqrt{7+5x}}$$

$$= -\frac{50\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{83421(7+5x)^{3/2}} - \frac{895300\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1}}{2319687747\sqrt{7+5x}}$$

$$= -\frac{50\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1+4x}}{83421(7+5x)^{3/2}} - \frac{895300\sqrt{2-3x}\sqrt{-5+2x}\sqrt{1}}{2319687747\sqrt{7+5x}}$$

Mathematica [A] time = 1.83, size = 246, normalized size = 0.85

$$-\frac{2\sqrt{2x-5}\sqrt{4x+1}\left(-28819\sqrt{682}(3x-2)\sqrt{\frac{8x^2-18x-5}{(2-3x)^2}}(5x+7)^2\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{31}{39}}\sqrt{\frac{2x-5}{3x-2}}\right),\frac{39}{62}\right)-984830\sqrt{6}\right.}{25516565217\sqrt{2-3x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 3*x]*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(7 + 5*x)^(5/2)),x]
```

```
[Out] (-2*Sqrt[-5 + 2*x]*Sqrt[1 + 4*x]*(1705*Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-671560
- 2797991*x - 294854*x^2 + 608600*x^3) - 984830*Sqrt[682]*(-2 + 3*x)*(7 + 5
*x)^2*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticE[ArcSin[Sqrt[31/39]*Sq
rt[(-5 + 2*x)/(-2 + 3*x)]], 39/62] - 28819*Sqrt[682]*(-2 + 3*x)*(7 + 5*x)^2
*Sqrt[(-5 - 18*x + 8*x^2)/(2 - 3*x)^2]*EllipticF[ArcSin[Sqrt[31/39]*Sqrt[(-
5 + 2*x)/(-2 + 3*x)]], 39/62]))/(25516565217*Sqrt[2 - 3*x]*(7 + 5*x)^(3/2)*
Sqrt[(7 + 5*x)/(-2 + 3*x)]*(-5 - 18*x + 8*x^2))
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{5x+7}\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}{3000x^6+3850x^5-16485x^4-30943x^3-3325x^2+14553x+3430}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)^(5/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral(-sqrt(5*x + 7)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)/(3000*x^
6 + 3850*x^5 - 16485*x^4 - 30943*x^3 - 3325*x^2 + 14553*x + 3430), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(5x+7)^{\frac{5}{2}}\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)^(5/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate(1/((5*x + 7)^(5/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x
)
```

maple [B] time = 0.04, size = 786, normalized size = 2.73

$$2\left(78786400\sqrt{11}\,\sqrt{\frac{5x+7}{4x+1}}\,\sqrt{3}\,\sqrt{13}\,\sqrt{\frac{2x-5}{4x+1}}\,\sqrt{\frac{3x-2}{4x+1}}\,x^{3}\,\mathrm{EllipticE}\left(\frac{\sqrt{31}\,\sqrt{11}\,\sqrt{\frac{5x+7}{4x+1}}}{31},\frac{\sqrt{31}\,\sqrt{78}}{39}\right)+50128960\sqrt{11}\,\sqrt{\frac{5x+}{4x+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x+7)^(5/2)/(-3*x+2)^(1/2)/(2*x-5)^(1/2)/(4*x+1)^(1/2),x)
```

```
[Out] 2/25516565217*(50128960*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*(
(2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticF(1/31*31^(1/2)*11^(
1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))*x^3+78786400*11^(1/2)*
((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(
4*x+1))^(1/2)*EllipticE(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39
*31^(1/2)*78^(1/2))*x^3+95245024*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*1
3^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x^2*EllipticF(1/31*
31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+149694160
*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*
((3*x-2)/(4*x+1))^(1/2)*x^2*EllipticE(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+
1))^(1/2),1/39*31^(1/2)*78^(1/2))+38223332*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)
*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x*Ellipti
cF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+6
0074630*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))
^(1/2)*((3*x-2)/(4*x+1))^(1/2)*x*EllipticE(1/31*31^(1/2)*11^(1/2)*((5*x+7)/
(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+4386284*11^(1/2)*((5*x+7)/(4*x+1))^(
1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1))^(1/2)*((3*x-2)/(4*x+1))^(1/2)*Ellip
ticF(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))
+6893810*11^(1/2)*((5*x+7)/(4*x+1))^(1/2)*3^(1/2)*13^(1/2)*((2*x-5)/(4*x+1)
)^(1/2)*((3*x-2)/(4*x+1))^(1/2)*EllipticE(1/31*31^(1/2)*11^(1/2)*((5*x+7)/(
4*x+1))^(1/2),1/39*31^(1/2)*78^(1/2))+496006500*x^3-665223020*x^2-204062589
5*x+1509107050)*(4*x+1)^(1/2)*(2*x-5)^(1/2)*(-3*x+2)^(1/2)/(120*x^4-182*x^3
-385*x^2+197*x+70)/(5*x+7)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(5x+7)^{\frac{5}{2}}\sqrt{4x+1}\sqrt{2x-5}\sqrt{-3x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)^(5/2)/(2-3*x)^(1/2)/(-5+2*x)^(1/2)/(1+4*x)^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(1/((5*x + 7)^(5/2)*sqrt(4*x + 1)*sqrt(2*x - 5)*sqrt(-3*x + 2)), x
)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{2-3x}\,\sqrt{4x+1}\,\sqrt{2x-5}\,(5x+7)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(5/2)),x)
[Out] int(1/((2 - 3*x)^(1/2)*(4*x + 1)^(1/2)*(2*x - 5)^(1/2)*(5*x + 7)^(5/2)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(7+5*x)**(5/2)/(2-3*x)**(1/2)/(-5+2*x)**(1/2)/(1+4*x)**(1/2),x)
[Out] Timed out
```

3.107 $\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal. Leaf size=968

$$-\frac{\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{\frac{(de-cf)(g+hx)}{(dg-ch)(e+fx)}}\,E\left(\sin^{-1}\left(\frac{\sqrt{fg-eh}\sqrt{c+dx}}{\sqrt{dg-ch}\sqrt{e+fx}}\right)|\frac{(be-af)(dg-ch)}{(bc-ad)(fg-eh)}\right)b}{dfh\sqrt{-\frac{(de-cf)(a+bx)}{(bc-ad)(e+fx)}}\sqrt{g+hx}}+\frac{(de-cf)(bfg+beh-2a}{}$$

```
[Out] b*(a*d*f*h-b*(-c*f*h+d*e*h+d*f*g))*(f*x+e)*EllipticPi((-a*f+b*e)^(1/2)*(h*x
+g)^(1/2)/(-a*h+b*g)^(1/2)/(f*x+e)^(1/2),f*(-a*h+b*g)/(-a*f+b*e)/h,((-c*f+d
*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))*(-a*h+b*g)^(1/2)*((-e*h+f*g)*(
b*x+a)/(-a*h+b*g)/(f*x+e))^(1/2)*((-e*h+f*g)*(d*x+c)/(-c*h+d*g)/(f*x+e))^(1
/2)/d/f^2/h^2/(-a*f+b*e)^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)-2*(b*x+a)*Ellipt
icPi((-a*d+b*c)^(1/2)*(h*x+g)^(1/2)/(c*h-d*g)^(1/2)/(b*x+a)^(1/2),-b*(-c*h+
d*g)/(-a*d+b*c)/h,((-a*f+b*e)*(-c*h+d*g)/(-a*d+b*c)/(-e*h+f*g))^(1/2))*(-a*
d+b*c)^(1/2)*(c*h-d*g)^(1/2)*((-a*h+b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))^(1/2)*
((-a*h+b*g)*(f*x+e)/(-e*h+f*g)/(b*x+a))^(1/2)/d/h/(d*x+c)^(1/2)/(f*x+e)^(1/
2)+b*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(h*x+g)^(1/2)/d/h/(f*x+e)^(1/2)+b*(-c*f+d*
e)*(-2*a*f*h+b*e*h+b*f*g)*EllipticF((-a*h+b*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*
g)^(1/2)/(b*x+a)^(1/2),(-(-a*d+b*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*h+b*g))^(1/2)
)*((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^(1/2)*(h*x+g)^(1/2)/d/f^2/h/(-a*h
+b*g)^(1/2)/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2)/(-(-a*f+b*e)*(h*x+g)/(-e*h+f*g)/
(b*x+a))^(1/2)-b*EllipticE((-e*h+f*g)^(1/2)*(d*x+c)^(1/2)/(-c*h+d*g)^(1/2)/
(f*x+e)^(1/2),((-a*f+b*e)*(-c*h+d*g)/(-a*d+b*c)/(-e*h+f*g))^(1/2))*(-c*h+d*
g)^(1/2)*(-e*h+f*g)^(1/2)*(b*x+a)^(1/2)*((-c*f+d*e)*(h*x+g)/(-c*h+d*g)/(f*x
+e))^(1/2)/d/f/h/(-(-c*f+d*e)*(b*x+a)/(-a*d+b*c)/(f*x+e))^(1/2)/(h*x+g)^(1/
2)
```

Rubi [A] time = 0.88, antiderivative size = 968, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.216$, Rules used = {166, 173, 176, 424, 170, 419, 165, 537}

$$-\frac{\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{\frac{(de-cf)(g+hx)}{(dg-ch)(e+fx)}}\,E\left(\sin^{-1}\left(\frac{\sqrt{fg-eh}\sqrt{c+dx}}{\sqrt{dg-ch}\sqrt{e+fx}}\right)|\frac{(be-af)(dg-ch)}{(bc-ad)(fg-eh)}\right)b}{dfh\sqrt{-\frac{(de-cf)(a+bx)}{(bc-ad)(e+fx)}}\sqrt{g+hx}}+\frac{(de-cf)(bfg+beh-2a}{df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (b*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[g + h*x])/(d*h*Sqrt[e + f*x]) - (b*Sqrt
[d*g - c*h]*Sqrt[f*g - e*h]*Sqrt[a + b*x]*Sqrt[((d*e - c*f)*(g + h*x))/((d*
g - c*h)*(e + f*x))]*EllipticE[ArcSin[(Sqrt[f*g - e*h]*Sqrt[c + d*x])/(Sqrt
[d*g - c*h]*Sqrt[e + f*x])], ((b*e - a*f)*(d*g - c*h))/((b*c - a*d)*(f*g -
e*h))])/(d*f*h*Sqrt[-(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))]*Sqr
t[g + h*x]) + (b*(d*e - c*f)*(b*f*g + b*e*h - 2*a*f*h)*Sqrt[((b*e - a*f)*(c
 + d*x))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g
- a*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g
 - e*h))/((d*e - c*f)*(b*g - a*h)))])/(d*f^2*h*Sqrt[b*g - a*h]*Sqrt[f*g - e
*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])
 + (b*Sqrt[b*g - a*h]*(a*d*f*h - b*(d*f*g + d*e*h - c*f*h))*Sqrt[((f*g - e*
h)*(a + b*x))/((b*g - a*h)*(e + f*x))]*Sqrt[((f*g - e*h)*(c + d*x))/((d*g -
 c*h)*(e + f*x))]*(e + f*x)*EllipticPi[(f*(b*g - a*h))/((b*e - a*f)*h), Arc
Sin[(Sqrt[b*e - a*f]*Sqrt[g + h*x])/(Sqrt[b*g - a*h]*Sqrt[e + f*x])], ((d*e
 - c*f)*(b*g - a*h))/((b*e - a*f)*(d*g - c*h))])/(d*f^2*Sqrt[b*e - a*f]*h^2
*Sqrt[a + b*x]*Sqrt[c + d*x]) - (2*Sqrt[b*c - a*d]*Sqrt[-(d*g) + c*h]*(a +
```

```
b*x)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h
)*(e + f*x))/((f*g - e*h)*(a + b*x))]*EllipticPi[-((b*(d*g - c*h))/((b*c -
a*d)*h)), ArcSin[(Sqrt[b*c - a*d]*Sqrt[g + h*x])/(Sqrt[-(d*g) + c*h]*Sqrt[a
 + b*x])], ((b*e - a*f)*(d*g - c*h))/((b*c - a*d)*(f*g - e*h))])/(d*h*Sqrt[
c + d*x]*Sqrt[e + f*x])
```

Rule 165

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*(a + b*x)*Sqrt[((b*g -
 a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*
g - e*h)*(a + b*x))])/(Sqrt[c + d*x]*Sqrt[e + f*x]), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + ((b*c - a*d)*x^2)/(d*g - c*h)]*Sqrt[1 + ((b*e - a*f)*x^2)/(f*g
- e*h)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 166

```
Int[((a_.) + (b_.)*(x_))^(3/2)/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[b/d, Int[(Sqrt[a + b*x]
*Sqrt[c + d*x])/(Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] - Dist[(b*c - a*d)/d,
 Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] /; F
reeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 173

```
Int[(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)])/(Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(Sqrt[a + b*x]*Sqrt[c +
 d*x]*Sqrt[g + h*x])/(h*Sqrt[e + f*x]), x] + (-Dist[((d*e - c*f)*(f*g - e*h
))/(2*f*h), Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*(e + f*x)^(3/2)*Sqrt[g + h*x])
, x], x] + Dist[((d*e - c*f)*(b*f*g + b*e*h - 2*a*f*h))/(2*f^2*h), Int[1/(S
qrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[(a*d
*f*h - b*(d*f*g + d*e*h - c*f*h))/(2*f^2*h), Int[Sqrt[e + f*x]/(Sqrt[a + b*
x]*Sqrt[c + d*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h},
 x]
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
```

```
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{b\int \frac{\sqrt{a+bx}\sqrt{c+dx}}{\sqrt{e+fx}\sqrt{g+hx}}\,dx}{d} - \frac{(bc-ad)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{d}$$

$$= \frac{b\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}}{dh\sqrt{e+fx}} - \frac{(b(de-cf)(fg-eh))\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\,(e+fx)^{3/2}\sqrt{g+hx}}\,d}{2dfh}$$

$$= \frac{b\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}}{dh\sqrt{e+fx}} - \frac{2\sqrt{bc-ad}\sqrt{-dg+ch}\,(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}}{}$$

$$= \frac{b\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}}{dh\sqrt{e+fx}} - \frac{b\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{\frac{(de-cf)(g+hx)}{(dg-ch)(e+fx)}}}{dfh\sqrt{-\frac{(de-cf)(a+b}{(bc-ad)(e+f}}}$$

Mathematica [B] time = 14.16, size = 6638, normalized size = 6.86

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x)^(3/2)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
[Out] Result too large to show
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algori
thm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{3}{2}}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algori
thm="giac")
[Out] integrate((b*x + a)^(3/2)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

maple [B] time = 0.10, size = 16526, normalized size = 17.07

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{3}{2}}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algori
thm="maxima")
[Out] integrate((b*x + a)^(3/2)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{3/2}}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)),x)
[Out] int((a + b*x)^(3/2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{3}{2}}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
[Out] Integral((a + b*x)**(3/2)/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

3.108 $\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+hx}}\,dx$

Optimal. Leaf size=228

$$\frac{2(a+bx)\sqrt{ch-dg}\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\,\Pi\left(-\frac{b(dg-ch)}{(bc-ad)h};\sin^{-1}\left(\frac{\sqrt{bc-ad}\,\sqrt{g+hx}}{\sqrt{ch-dg}\,\sqrt{a+bx}}\right)|\frac{(be-af)(dg-ch)}{(bc-ad)(fg-eh)}\right)}{h\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{bc-ad}}$$

```
[Out] 2*(b*x+a)*EllipticPi((-a*d+b*c)^(1/2)*(h*x+g)^(1/2)/(c*h-d*g)^(1/2)/(b*x+a)
^(1/2),-b*(-c*h+d*g)/(-a*d+b*c)/h,((-a*f+b*e)*(-c*h+d*g)/(-a*d+b*c)/(-e*h+f
*g))^(1/2))*(c*h-d*g)^(1/2)*((-a*h+b*g)*(d*x+c)/(-c*h+d*g)/(b*x+a))^(1/2)*(
(-a*h+b*g)*(f*x+e)/(-e*h+f*g)/(b*x+a))^(1/2)/h/(-a*d+b*c)^(1/2)/(d*x+c)^(1/
2)/(f*x+e)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {165, 537}

$$\frac{2(a+bx)\sqrt{ch-dg}\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\,\Pi\left(-\frac{b(dg-ch)}{(bc-ad)h};\sin^{-1}\left(\frac{\sqrt{bc-ad}\,\sqrt{g+hx}}{\sqrt{ch-dg}\,\sqrt{a+bx}}\right)|\frac{(be-af)(dg-ch)}{(bc-ad)(fg-eh)}\right)}{h\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*Sqrt[-(d*g) + c*h]*(a + b*x)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(
a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*EllipticPi
[-((b*(d*g - c*h))/((b*c - a*d)*h)), ArcSin[(Sqrt[b*c - a*d]*Sqrt[g + h*x])
/(Sqrt[-(d*g) + c*h]*Sqrt[a + b*x])], ((b*e - a*f)*(d*g - c*h))/((b*c - a*d
)*(f*g - e*h))])/(Sqrt[b*c - a*d]*h*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rule 165

```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*(a + b*x)*Sqrt[((b*g -
 a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*
g - e*h)*(a + b*x))])/(Sqrt[c + d*x]*Sqrt[e + f*x]), Subst[Int[1/((h - b*x^
2)*Sqrt[1 + ((b*c - a*d)*x^2)/(d*g - c*h)]*Sqrt[1 + ((b*e - a*f)*x^2)/(f*g
- e*h)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e,
f, g, h}, x]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+hx}}\,dx = \frac{\left(2(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\,\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\right)\text{Subst}\left(\int \frac{1}{\left(h-bx^2\right)\sqrt{1+\frac{(bc-ad)x^2}{dg-ch}}\,\sqrt{1+\frac{(be-af)x}{fg-eh}}}\right.}{\sqrt{c+dx}\,\sqrt{e+fx}}$$

$$= \frac{2\sqrt{-dg+ch}\,(a+bx)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\,\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}\,\Pi\left(-\frac{b(dg-ch)}{(bc-ad)h};\sin^{-1}\left(\frac{\sqrt{bc-ad}}{\sqrt{-dg+ch}}\right.\right.}{\sqrt{bc-ad}\,h\sqrt{c+dx}\,\sqrt{e+fx}}$$

Mathematica [B] time = 5.58, size = 584, normalized size = 2.56

$$-\frac{2(c+dx)^{3/2}\sqrt{\frac{(a+bx)(dg-ch)}{(c+dx)(bg-ah)}}\left(\frac{ad(g+hx)\sqrt{\frac{(e+fx)(dg-ch)}{(c+dx)(fg-eh)}}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{(g+hx)(cf-de)}{(c+dx)(fg-eh)}}\right),\frac{(bc-ad)(eh-fg)}{(bg-ah)(de-cf)}\right)}{(c+dx)(dg-ch)\sqrt{\frac{(g+hx)(cf-de)}{(c+dx)(fg-eh)}}}+\frac{bc(g+hx)\sqrt{\frac{(e+fx)(dg-ch)}{(c+dx)(fg-eh)}}\,\text{EllipticF}}{(c+dx)(ch-}\right.}{d\sqrt{a+bx}\,\sqrt{e+fx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (-2*Sqrt[((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x))]*(c + d*x)^(3/2)*(
(a*d*Sqrt[((d*g - c*h)*(e + f*x))/((f*g - e*h)*(c + d*x))]*(g + h*x)*Ellipt
icF[ArcSin[Sqrt[((-(d*e) + c*f)*(g + h*x))/((f*g - e*h)*(c + d*x))]], ((b*c
 - a*d)*(-(f*g) + e*h))/((d*e - c*f)*(b*g - a*h))])/((d*g - c*h)*(c + d*x)*
Sqrt[((-(d*e) + c*f)*(g + h*x))/((f*g - e*h)*(c + d*x))]) + (b*c*Sqrt[((d*g
 - c*h)*(e + f*x))/((f*g - e*h)*(c + d*x))]*(g + h*x)*EllipticF[ArcSin[Sqrt
[((-(d*e) + c*f)*(g + h*x))/((f*g - e*h)*(c + d*x))]], ((b*c - a*d)*(-(f*g)
 + e*h))/((d*e - c*f)*(b*g - a*h))])/((-(d*g) + c*h)*(c + d*x)*Sqrt[((-(d*e
) + c*f)*(g + h*x))/((f*g - e*h)*(c + d*x))]) + (b*(f*g - e*h)*Sqrt[-(((d*e
 - c*f)*(d*g - c*h)*(e + f*x)*(g + h*x))/((f*g - e*h)^2*(c + d*x)^2))]*Elli
pticPi[(d*(-(f*g) + e*h))/((d*e - c*f)*h), ArcSin[Sqrt[((-(d*e) + c*f)*(g +
 h*x))/((f*g - e*h)*(c + d*x))]], ((b*c - a*d)*(-(f*g) + e*h))/((d*e - c*f)
*(b*g - a*h))])/((d*e - c*f)*h)))/(d*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h
*x])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algori
thm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algori
thm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.06, size = 2465, normalized size = 10.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

```
[Out] 2*(EllipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*
g)/(c*h-d*g)/(a*f-b*e))^(1/2))*x^2*a^2*f^3*h^2-EllipticF(((a*f-b*e)*(h*x+g)
/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*
x^2*a*b*e*f^2*h^2-EllipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c
*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*x^2*a*b*f^3*g*h+EllipticF(((a
*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*
f-b*e))^(1/2))*x^2*b^2*e*f^2*g*h+EllipticPi(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f
*x+e))^(1/2),(a*h-b*g)*f/h/(a*f-b*e),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*
e))^(1/2))*x^2*a*b*e*f^2*h^2-EllipticPi(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e
))^(1/2),(a*h-b*g)*f/h/(a*f-b*e),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^
(1/2))*x^2*a*b*f^3*g*h-EllipticPi(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/
2),(a*h-b*g)*f/h/(a*f-b*e),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))
*x^2*b^2*e*f^2*g*h+EllipticPi(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),(
a*h-b*g)*f/h/(a*f-b*e),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*x^2
*b^2*f^3*g^2+2*EllipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-
d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*x*a^2*e*f^2*h^2-2*EllipticF(((a*
f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f
-b*e))^(1/2))*x*a*b*e^2*f*h^2-2*EllipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x
+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*x*a*b*e*f^2*g*h
+2*EllipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*
g)/(c*h-d*g)/(a*f-b*e))^(1/2))*x*b^2*e^2*f*g*h+2*EllipticPi(((a*f-b*e)*(h*x
+g)/(a*h-b*g)/(f*x+e))^(1/2),(a*h-b*g)*f/h/(a*f-b*e),((c*f-d*e)*(a*h-b*g)/(
c*h-d*g)/(a*f-b*e))^(1/2))*x*a*b*e^2*f*h^2-2*EllipticPi(((a*f-b*e)*(h*x+g)/
(a*h-b*g)/(f*x+e))^(1/2),(a*h-b*g)*f/h/(a*f-b*e),((c*f-d*e)*(a*h-b*g)/(c*h-
d*g)/(a*f-b*e))^(1/2))*x*a*b*e*f^2*g*h-2*EllipticPi(((a*f-b*e)*(h*x+g)/(a*h
-b*g)/(f*x+e))^(1/2),(a*h-b*g)*f/h/(a*f-b*e),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)
/(a*f-b*e))^(1/2))*x*b^2*e^2*f*g*h+2*EllipticPi(((a*f-b*e)*(h*x+g)/(a*h-b*g
)/(f*x+e))^(1/2),(a*h-b*g)*f/h/(a*f-b*e),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*
f-b*e))^(1/2))*x*b^2*e*f^2*g^2+EllipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+
e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*a^2*e^2*f*h^2-El
lipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c
*h-d*g)/(a*f-b*e))^(1/2))*a*b*e^3*h^2-EllipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g
)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*a*b*e^2*f
*g*h+EllipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-
b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*b^2*e^3*g*h+EllipticPi(((a*f-b*e)*(h*x+g)/
(a*h-b*g)/(f*x+e))^(1/2),(a*h-b*g)*f/h/(a*f-b*e),((c*f-d*e)*(a*h-b*g)/(c*h-
d*g)/(a*f-b*e))^(1/2))*a*b*e^3*h^2-EllipticPi(((a*f-b*e)*(h*x+g)/(a*h-b*g)/
(f*x+e))^(1/2),(a*h-b*g)*f/h/(a*f-b*e),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-
b*e))^(1/2))*a*b*e^2*f*g*h-EllipticPi(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))
^(1/2),(a*h-b*g)*f/h/(a*f-b*e),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1
/2))*b^2*e^3*g*h+EllipticPi(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),(a*
h-b*g)*f/h/(a*f-b*e),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*b^2*e
^2*f*g^2)*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)*((e*h-f*g)*(d*x+c)/(c
*h-d*g)/(f*x+e))^(1/2)*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)/h/f/(h*x
+g)^(1/2)/(f*x+e)^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)/(e*h-f*g)/(a*f-b*e)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx+a}}{\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate(sqrt(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+b\,x}}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)),x)
```

```
[Out] int((a + b*x)^(1/2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+hx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x)/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

3.109 $\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal. Leaf size=161

$$-\frac{2\sqrt{e+fx}\sqrt{\frac{(c+dx)(fg-eh)}{(e+fx)(dg-ch)}}\,\text{EllipticF}\left(\tan^{-1}\left(\frac{\sqrt{g+hx}\sqrt{be-af}}{\sqrt{a+bx}\sqrt{fg-eh}}\right),\frac{(bg-ah)(de-cf)}{(be-af)(dg-ch)}\right)}{\sqrt{c+dx}\sqrt{be-af}\sqrt{fg-eh}}$$

```
[Out] -2*(1/(1+(-a*f+b*e)*(h*x+g)/(-e*h+f*g)/(b*x+a)))^(1/2)*(1+(-a*f+b*e)*(h*x+g
)/(-e*h+f*g)/(b*x+a))^(1/2)*EllipticF((-a*f+b*e)^(1/2)*(h*x+g)^(1/2)/(-e*h+
f*g)^(1/2)/(b*x+a)^(1/2)/(1+(-a*f+b*e)*(h*x+g)/(-e*h+f*g)/(b*x+a))^(1/2),((
-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))*((-e*h+f*g)*(d*x+c)/(-c*
h+d*g)/(f*x+e))^(1/2)*(f*x+e)^(1/2)/(-a*f+b*e)^(1/2)/(-e*h+f*g)^(1/2)/(d*x+
c)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 198, normalized size of antiderivative = 1.23, number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {170, 419}

$$\frac{2\sqrt{g+hx}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\,F\left(\sin^{-1}\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right)|-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{\sqrt{c+dx}\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (2*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*Elli
pticF[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x]
)], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*g - a*h)))])/(Sqrt[b*g - a*
h]*Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h
)*(a + b*x)))])
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{\left(2\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{(bc-ad)x^2}{de-cf}}\sqrt{1-\frac{(bg-ah)x^2}{fg-eh}}}\,dx, x, \frac{\sqrt{e}}{\sqrt{a}}\right.}{(fg-eh)\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}}$$

$$= \frac{2\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\,F\left(\sin^{-1}\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right)|-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}}$$

Mathematica [A] time = 1.34, size = 227, normalized size = 1.41

$$-\frac{2\sqrt{a+bx}\sqrt{g+hx}\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\,\text{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{(g+hx)(af-be)}{(a+bx)(fg-eh)}}\right), \frac{(ad-bc)(eh-fg)}{(be-af)(dg-ch)}\right)}{\sqrt{c+dx}\sqrt{e+fx}(bg-ah)\sqrt{\frac{(g+hx)(af-be)}{(a+bx)(fg-eh)}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (-2*Sqrt[a + b*x]*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqr
t[((b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*Sqrt[g + h*x]*EllipticF[
ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], ((-(b*c)
+ a*d)*(-(f*g) + e*h))/((b*e - a*f)*(d*g - c*h))])/((b*g - a*h)*Sqrt[c + d*
x]*Sqrt[e + f*x]*Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))])
```

fricas [F] time = 5.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}{bdfhx^4+aceg+\left(bdfg+\left(bde+(bc+ad)f\right)h\right)x^3+\left(\left(bde+(bc+ad)f\right)g+\left(acf+(bc+ad)e\right)h\right)x^2+\left(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algo
rithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)/(b*d*f*h*x
^4 + a*c*e*g + (b*d*f*g + (b*d*e + (b*c + a*d)*f)*h)*x^3 + ((b*d*e + (b*c +
 a*d)*f)*g + (a*c*f + (b*c + a*d)*e)*h)*x^2 + (a*c*e*h + (a*c*f + (b*c + a*
d)*e)*g)*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algo
rithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

maple [A] time = 0.07, size = 270, normalized size = 1.68

$$\frac{2\sqrt{\frac{(af-be)(hx+g)}{(ah-bg)(fx+e)}}\sqrt{\frac{(eh-fg)(dx+c)}{(ch-dg)(fx+e)}}\sqrt{\frac{(eh-fg)(bx+a)}{(ah-bg)(fx+e)}}\left(a f^2h x^2-b f^2g x^2+2aefhx-2befgx+a e^2h-b e^2g\right)\mathrm{Ellipti}}{\sqrt{hx+g}\sqrt{fx+e}\sqrt{dx+c}\sqrt{bx+a}\left(eh-fg\right)\left(af-be\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

```
[Out] 2/(h*x+g)^(1/2)/(f*x+e)^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*((a*f-b*e)*(h*x+g
)/(a*h-b*g)/(f*x+e))^(1/2)*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)*((e*
h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)*EllipticF(((a*f-b*e)*(h*x+g)/(a*h-b
*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*(a*f^2*
h*x^2-b*f^2*g*x^2+2*a*e*f*h*x-2*b*e*f*g*x+a*e^2*h-b*e^2*g)/(e*h-f*g)/(a*f-b
*e)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algo
rithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{a+b\,x}\,\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(1/2)),x)
```

```
[Out] int(1/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

3.110 $\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+hx}}\,dx$

Optimal. Leaf size=429

$$-\frac{2d\sqrt{g+hx}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{e+fx}\,\sqrt{bg-ah}}{\sqrt{a+bx}\,\sqrt{fg-eh}}\right),-\frac{(bc-ad)(fg-eh)}{(bg-ah)(de-cf)}\right)}{\sqrt{c+dx}\,(bc-ad)\sqrt{bg-ah}\,\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}-\frac{2b\sqrt{c+dx}\,\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}{\sqrt{g+hx}\,(bc-ad)(be-}$$

```
[Out] -2*d*EllipticF((-a*h+b*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(b*x+a)^(1/2
),(-(-a*d+b*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*h+b*g))^(1/2))*((-a*f+b*e)*(d*x+c)
/(-c*f+d*e)/(b*x+a))^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/(-a*h+b*g)^(1/2)/(-e*h+
f*g)^(1/2)/(d*x+c)^(1/2)/(-(-a*f+b*e)*(h*x+g)/(-e*h+f*g)/(b*x+a))^(1/2)-2*b
*EllipticE((-a*h+b*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(b*x+a)^(1/2),(-
(-a*d+b*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*h+b*g))^(1/2))*(-e*h+f*g)^(1/2)*(d*x+c
)^(1/2)*(-(-a*f+b*e)*(h*x+g)/(-e*h+f*g)/(b*x+a))^(1/2)/(-a*d+b*c)/(-a*f+b*e
)/(-a*h+b*g)^(1/2)/((-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))^(1/2)/(h*x+g)^(1
/2)
```

Rubi [A] time = 0.26, antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {171, 170, 419, 176, 424}

$$-\frac{2d\sqrt{g+hx}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\,F\left(\sin^{-1}\left(\frac{\sqrt{bg-ah}\,\sqrt{e+fx}}{\sqrt{fg-eh}\,\sqrt{a+bx}}\right)|-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{\sqrt{c+dx}\,(bc-ad)\sqrt{bg-ah}\,\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}-\frac{2b\sqrt{c+dx}\,\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}\,E\left(\sin\right.}{\sqrt{g+hx}\,(bc-ad)(be-af)\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (-2*b*Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g -
e*h)*(a + b*x)))]*EllipticE[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*
g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*g - a
*h)))])/((b*c - a*d)*(b*e - a*f)*Sqrt[b*g - a*h]*Sqrt[((b*e - a*f)*(c + d*x
))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]) - (2*d*Sqrt[((b*e - a*f)*(c + d*
x))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g - a*h
]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*
h))/((d*e - c*f)*(b*g - a*h)))])/((b*c - a*d)*Sqrt[b*g - a*h]*Sqrt[f*g - e*
h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])
```

Rule 170

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(2*Sqrt[g + h*x]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))])/((f*g - e*h)*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]), Subst[Int[1/(Sq
rt[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]*Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h
)]), x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

Rule 171

```
Int[1/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_
.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> -Dist[d/(b*c - a*d), Int[1
/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Dist[b
/(b*c - a*d), Int[Sqrt[c + d*x]/((a + b*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x
```

```
]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 176

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[(-2*Sqrt[c + d*x]*Sqrt[
-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))])/((b*e - a*f)*Sqrt[g +
h*x]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]), Subst[Int[Sqrt
[1 + ((b*c - a*d)*x^2)/(d*e - c*f)]/Sqrt[1 - ((b*g - a*h)*x^2)/(f*g - e*h)]
, x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}
, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{b\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{bc-ad} - \frac{d\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx}{bc-ad}$$

$$= -\frac{\left(2d\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{(bc-ad)x^2}{de-cf}}\sqrt{1-\frac{(bg-ah)x^2}{fg-eh}}}\,d\right.}{(bc-ad)(fg-eh)\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}}$$

$$= -\frac{2b\sqrt{fg-eh}\sqrt{c+dx}\sqrt{-\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}}\,E\left(\sin^{-1}\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right)\right|}{(bc-ad)(be-af)\sqrt{bg-ah}\sqrt{\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}}\sqrt{g+h}}$$

Mathematica [B] time = 14.14, size = 3247, normalized size = 7.57

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] (-2*b^2*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*d)*(b*e - a*f)
*(b*g - a*h)*Sqrt[a + b*x]) - (2*(-((b*(c + d*x)^(3/2)*(f + (d*e)/(c + d*x)
 - (c*f)/(c + d*x))*(h + (d*g)/(c + d*x) - (c*h)/(c + d*x))*Sqrt[a + ((c +
d*x)*(b - (b*c)/(c + d*x)))/d]))/(Sqrt[e + ((c + d*x)*(f - (c*f)/(c + d*x)))
/d]*Sqrt[g + ((c + d*x)*(h - (c*h)/(c + d*x)))/d])) + ((b*c - a*d)*f*(b*g -
 a*h)*(-(d*g) + c*h)*Sqrt[c + d*x]*Sqrt[(b - (b*c)/(c + d*x) + (a*d)/(c + d
*x))*(f + (d*e)/(c + d*x) - (c*f)/(c + d*x))*(h + (d*g)/(c + d*x) - (c*h)/(
```

```
c + d*x))]*Sqrt[a + ((c + d*x)*(b - (b*c)/(c + d*x)))/d]*((d*e*Sqrt[((b*c -
 a*d)*(-(d*g) + c*h)*(b/(b*c - a*d) - (c + d*x)^(-1)))/(-(b*d*g) + a*d*h)]*
(-(f/(-(d*e) + c*f)) + (c + d*x)^(-1))*Sqrt[(-(h/(-(d*g) + c*h)) + (c + d*x
)^(-1))/(f/(-(d*e) + c*f) - h/(-(d*g) + c*h))]*(((-(b*d*g) + a*d*h)*Ellipti
cE[ArcSin[Sqrt[((d*e - c*f)*(h + (d*g)/(c + d*x) - (c*h)/(c + d*x)))/(d*(-(
f*g) + e*h))]], ((b*c - a*d)*(-(f*g) + e*h))/((-(d*e) + c*f)*(-(b*g) + a*h)
)])/((b*c - a*d)*(-(d*g) + c*h)) - (b*EllipticF[ArcSin[Sqrt[((d*e - c*f)*(h
 + (d*g)/(c + d*x) - (c*h)/(c + d*x)))/(d*(-(f*g) + e*h))]], ((b*c - a*d)*(
-(f*g) + e*h))/((-(d*e) + c*f)*(-(b*g) + a*h))])/(b*c - a*d)))/(Sqrt[(-(f/(
-(d*e) + c*f)) + (c + d*x)^(-1))/(-(f/(-(d*e) + c*f)) + h/(-(d*g) + c*h))]*
Sqrt[(b + (-(b*c) + a*d)/(c + d*x))*(f + (d*e - c*f)/(c + d*x))*(h + (d*g -
 c*h)/(c + d*x))]) - (c*f*Sqrt[((b*c - a*d)*(-(d*g) + c*h)*(b/(b*c - a*d) -
 (c + d*x)^(-1)))/(-(b*d*g) + a*d*h)]*(-(f/(-(d*e) + c*f)) + (c + d*x)^(-1)
)*Sqrt[(-(h/(-(d*g) + c*h)) + (c + d*x)^(-1))/(f/(-(d*e) + c*f) - h/(-(d*g)
 + c*h))]*(((-(b*d*g) + a*d*h)*EllipticE[ArcSin[Sqrt[((d*e - c*f)*(h + (d*g
)/(c + d*x) - (c*h)/(c + d*x)))/(d*(-(f*g) + e*h))]], ((b*c - a*d)*(-(f*g)
+ e*h))/((-(d*e) + c*f)*(-(b*g) + a*h))])/((b*c - a*d)*(-(d*g) + c*h)) - (b
*EllipticF[ArcSin[Sqrt[((d*e - c*f)*(h + (d*g)/(c + d*x) - (c*h)/(c + d*x))
)/(d*(-(f*g) + e*h))]], ((b*c - a*d)*(-(f*g) + e*h))/((-(d*e) + c*f)*(-(b*g
) + a*h))])/(b*c - a*d)))/(Sqrt[(-(f/(-(d*e) + c*f)) + (c + d*x)^(-1))/(-(f
/(-(d*e) + c*f)) + h/(-(d*g) + c*h))]*Sqrt[(b + (-(b*c) + a*d)/(c + d*x))*(
f + (d*e - c*f)/(c + d*x))*(h + (d*g - c*h)/(c + d*x))]) + (f*Sqrt[(-(b/(b*
c - a*d)) + (c + d*x)^(-1))/(-(b/(b*c - a*d)) + h/(-(d*g) + c*h))]*Sqrt[(-(
f/(-(d*e) + c*f)) + (c + d*x)^(-1))/(-(f/(-(d*e) + c*f)) + h/(-(d*g) + c*h)
)]*(-(h/(-(d*g) + c*h)) + (c + d*x)^(-1))*EllipticF[ArcSin[Sqrt[((-(d*e) +
c*f)*(-h - (d*g)/(c + d*x) + (c*h)/(c + d*x)))/(d*(-(f*g) + e*h))]], ((b*c
- a*d)*(-(f*g) + e*h))/((-(d*e) + c*f)*(-(b*g) + a*h))])/(Sqrt[(-(h/(-(d*g)
 + c*h)) + (c + d*x)^(-1))/(f/(-(d*e) + c*f) - h/(-(d*g) + c*h))]*Sqrt[(b +
 (-(b*c) + a*d)/(c + d*x))*(f + (d*e - c*f)/(c + d*x))*(h + (d*g - c*h)/(c
+ d*x))]))))/((f*g - e*h)*(b - (b*c)/(c + d*x) + (a*d)/(c + d*x))*Sqrt[e + (
(c + d*x)*(f - (c*f)/(c + d*x)))/d]*Sqrt[g + ((c + d*x)*(h - (c*h)/(c + d*x
)))/d]) - ((b*c - a*d)*(b*e - a*f)*(-(d*e) + c*f)*h*Sqrt[c + d*x]*Sqrt[(b -
 (b*c)/(c + d*x) + (a*d)/(c + d*x))*(f + (d*e)/(c + d*x) - (c*f)/(c + d*x))
*(h + (d*g)/(c + d*x) - (c*h)/(c + d*x))]*Sqrt[a + ((c + d*x)*(b - (b*c)/(c
 + d*x)))/d]*((d*g*Sqrt[((b*c - a*d)*(-(d*g) + c*h)*(b/(b*c - a*d) - (c + d
*x)^(-1)))/(-(b*d*g) + a*d*h)]*(-(f/(-(d*e) + c*f)) + (c + d*x)^(-1))*Sqrt[
(-(h/(-(d*g) + c*h)) + (c + d*x)^(-1))/(f/(-(d*e) + c*f) - h/(-(d*g) + c*h)
)]*(((-(b*d*g) + a*d*h)*EllipticE[ArcSin[Sqrt[((d*e - c*f)*(h + (d*g)/(c +
d*x) - (c*h)/(c + d*x)))/(d*(-(f*g) + e*h))]], ((b*c - a*d)*(-(f*g) + e*h))
/((-(d*e) + c*f)*(-(b*g) + a*h))])/((b*c - a*d)*(-(d*g) + c*h)) - (b*Ellipt
icF[ArcSin[Sqrt[((d*e - c*f)*(h + (d*g)/(c + d*x) - (c*h)/(c + d*x)))/(d*(-
(f*g) + e*h))]], ((b*c - a*d)*(-(f*g) + e*h))/((-(d*e) + c*f)*(-(b*g) + a*h
))])/(b*c - a*d)))/(Sqrt[(-(f/(-(d*e) + c*f)) + (c + d*x)^(-1))/(-(f/(-(d*e
) + c*f)) + h/(-(d*g) + c*h))]*Sqrt[(b + (-(b*c) + a*d)/(c + d*x))*(f + (d*
e - c*f)/(c + d*x))*(h + (d*g - c*h)/(c + d*x))]) - (c*h*Sqrt[((b*c - a*d)*
(-(d*g) + c*h)*(b/(b*c - a*d) - (c + d*x)^(-1)))/(-(b*d*g) + a*d*h)]*(-(f/(
-(d*e) + c*f)) + (c + d*x)^(-1))*Sqrt[(-(h/(-(d*g) + c*h)) + (c + d*x)^(-1)
)/(f/(-(d*e) + c*f) - h/(-(d*g) + c*h))]*(((-(b*d*g) + a*d*h)*EllipticE[Arc
Sin[Sqrt[((d*e - c*f)*(h + (d*g)/(c + d*x) - (c*h)/(c + d*x)))/(d*(-(f*g) +
 e*h))]], ((b*c - a*d)*(-(f*g) + e*h))/((-(d*e) + c*f)*(-(b*g) + a*h))])/((
b*c - a*d)*(-(d*g) + c*h)) - (b*EllipticF[ArcSin[Sqrt[((d*e - c*f)*(h + (d*
g)/(c + d*x) - (c*h)/(c + d*x)))/(d*(-(f*g) + e*h))]], ((b*c - a*d)*(-(f*g)
 + e*h))/((-(d*e) + c*f)*(-(b*g) + a*h))])/(b*c - a*d)))/(Sqrt[(-(f/(-(d*e)
 + c*f)) + (c + d*x)^(-1))/(-(f/(-(d*e) + c*f)) + h/(-(d*g) + c*h))]*Sqrt[(
b + (-(b*c) + a*d)/(c + d*x))*(f + (d*e - c*f)/(c + d*x))*(h + (d*g - c*h)/
(c + d*x))]) + (h*Sqrt[(-(b/(b*c - a*d)) + (c + d*x)^(-1))/(-(b/(b*c - a*d)
) + h/(-(d*g) + c*h))]*Sqrt[(-(f/(-(d*e) + c*f)) + (c + d*x)^(-1))/(-(f/(-(
d*e) + c*f)) + h/(-(d*g) + c*h))]*(-(h/(-(d*g) + c*h)) + (c + d*x)^(-1))*El
lipticF[ArcSin[Sqrt[((-(d*e) + c*f)*(-h - (d*g)/(c + d*x) + (c*h)/(c + d*x)
```

```
))/(d*(-(f*g) + e*h))]], ((b*c - a*d)*(-(f*g) + e*h))/((-(d*e) + c*f)*(-(b*
g) + a*h))])/(Sqrt[(-(h/(-(d*g) + c*h)) + (c + d*x)^(-1))/(f/(-(d*e) + c*f)
 - h/(-(d*g) + c*h))]*Sqrt[(b + (-(b*c) + a*d)/(c + d*x))*(f + (d*e - c*f)/
(c + d*x))*(h + (d*g - c*h)/(c + d*x))]))))/((f*g - e*h)*(b - (b*c)/(c + d*x
) + (a*d)/(c + d*x))*Sqrt[e + ((c + d*x)*(f - (c*f)/(c + d*x)))/d]*Sqrt[g +
 ((c + d*x)*(h - (c*h)/(c + d*x)))/d])))/(d*(b*c - a*d)*(b*e - a*f)*(b*g -
a*h))
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{}{b^2dfhx^5 + a^2ceg + \left(b^2dfg + \left(b^2de + \left(b^2c + 2\,abd\right)f\right)h\right)x^4 + \left(\left(b^2de + \left(b^2c + 2\,abd\right)f\right)g + \left(\left(b^2c + 2\,a\right.\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algo
rithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)/(b^2*d*f*h
*x^5 + a^2*c*e*g + (b^2*d*f*g + (b^2*d*e + (b^2*c + 2*a*b*d)*f)*h)*x^4 + ((
b^2*d*e + (b^2*c + 2*a*b*d)*f)*g + ((b^2*c + 2*a*b*d)*e + (2*a*b*c + a^2*d)
*f)*h)*x^3 + (((b^2*c + 2*a*b*d)*e + (2*a*b*c + a^2*d)*f)*g + (a^2*c*f + (2
*a*b*c + a^2*d)*e)*h)*x^2 + (a^2*c*e*h + (a^2*c*f + (2*a*b*c + a^2*d)*e)*g)
*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algo
rithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

maple [B] time = 0.11, size = 4660, normalized size = 10.86

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

```
[Out] 2/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)*(-2*EllipticF(((a
*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*
f-b*e))^(1/2))*x*a*b*d*e*f*g*h*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)*
((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f
*x+e))^(1/2)-2*EllipticE(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-
d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*x*a*b*d*e*f*g*h*((e*h-f*g)*(b*x+
a)/(a*h-b*g)/(f*x+e))^(1/2)*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)*((e
*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)+x*b^2*d*e^2*g*h-x*b^2*d*e*f*g^2+El
lipticE(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c
*h-d*g)/(a*f-b*e))^(1/2))*x^2*a*b*c*f^2*h^2*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f
*x+e))^(1/2)*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)*((e*h-f*g)*(d*x+c)
/(c*h-d*g)/(f*x+e))^(1/2)-EllipticE(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(
1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*x^2*b^2*c*f^2*g*h*((e
*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+
e))^(1/2)*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)+2*EllipticF(((a*f-b*e
)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e)
```

```
)^(1/2))*x*a^2*d*e*f*h^2*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)*((a*f-
b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))
^(1/2)+2*EllipticE(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(
a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*x*b^2*d*e*f*g^2*((e*h-f*g)*(b*x+a)/(a*
h-b*g)/(f*x+e))^(1/2)*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)*((e*h-f*g
)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)-EllipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(
f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*a*b*d*e^2*g*
h*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)*((a*f-b*e)*(h*x+g)/(a*h-b*g)/
(f*x+e))^(1/2)*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)-EllipticE(((a*f-
b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b
*e))^(1/2))*a*b*d*e^2*g*h*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)*((a*f
-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e)
)^(1/2)-EllipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a
*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*x^2*a*b*c*f^2*h^2*((e*h-f*g)*(b*x+a)/(a
*h-b*g)/(f*x+e))^(1/2)*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)*((e*h-f*
g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)+EllipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g)/
(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*x^2*b^2*c*f
^2*g*h*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)*((a*f-b*e)*(h*x+g)/(a*h-
b*g)/(f*x+e))^(1/2)*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)+x^2*b^2*d*e
^2*h^2+x*b^2*c*e^2*h^2+a*b*c*f^2*g^2+b^2*c*e^2*g*h-b^2*c*e*f*g^2-EllipticF(
((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/
(a*f-b*e))^(1/2))*x^2*a*b*d*f^2*g*h*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(
1/2)*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)*((e*h-f*g)*(d*x+c)/(c*h-d*
g)/(f*x+e))^(1/2)-EllipticE(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c
*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*x^2*a*b*d*f^2*g*h*((e*h-f*g)*
(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2
)*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)-2*EllipticF(((a*f-b*e)*(h*x+g
)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))
*x*a*b*c*e*f*h^2*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)*((a*f-b*e)*(h*
x+g)/(a*h-b*g)/(f*x+e))^(1/2)*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)+2
*EllipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)
/(c*h-d*g)/(a*f-b*e))^(1/2))*x*b^2*c*e*f*g*h*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(
f*x+e))^(1/2)*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)*((e*h-f*g)*(d*x+c
)/(c*h-d*g)/(f*x+e))^(1/2)+2*EllipticE(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e)
)^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*x*a*b*c*e*f*h^2*((
e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x
+e))^(1/2)*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)-2*EllipticE(((a*f-b*
e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e
))^(1/2))*x*b^2*c*e*f*g*h*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)*((a*f
-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e)
)^(1/2)+x*a*b*d*f^2*g^2+EllipticE(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/
2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*x^2*b^2*d*f^2*g^2*((e*h
-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e)
)^(1/2)*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)-EllipticF(((a*f-b*e)*(h
*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1
/2))*a*b*c*e^2*h^2*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)*((a*f-b*e)*(
h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)
+EllipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)
/(c*h-d*g)/(a*f-b*e))^(1/2))*b^2*c*e^2*g*h*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*
x+e))^(1/2)*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)*((e*h-f*g)*(d*x+c)/
(c*h-d*g)/(f*x+e))^(1/2)+EllipticE(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1
/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*a*b*c*e^2*h^2*((e*h-f*
g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(
1/2)*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)-EllipticE(((a*f-b*e)*(h*x+
g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2)
)*b^2*c*e^2*g*h*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)*((a*f-b*e)*(h*x
+g)/(a*h-b*g)/(f*x+e))^(1/2)*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)+El
lipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c
*h-d*g)/(a*f-b*e))^(1/2))*x^2*a^2*d*f^2*h^2*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f
```

```
*x+e))^(1/2)*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(1/2)*((e*h-f*g)*(d*x+c)
/(c*h-d*g)/(f*x+e))^(1/2)+EllipticF(((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^(
1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2))*a^2*d*e^2*h^2*((e*h-f
*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)*((a*f-b*e)*(h*x+g)/(a*h-b*g)/(f*x+e))^
(1/2)*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)+EllipticE(((a*f-b*e)*(h*x
+g)/(a*h-b*g)/(f*x+e))^(1/2),((c*f-d*e)*(a*h-b*g)/(c*h-d*g)/(a*f-b*e))^(1/2
))*b^2*d*e^2*g^2*((e*h-f*g)*(b*x+a)/(a*h-b*g)/(f*x+e))^(1/2)*((a*f-b*e)*(h*
x+g)/(a*h-b*g)/(f*x+e))^(1/2)*((e*h-f*g)*(d*x+c)/(c*h-d*g)/(f*x+e))^(1/2)-x
*a*b*d*e*f*g*h-x^2*a*b*d*e*f*h^2+x^2*a*b*d*f^2*g*h-x^2*b^2*d*e*f*g*h-x*a*b*
c*e*f*h^2+x*a*b*c*f^2*g*h-x*b^2*c*e*f*g*h-a*b*c*e*f*g*h)/(e*h-f*g)/(a*h-b*g
)/(a*f-b*e)/(a*d-b*c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algo
rithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,{\left(a+b\,x\right)}^{3/2}\,\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(3/2)*(c + d*x)^(1/2)),x)
```

```
[Out] int(1/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(3/2)*(c + d*x)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{3}{2}}\,\sqrt{c+dx}\,\sqrt{e+fx}\,\sqrt{g+hx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x)**(3/2)*sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

3.111 $\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal. Leaf size=786

$$-\frac{4bd\sqrt{g+hx}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{e+fx}\sqrt{bg-ah}}{\sqrt{a+bx}\sqrt{fg-eh}}\right),-\frac{(bc-ad)(fg-eh)}{(bg-ah)(de-cf)}\right)}{\sqrt{c+dx}\,(bc-ad)^2\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}+\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\,(a^2d^2}{\sqrt{a+bx}\,(bc-ad}$$

```
[Out] -2*d^3*(b*x+a)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)^2/(-c*f+d*e)/(-
c*h+d*g)/(d*x+c)^(1/2)-2*b^3*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*
d+b*c)^2/(-a*f+b*e)/(-a*h+b*g)/(b*x+a)^(1/2)+2*b*(a^2*d^2*f*h-a*b*d^2*(e*h+
f*g)+b^2*(2*d^2*e*g+c^2*f*h-c*d*(e*h+f*g)))*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*
x+g)^(1/2)/(-a*d+b*c)^2/(-a*f+b*e)/(-c*f+d*e)/(-a*h+b*g)/(-c*h+d*g)/(b*x+a)
^(1/2)-4*b*d*EllipticF((-a*h+b*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(b*x
+a)^(1/2),(-(-a*d+b*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*h+b*g))^(1/2))*((-a*f+b*e)
*(d*x+c)/(-c*f+d*e)/(b*x+a))^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)^2/(-a*h+b*g)^(1
/2)/(-e*h+f*g)^(1/2)/(d*x+c)^(1/2)/(-(-a*f+b*e)*(h*x+g)/(-e*h+f*g)/(b*x+a))
^(1/2)-2*(a^2*d^2*f*h-a*b*d^2*(e*h+f*g)+b^2*(2*d^2*e*g+c^2*f*h-c*d*(e*h+f*g
)))*EllipticE((-a*h+b*g)^(1/2)*(f*x+e)^(1/2)/(-e*h+f*g)^(1/2)/(b*x+a)^(1/2)
,(-(-a*d+b*c)*(-e*h+f*g)/(-c*f+d*e)/(-a*h+b*g))^(1/2))*(-e*h+f*g)^(1/2)*(d*
x+c)^(1/2)*(-(-a*f+b*e)*(h*x+g)/(-e*h+f*g)/(b*x+a))^(1/2)/(-a*d+b*c)^2/(-a*
f+b*e)/(-c*f+d*e)/(-c*h+d*g)/(-a*h+b*g)^(1/2)/((-a*f+b*e)*(d*x+c)/(-c*f+d*e
)/(b*x+a))^(1/2)/(h*x+g)^(1/2)
```

Rubi [F] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((a + b*x)^(3/2)*(c + d*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

```
[Out] Defer[Int][1/((a + b*x)^(3/2)*(c + d*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

Mathematica [B] time = 17.29, size = 7075, normalized size = 9.00

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),x
]
```

```
[Out] Result too large to show
```

fricas [F] time = 6.36, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{}{b^2d^2fhx^6+a^2c^2eg+\left(b^2d^2fg+\left(b^2d^2e+2\left(b^2cd+abd^2\right)f\right)h\right)x^5+\left(\left(b^2d^2e+2\left(b^2cd+abd^2\right)f\right)g+\left(2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algo
rithm="fricas")
```

```
[Out] integral(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)/(b^2*d^2*f
*h*x^6 + a^2*c^2*e*g + (b^2*d^2*f*g + (b^2*d^2*e + 2*(b^2*c*d + a*b*d^2)*f)
*h)*x^5 + ((b^2*d^2*e + 2*(b^2*c*d + a*b*d^2)*f)*g + (2*(b^2*c*d + a*b*d^2)
*e + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f)*h)*x^4 + ((2*(b^2*c*d + a*b*d^2)*e
+ (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f)*g + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*e
 + 2*(a*b*c^2 + a^2*c*d)*f)*h)*x^3 + (((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*e +
2*(a*b*c^2 + a^2*c*d)*f)*g + (a^2*c^2*f + 2*(a*b*c^2 + a^2*c*d)*e)*h)*x^2 +
 (a^2*c^2*e*h + (a^2*c^2*f + 2*(a*b*c^2 + a^2*c*d)*e)*g)*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algo
rithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(3/2)*sqrt(f*x + e)*sqrt(h*x + g)),
x)
```

maple [B] time = 0.25, size = 21102, normalized size = 26.85

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(3/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algo
rithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(3/2)*(d*x + c)^(3/2)*sqrt(f*x + e)*sqrt(h*x + g)),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,{\left(a+b\,x\right)}^{3/2}\,{\left(c+d\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(3/2)*(c + d*x)^(3/2)),x)
```

```
[Out] int(1/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(3/2)*(c + d*x)^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(3/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

```
[Out] Timed out
```

3.112 $\int \frac{x^4(e+fx)^n}{(a+bx)(c+dx)} \, dx$

Optimal. Leaf size=319

$$-\frac{a^4(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b(e+fx)}{be-af}\right)}{b^3(n+1)(bc-ad)(be-af)}+\frac{\left(a^2d^2+abcd+b^2c^2\right)(e+fx)^{n+1}}{b^3d^3f(n+1)}+\frac{e(ad+bc)(e+fx)^{n+1}}{b^2d^2f^2(n+1)}-\frac{(ad+b}{b^2d}$$

```
[Out] e^2*(f*x+e)^(1+n)/b/d/f^3/(1+n)+(a*d+b*c)*e*(f*x+e)^(1+n)/b^2/d^2/f^2/(1+n)
+(a^2*d^2+a*b*c*d+b^2*c^2)*(f*x+e)^(1+n)/b^3/d^3/f/(1+n)-2*e*(f*x+e)^(2+n)/
b/d/f^3/(2+n)-(a*d+b*c)*(f*x+e)^(2+n)/b^2/d^2/f^2/(2+n)+(f*x+e)^(3+n)/b/d/f
^3/(3+n)-a^4*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b*(f*x+e)/(-a*f+b*e))/b
^3/(-a*d+b*c)/(-a*f+b*e)/(1+n)+c^4*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],d
*(f*x+e)/(-c*f+d*e))/d^3/(-a*d+b*c)/(-c*f+d*e)/(1+n)
```

Rubi [A] time = 0.28, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {180, 43, 68}

$$\frac{\left(a^2d^2+abcd+b^2c^2\right)(e+fx)^{n+1}}{b^3d^3f(n+1)}-\frac{a^4(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b(e+fx)}{be-af}\right)}{b^3(n+1)(bc-ad)(be-af)}+\frac{e(ad+bc)(e+fx)^{n+1}}{b^2d^2f^2(n+1)}-\frac{(ad+bc}{b^2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
```

```
[Out] (e^2*(e + f*x)^(1 + n))/(b*d*f^3*(1 + n)) + ((b*c + a*d)*e*(e + f*x)^(1 + n
))/(b^2*d^2*f^2*(1 + n)) + ((b^2*c^2 + a*b*c*d + a^2*d^2)*(e + f*x)^(1 + n)
)/(b^3*d^3*f*(1 + n)) - (2*e*(e + f*x)^(2 + n))/(b*d*f^3*(2 + n)) - ((b*c +
 a*d)*(e + f*x)^(2 + n))/(b^2*d^2*f^2*(2 + n)) + (e + f*x)^(3 + n)/(b*d*f^3
*(3 + n)) - (a^4*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e
 + f*x))/(b*e - a*f)])/(b^3*(b*c - a*d)*(b*e - a*f)*(1 + n)) + (c^4*(e + f*
x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])/(
d^3*(b*c - a*d)*(d*e - c*f)*(1 + n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rubi steps

$$\int \frac{x^4(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \left(\frac{(b^2c^2+abcd+a^2d^2)(e+fx)^n}{b^3d^3} - \frac{(bc+ad)x(e+fx)^n}{b^2d^2} + \frac{x^2(e+fx)^n}{bd} + \frac{a^4(e+f\ldots}{b^3(bc-ad)(\ldots} \right.$$

$$= \frac{(b^2c^2+abcd+a^2d^2)(e+fx)^{1+n}}{b^3d^3f(1+n)} + \frac{\int x^2(e+fx)^n\,dx}{bd} + \frac{a^4\int\frac{(e+fx)^n}{a+bx}\,dx}{b^3(bc-ad)} - \frac{c^4\int\frac{(e+fx)^n}{c+dx}\,dx}{d^3(bc-ad)}$$

$$= \frac{(b^2c^2+abcd+a^2d^2)(e+fx)^{1+n}}{b^3d^3f(1+n)} - \frac{a^4(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{b(e+fx)}{be-af}\right)}{b^3(bc-ad)(be-af)(1+n)} + \frac{c^4(e+\ldots}{\ldots}$$

$$= \frac{e^2(e+fx)^{1+n}}{bdf^3(1+n)} + \frac{(bc+ad)e(e+fx)^{1+n}}{b^2d^2f^2(1+n)} + \frac{(b^2c^2+abcd+a^2d^2)(e+fx)^{1+n}}{b^3d^3f(1+n)} - \frac{2e(e+fx)\ldots}{bdf^3(2+\ldots}$$

Mathematica [A] time = 0.99, size = 285, normalized size = 0.89

$$\frac{(e+fx)^{n+1}\left(\frac{b^3c^4f^3(n^2+5n+6)\,{}_2F_1\left(1,n+1;n+2;\frac{d(e+fx)}{de-cf}\right)-(bc-ad)(cf-de)\left(a^2d^2f^2(n^2+5n+6)+abdf(n+3)(cf(n+2)+d(e-f(n+1)x))+b^2\left(c^2f^2(n^2+5n\ldots\right.\right.}{f^3(n+2)(n+3)(ad-bc)(cf-de)}\right.}{b^3d^3(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
```

```
[Out] ((e + f*x)^(1 + n)*(-((a^4*d^3*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e + f
*x))/(b*e - a*f)])/((b*c - a*d)*(b*e - a*f))) + (-((b*c - a*d)*(-(d*e) + c*
f)*(a^2*d^2*f^2*(6 + 5*n + n^2) + a*b*d*f*(3 + n)*(c*f*(2 + n) + d*(e - f*(
1 + n)*x)) + b^2*(c^2*f^2*(6 + 5*n + n^2) + c*d*f*(3 + n)*(e - f*(1 + n)*x)
 + d^2*(2*e^2 - 2*e*f*(1 + n)*x + f^2*(2 + 3*n + n^2)*x^2)))) + b^3*c^4*f^3
*(6 + 5*n + n^2)*Hypergeometric2F1[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*
f)])/((-(b*c) + a*d)*f^3*(-(d*e) + c*f)*(2 + n)*(3 + n))))/(b^3*d^3*(1 + n)
)
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(fx+e\right)^n x^4}{bdx^2+ac+(bc+ad)x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n*x^4/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx+e\right)^n x^4}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n*x^4/((b*x + a)*(d*x + c)), x)
```

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(fx+e\right)^n}{(bx+a)\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

```
[Out] int(x^4*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx+e\right)^{n}x^{4}}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n*x^4/((b*x + a)*(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\left(e+f\,x\right)}^n}{\left(a+b\,x\right)\,\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(e + f*x)^n)/((a + b*x)*(c + d*x)),x)
```

```
[Out] int((x^4*(e + f*x)^n)/((a + b*x)*(c + d*x)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(f*x+e)**n/(b*x+a)/(d*x+c),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.113 $\int \frac{x^3(e+fx)^n}{(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=216

$$\frac{a^3(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b(e+fx)}{be-af}\right)}{b^2(n+1)(bc-ad)(be-af)}-\frac{(ad+bc)(e+fx)^{n+1}}{b^2d^2f(n+1)}-\frac{c^3(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{d(e+fx)}{de-cf}\right)}{d^2(n+1)(bc-ad)(de-cf)}-\frac{e(e\ldots}{bdf\ldots}$$

```
[Out] -e*(f*x+e)^(1+n)/b/d/f^2/(1+n)-(a*d+b*c)*(f*x+e)^(1+n)/b^2/d^2/f/(1+n)+(f*x
+e)^(2+n)/b/d/f^2/(2+n)+a^3*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b*(f*x+e
)/(-a*f+b*e))/b^2/(-a*d+b*c)/(-a*f+b*e)/(1+n)-c^3*(f*x+e)^(1+n)*hypergeom([
1, 1+n],[2+n],d*(f*x+e)/(-c*f+d*e))/d^2/(-a*d+b*c)/(-c*f+d*e)/(1+n)
```

Rubi [A] time = 0.15, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {180, 43, 68}

$$\frac{a^3(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b(e+fx)}{be-af}\right)}{b^2(n+1)(bc-ad)(be-af)}-\frac{(ad+bc)(e+fx)^{n+1}}{b^2d^2f(n+1)}-\frac{c^3(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{d(e+fx)}{de-cf}\right)}{d^2(n+1)(bc-ad)(de-cf)}-\frac{e(e\ldots}{bdf\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
```

```
[Out] -((e*(e + f*x)^(1 + n))/(b*d*f^2*(1 + n))) - ((b*c + a*d)*(e + f*x)^(1 + n)
)/(b^2*d^2*f*(1 + n)) + (e + f*x)^(2 + n)/(b*d*f^2*(2 + n)) + (a^3*(e + f*x
)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e + f*x))/(b*e - a*f)])/(b
^2*(b*c - a*d)*(b*e - a*f)*(1 + n)) - (c^3*(e + f*x)^(1 + n)*Hypergeometric
2F1[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])/(d^2*(b*c - a*d)*(d*e - c*
f)*(1 + n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rubi steps

$$\int \frac{x^3(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \left(\frac{(-bc-ad)(e+fx)^n}{b^2d^2} + \frac{x(e+fx)^n}{bd} - \frac{a^3(e+fx)^n}{b^2(bc-ad)(a+bx)} - \frac{c^3(e+fx)^n}{d^2(-bc+ad)(c+dx)}\right.$$

$$= -\frac{(bc+ad)(e+fx)^{1+n}}{b^2d^2f(1+n)} + \frac{\int x(e+fx)^n\,dx}{bd} - \frac{a^3\int \frac{(e+fx)^n}{a+bx}\,dx}{b^2(bc-ad)} + \frac{c^3\int \frac{(e+fx)^n}{c+dx}\,dx}{d^2(bc-ad)}$$

$$= -\frac{(bc+ad)(e+fx)^{1+n}}{b^2d^2f(1+n)} + \frac{a^3(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{b(e+fx)}{be-af}\right)}{b^2(bc-ad)(be-af)(1+n)} - \frac{c^3(e+fx)^{1+n}\,{}_2}{d^2(bc-}$$

$$= -\frac{e(e+fx)^{1+n}}{bdf^2(1+n)} - \frac{(bc+ad)(e+fx)^{1+n}}{b^2d^2f(1+n)} + \frac{(e+fx)^{2+n}}{bdf^2(2+n)} + \frac{a^3(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2\right.}{b^2(bc-ad)(be-af)(}$$

Mathematica [A] time = 0.43, size = 174, normalized size = 0.81

$$\frac{(e+fx)^{n+1}\left(\frac{a^3\,{}_2F_1\left(1,n+1;n+2;\frac{b(e+fx)}{be-af}\right)}{be-af} + \frac{(bc-ad)(cf-de)(adf(n+2)+bcf(n+2)+bd(e-f(n+1)x))-b^2c^3f^2(n+2)\,{}_2F_1\left(1,n+1;n+2;\frac{d(e+fx)}{de-cf}\right)}{d^2f^2(n+2)(de-cf)}\right)}{b^2(n+1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
```

```
[Out] ((e + f*x)^(1 + n)*((a^3*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e + f*x))/(
b*e - a*f)])/(b*e - a*f) + ((b*c - a*d)*(-(d*e) + c*f)*(b*c*f*(2 + n) + a*d
*f*(2 + n) + b*d*(e - f*(1 + n)*x)) - b^2*c^3*f^2*(2 + n)*Hypergeometric2F1
[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])/(d^2*f^2*(d*e - c*f)*(2 + n))
))/(b^2*(b*c - a*d)*(1 + n))
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(fx+e\right)^n x^3}{bdx^2+ac+(bc+ad)x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n*x^3/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx+e\right)^n x^3}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n*x^3/((b*x + a)*(d*x + c)), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(fx+e\right)^n}{(bx+a)\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

```
[Out] int(x^3*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx+e\right)^n x^3}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n*x^3/((b*x + a)*(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\left(e+f\,x\right)^n}{\left(a+b\,x\right)\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(e + f*x)^n)/((a + b*x)*(c + d*x)),x)
```

```
[Out] int((x^3*(e + f*x)^n)/((a + b*x)*(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(e+fx\right)^n}{\left(a+bx\right)\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x+e)**n/(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(x**3*(e + f*x)**n/((a + b*x)*(c + d*x)), x)
```

3.114 $\int \frac{x^2(e+fx)^n}{(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=158

$$-\frac{a^2(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b(e+fx)}{be-af}\right)}{b(n+1)(bc-ad)(be-af)}+\frac{c^2(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{d(e+fx)}{de-cf}\right)}{d(n+1)(bc-ad)(de-cf)}+\frac{(e+fx)^{n+1}}{bdf(n+1)}$$

```
[Out] (f*x+e)^(1+n)/b/d/f/(1+n)-a^2*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b*(f*x
+e)/(-a*f+b*e))/b/(-a*d+b*c)/(-a*f+b*e)/(1+n)+c^2*(f*x+e)^(1+n)*hypergeom([
1, 1+n],[2+n],d*(f*x+e)/(-c*f+d*e))/d/(-a*d+b*c)/(-c*f+d*e)/(1+n)
```

Rubi [A] time = 0.11, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {180, 68}

$$-\frac{a^2(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b(e+fx)}{be-af}\right)}{b(n+1)(bc-ad)(be-af)}+\frac{c^2(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{d(e+fx)}{de-cf}\right)}{d(n+1)(bc-ad)(de-cf)}+\frac{(e+fx)^{n+1}}{bdf(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
```

```
[Out] (e + f*x)^(1 + n)/(b*d*f*(1 + n)) - (a^2*(e + f*x)^(1 + n)*Hypergeometric2F
1[1, 1 + n, 2 + n, (b*(e + f*x))/(b*e - a*f)])/(b*(b*c - a*d)*(b*e - a*f)*(
1 + n)) + (c^2*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (d*(e +
 f*x))/(d*e - c*f)])/(d*(b*c - a*d)*(d*e - c*f)*(1 + n))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(e+fx)^n}{(a+bx)(c+dx)}\,dx &= \int \left(\frac{(e+fx)^n}{bd}+\frac{a^2(e+fx)^n}{b(bc-ad)(a+bx)}+\frac{c^2(e+fx)^n}{d(-bc+ad)(c+dx)}\right) dx\\ &= \frac{(e+fx)^{1+n}}{bdf(1+n)}+\frac{a^2\int \frac{(e+fx)^n}{a+bx}\,dx}{b(bc-ad)}-\frac{c^2\int \frac{(e+fx)^n}{c+dx}\,dx}{d(bc-ad)}\\ &= \frac{(e+fx)^{1+n}}{bdf(1+n)}-\frac{a^2(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{b(e+fx)}{be-af}\right)}{b(bc-ad)(be-af)(1+n)}+\frac{c^2(e+fx)^{1+n}\,{}_2F_1\left(1,1+n\right.}{d(bc-ad)(de-cf}\end{aligned}$$

Mathematica [A] time = 0.07, size = 153, normalized size = 0.97

$$\frac{(e+fx)^{n+1}\left(a^2df(cf-de)\,{}_2F_1\left(1,n+1;n+2;\frac{b(e+fx)}{be-af}\right)+(be-af)\left(bc^2f\,{}_2F_1\left(1,n+1;n+2;\frac{d(e+fx)}{de-cf}\right)-(bc-a\right.\right.}{bdf(n+1)(bc-ad)(be-af)(de-cf)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
```

```
[Out] ((e + f*x)^(1 + n)*(a^2*d*f*(-(d*e) + c*f)*Hypergeometric2F1[1, 1 + n, 2 +
n, (b*(e + f*x))/(b*e - a*f)] + (b*e - a*f)*(-((b*c - a*d)*(-(d*e) + c*f))
+ b*c^2*f*Hypergeometric2F1[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])))/
(b*d*(b*c - a*d)*f*(b*e - a*f)*(d*e - c*f)*(1 + n))
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(fx+e\right)^{n}x^{2}}{bdx^{2}+ac+(bc+ad)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n*x^2/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx+e\right)^{n}x^{2}}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n*x^2/((b*x + a)*(d*x + c)), x)
```

maple [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{x^{2}\left(fx+e\right)^{n}}{(bx+a)\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

```
[Out] int(x^2*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx+e\right)^{n}x^{2}}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n*x^2/((b*x + a)*(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2}\left(e+f\,x\right)^{n}}{\left(a+b\,x\right)\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(e + f*x)^n)/((a + b*x)*(c + d*x)),x)
```

```
[Out] int((x^2*(e + f*x)^n)/((a + b*x)*(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \left(e + fx\right)^n}{(a + bx)(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(f*x+e)**n/(b*x+a)/(d*x+c),x)
[Out] Integral(x**2*(e + f*x)**n/((a + b*x)*(c + d*x)), x)
```

3.115 $\int \frac{x(e+fx)^n}{(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=124

$$\frac{a(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b(e+fx)}{be-af}\right)}{(n+1)(bc-ad)(be-af)}-\frac{c(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{d(e+fx)}{de-cf}\right)}{(n+1)(bc-ad)(de-cf)}$$

```
[Out] a*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b*(f*x+e)/(-a*f+b*e))/(-a*d+b*c)/(
-a*f+b*e)/(1+n)-c*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],d*(f*x+e)/(-c*f+d*
e))/(-a*d+b*c)/(-c*f+d*e)/(1+n)
```

Rubi [A] time = 0.04, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {156, 68}

$$\frac{a(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b(e+fx)}{be-af}\right)}{(n+1)(bc-ad)(be-af)}-\frac{c(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{d(e+fx)}{de-cf}\right)}{(n+1)(bc-ad)(de-cf)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
```

```
[Out] (a*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e + f*x))/(b*e
- a*f)])/((b*c - a*d)*(b*e - a*f)*(1 + n)) - (c*(e + f*x)^(1 + n)*Hypergeom
etric2F1[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])/((b*c - a*d)*(d*e - c
*f)*(1 + n))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x(e+fx)^n}{(a+bx)(c+dx)}\,dx &= -\frac{a\int \frac{(e+fx)^n}{a+bx}\,dx}{bc-ad}+\frac{c\int \frac{(e+fx)^n}{c+dx}\,dx}{bc-ad}\\ &= \frac{a(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{b(e+fx)}{be-af}\right)}{(bc-ad)(be-af)(1+n)}-\frac{c(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{d(e+fx)}{de-cf}\right)}{(bc-ad)(de-cf)(1+n)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 116, normalized size = 0.94

$$\frac{(e+fx)^{n+1}\left(a(cf-de)\,{}_2F_1\left(1,n+1;n+2;\frac{b(e+fx)}{be-af}\right)+c(be-af)\,{}_2F_1\left(1,n+1;n+2;\frac{d(e+fx)}{de-cf}\right)\right)}{(n+1)(bc-ad)(be-af)(cf-de)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
```

```
[Out] ((e + f*x)^(1 + n)*(a*(-(d*e) + c*f)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*
(e + f*x))/(b*e - a*f)] + c*(b*e - a*f)*Hypergeometric2F1[1, 1 + n, 2 + n,
(d*(e + f*x))/(d*e - c*f)]))/((b*c - a*d)*(b*e - a*f)*(-(d*e) + c*f)*(1 + n
))
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(fx+e\right)^{n}x}{bdx^{2}+ac+(bc+ad)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n*x/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx+e\right)^{n}x}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n*x/((b*x + a)*(d*x + c)), x)
```

maple [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{x\left(fx+e\right)^{n}}{(bx+a)\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

```
[Out] int(x*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx+e\right)^{n}x}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n*x/((b*x + a)*(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,{\left(e+f\,x\right)}^n}{\left(a+b\,x\right)\,\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(e + f*x)^n)/((a + b*x)*(c + d*x)),x)
```

```
[Out] int((x*(e + f*x)^n)/((a + b*x)*(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(e+fx\right)^{n}}{\left(a+bx\right)\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x+e)**n/(b*x+a)/(d*x+c),x)
[Out] Integral(x*(e + f*x)**n/((a + b*x)*(c + d*x)), x)
```

3.116 $\int \frac{(e+fx)^n}{(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=124

$$\frac{d(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{d(e+fx)}{de-cf}\right)}{(n+1)(bc-ad)(de-cf)}-\frac{b(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b(e+fx)}{be-af}\right)}{(n+1)(bc-ad)(be-af)}$$

```
[Out] -b*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b*(f*x+e)/(-a*f+b*e))/(-a*d+b*c)/
(-a*f+b*e)/(1+n)+d*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],d*(f*x+e)/(-c*f+d
*e))/(-a*d+b*c)/(-c*f+d*e)/(1+n)
```

Rubi [A] time = 0.03, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {86, 68}

$$\frac{d(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{d(e+fx)}{de-cf}\right)}{(n+1)(bc-ad)(de-cf)}-\frac{b(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b(e+fx)}{be-af}\right)}{(n+1)(bc-ad)(be-af)}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^n/((a + b*x)*(c + d*x)),x]
```

```
[Out] -((b*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e + f*x))/(b*
e - a*f)])/((b*c - a*d)*(b*e - a*f)*(1 + n))) + (d*(e + f*x)^(1 + n)*Hyperg
eometric2F1[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])/((b*c - a*d)*(d*e
- c*f)*(1 + n))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{(e+fx)^n}{(a+bx)(c+dx)}\,dx &= \frac{b\int \frac{(e+fx)^n}{a+bx}\,dx}{bc-ad}-\frac{d\int \frac{(e+fx)^n}{c+dx}\,dx}{bc-ad}\\ &= -\frac{b(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{b(e+fx)}{be-af}\right)}{(bc-ad)(be-af)(1+n)}+\frac{d(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{d(e+fx)}{de-cf}\right)}{(bc-ad)(de-cf)(1+n)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 116, normalized size = 0.94

$$\frac{(e+fx)^{n+1}\left(b(de-cf)\,{}_2F_1\left(1,n+1;n+2;\frac{b(e+fx)}{be-af}\right)+d(af-be)\,{}_2F_1\left(1,n+1;n+2;\frac{d(e+fx)}{de-cf}\right)\right)}{(n+1)(bc-ad)(be-af)(cf-de)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^n/((a + b*x)*(c + d*x)),x]
```

```
[Out] ((e + f*x)^(1 + n)*(b*(d*e - c*f)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e
+ f*x))/(b*e - a*f)] + d*(-(b*e) + a*f)*Hypergeometric2F1[1, 1 + n, 2 + n,
(d*(e + f*x))/(d*e - c*f)]))/((b*c - a*d)*(b*e - a*f)*(-(d*e) + c*f)*(1 + n
))
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(fx+e\right)^n}{bdx^2+ac+(bc+ad)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx+e\right)^n}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n/((b*x + a)*(d*x + c)), x)
```

maple [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{\left(fx+e\right)^n}{(bx+a)\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^n/(b*x+a)/(d*x+c),x)
```

```
[Out] int((f*x+e)^n/(b*x+a)/(d*x+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx+e\right)^n}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n/((b*x + a)*(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(e+f\,x\right)^n}{\left(a+b\,x\right)\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^n/((a + b*x)*(c + d*x)),x)
```

```
[Out] int((e + f*x)^n/((a + b*x)*(c + d*x)), x)
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(e+fx\right)^{n}}{\left(a+bx\right)\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**n/(b*x+a)/(d*x+c),x)
```

```
[Out] Integral((e + f*x)**n/((a + b*x)*(c + d*x)), x)
```

3.117 $\int \frac{(e+fx)^n}{x(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=175

$$\frac{b^2(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b(e+fx)}{be-af}\right)}{a(n+1)(bc-ad)(be-af)}-\frac{d^2(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{d(e+fx)}{de-cf}\right)}{c(n+1)(bc-ad)(de-cf)}-\frac{(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n\right.}{ace(n+1)}$$

```
[Out] b^2*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b*(f*x+e)/(-a*f+b*e))/a/(-a*d+b*
c)/(-a*f+b*e)/(1+n)-d^2*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],d*(f*x+e)/(-
c*f+d*e))/c/(-a*d+b*c)/(-c*f+d*e)/(1+n)-(f*x+e)^(1+n)*hypergeom([1, 1+n],[2
+n],1+f*x/e)/a/c/e/(1+n)
```

Rubi [A] time = 0.12, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {180, 65, 68}

$$\frac{b^2(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b(e+fx)}{be-af}\right)}{a(n+1)(bc-ad)(be-af)}-\frac{d^2(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{d(e+fx)}{de-cf}\right)}{c(n+1)(bc-ad)(de-cf)}-\frac{(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n\right.}{ace(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^n/(x*(a + b*x)*(c + d*x)),x]
```

```
[Out] (b^2*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e + f*x))/(b*
e - a*f)])/(a*(b*c - a*d)*(b*e - a*f)*(1 + n)) - (d^2*(e + f*x)^(1 + n)*Hyp
ergeometric2F1[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])/(c*(b*c - a*d)*
(d*e - c*f)*(1 + n)) - ((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n
, 1 + (f*x)/e])/(a*c*e*(1 + n))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rubi steps

$$\int \frac{(e+fx)^n}{x(a+bx)(c+dx)}\,dx = \int \left(\frac{(e+fx)^n}{acx} + \frac{b^2(e+fx)^n}{a(-bc+ad)(a+bx)} + \frac{d^2(e+fx)^n}{c(bc-ad)(c+dx)}\right) dx$$
$$= \frac{\int \frac{(e+fx)^n}{x}\,dx}{ac} - \frac{b^2 \int \frac{(e+fx)^n}{a+bx}\,dx}{a(bc-ad)} + \frac{d^2 \int \frac{(e+fx)^n}{c+dx}\,dx}{c(bc-ad)}$$
$$= \frac{b^2(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{b(e+fx)}{be-af}\right)}{a(bc-ad)(be-af)(1+n)} - \frac{d^2(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{d(e+f}{de-c}\right.}{c(bc-ad)(de-cf)(1+n)}$$

Mathematica [A] time = 0.19, size = 170, normalized size = 0.97

$$-\frac{(e+fx)^{n+1}\left(b^2ce(de-cf)\,{}_2F_1\left(1,n+1;n+2;\frac{b(e+fx)}{be-af}\right)+(af-be)\left(ad^2e\,{}_2F_1\left(1,n+1;n+2;\frac{d(e+fx)}{de-cf}\right)-(bc-a\right.\right.}{ace(n+1)(ad-bc)(af-be)(cf-de)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^n/(x*(a + b*x)*(c + d*x)),x]
```

```
[Out] -(((e + f*x)^(1 + n)*(b^2*c*e*(d*e - c*f)*Hypergeometric2F1[1, 1 + n, 2 + n
, (b*(e + f*x))/(b*e - a*f)] + (-(b*e) + a*f)*(a*d^2*e*Hypergeometric2F1[1,
 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)] - (b*c - a*d)*(-(d*e) + c*f)*Hype
rgeometric2F1[1, 1 + n, 2 + n, 1 + (f*x)/e])))/(a*c*(-(b*c) + a*d)*e*(-(b*e
) + a*f)*(-(d*e) + c*f)*(1 + n)))
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(fx+e)^n}{bdx^3+acx+(bc+ad)x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n/(b*d*x^3 + a*c*x + (b*c + a*d)*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^n}{(bx+a)(dx+c)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x/(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n/((b*x + a)*(d*x + c)*x), x)
```

maple [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^n}{(bx+a)(dx+c)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^n/x/(b*x+a)/(d*x+c),x)
```

```
[Out] int((f*x+e)^n/x/(b*x+a)/(d*x+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^n}{(bx+a)(dx+c)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x/(b*x+a)/(d*x+c),x, algorithm="maxima")
[Out] integrate((f*x + e)^n/((b*x + a)*(d*x + c)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e+f\,x)^n}{x\,(a+b\,x)\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^n/(x*(a + b*x)*(c + d*x)),x)
[Out] int((e + f*x)^n/(x*(a + b*x)*(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^n}{x\,(a+bx)\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**n/x/(b*x+a)/(d*x+c),x)
[Out] Integral((e + f*x)**n/(x*(a + b*x)*(c + d*x)), x)
```

3.118 $\int \frac{(e+fx)^n}{x^2(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=222

$$-\frac{b^3(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b(e+fx)}{be-af}\right)}{a^2(n+1)(bc-ad)(be-af)}+\frac{(ad+bc)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{fx}{e}+1\right)}{a^2c^2e(n+1)}+\frac{d^3(e+fx)^{n+1}\,{}_2}{c^2(n+}$$

```
[Out] -b^3*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b*(f*x+e)/(-a*f+b*e))/a^2/(-a*d
+b*c)/(-a*f+b*e)/(1+n)+d^3*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],d*(f*x+e)
/(-c*f+d*e))/c^2/(-a*d+b*c)/(-c*f+d*e)/(1+n)+(a*d+b*c)*(f*x+e)^(1+n)*hyperg
eom([1, 1+n],[2+n],1+f*x/e)/a^2/c^2/e/(1+n)+f*(f*x+e)^(1+n)*hypergeom([2, 1
+n],[2+n],1+f*x/e)/a/c/e^2/(1+n)
```

Rubi [A] time = 0.15, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {180, 65, 68}

$$-\frac{b^3(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{b(e+fx)}{be-af}\right)}{a^2(n+1)(bc-ad)(be-af)}+\frac{(ad+bc)(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{fx}{e}+1\right)}{a^2c^2e(n+1)}+\frac{d^3(e+fx)^{n+1}\,{}_2}{c^2(n+}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^n/(x^2*(a + b*x)*(c + d*x)),x]
```

```
[Out] -((b^3*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e + f*x))/(
b*e - a*f)])/(a^2*(b*c - a*d)*(b*e - a*f)*(1 + n))) + (d^3*(e + f*x)^(1 + n
)*Hypergeometric2F1[1, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])/(c^2*(b*c
- a*d)*(d*e - c*f)*(1 + n)) + ((b*c + a*d)*(e + f*x)^(1 + n)*Hypergeometric
2F1[1, 1 + n, 2 + n, 1 + (f*x)/e])/(a^2*c^2*e*(1 + n)) + (f*(e + f*x)^(1 +
n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (f*x)/e])/(a*c*e^2*(1 + n))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rubi steps

$$\int \frac{(e+fx)^n}{x^2(a+bx)(c+dx)}\,dx = \int \left(\frac{(e+fx)^n}{acx^2} + \frac{(-bc-ad)(e+fx)^n}{a^2c^2x} - \frac{b^3(e+fx)^n}{a^2(-bc+ad)(a+bx)} - \frac{d^3(e+fx)^n}{c^2(bc-ad)(c+dx)}\right) d$$

$$= \frac{\int \frac{(e+fx)^n}{x^2}\,dx}{ac} + \frac{b^3 \int \frac{(e+fx)^n}{a+bx}\,dx}{a^2(bc-ad)} - \frac{d^3 \int \frac{(e+fx)^n}{c+dx}\,dx}{c^2(bc-ad)} - \frac{(bc+ad)\int \frac{(e+fx)^n}{x}\,dx}{a^2c^2}$$

$$= -\frac{b^3(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{b(e+fx)}{be-af}\right)}{a^2(bc-ad)(be-af)(1+n)} + \frac{d^3(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{d(e+f}{de-c}\right.}{c^2(bc-ad)(de-cf)(1+n)}$$

Mathematica [A] time = 0.28, size = 177, normalized size = 0.80

$$\frac{(e+fx)^{n+1}\left(\frac{\frac{e(ad+bc)\,{}_2F_1\left(1,n+1;n+2;\frac{fx}{e}+1\right)+acf\,{}_2F_1\left(2,n+1;n+2;\frac{fx}{e}+1\right)}{a^2e^2}-\frac{d^3\,{}_2F_1\left(1,n+1;n+2;\frac{d(e+fx)}{de-cf}\right)}{(bc-ad)(cf-de)}}{c^2}-\frac{b^3\,{}_2F_1\left(1,n+1;n+2;\frac{b(e+fx)}{be-af}\right)}{a^2(bc-ad)(be-af)}\right)}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^n/(x^2*(a + b*x)*(c + d*x)),x]
```

```
[Out] ((e + f*x)^(1 + n)*(-((b^3*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(e + f*x))
/(b*e - a*f)])/(a^2*(b*c - a*d)*(b*e - a*f))) + (-((d^3*Hypergeometric2F1[1
, 1 + n, 2 + n, (d*(e + f*x))/(d*e - c*f)])/((b*c - a*d)*(-(d*e) + c*f))) +
 ((b*c + a*d)*e*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (f*x)/e] + a*c*f*Hyp
ergeometric2F1[2, 1 + n, 2 + n, 1 + (f*x)/e])/(a^2*e^2))/c^2))/(1 + n)
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(fx+e\right)^n}{bdx^4+acx^2+(bc+ad)x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x^2/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n/(b*d*x^4 + a*c*x^2 + (b*c + a*d)*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx+e\right)^n}{(bx+a)(dx+c)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x^2/(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n/((b*x + a)*(d*x + c)*x^2), x)
```

maple [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{\left(fx+e\right)^n}{(bx+a)\,(dx+c)\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^n/x^2/(b*x+a)/(d*x+c),x)
```

```
[Out] int((f*x+e)^n/x^2/(b*x+a)/(d*x+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^n}{(bx+a)(dx+c)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x^2/(b*x+a)/(d*x+c),x, algorithm="maxima")
[Out] integrate((f*x + e)^n/((b*x + a)*(d*x + c)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)^n}{x^2\,(a+b\,x)\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^n/(x^2*(a + b*x)*(c + d*x)),x)
[Out] int((e + f*x)^n/(x^2*(a + b*x)*(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**n/x**2/(b*x+a)/(d*x+c),x)
[Out] Timed out
```

3.119 $\int (a+bx)^m(c+dx)(e+fx)(g+hx)\,dx$

Optimal. Leaf size=167

$$\frac{(a+bx)^{m+2}\left(3a^2dfh-2ab(cfh+deh+dfg)+b^2(ceh+cfg+deg)\right)}{b^4(m+2)}+\frac{(bc-ad)(be-af)(bg-ah)(a+bx)^{m+1}}{b^4(m+1)}-$$

[Out] (-a*d+b*c)*(-a*f+b*e)*(-a*h+b*g)*(b*x+a)^(1+m)/b^4/(1+m)+(3*a^2*d*f*h+b^2*(c*e*h+c*f*g+d*e*g)-2*a*b*(c*f*h+d*e*h+d*f*g))*(b*x+a)^(2+m)/b^4/(2+m)-(3*a*d*f*h-b*(c*f*h+d*e*h+d*f*g))*(b*x+a)^(3+m)/b^4/(3+m)+d*f*h*(b*x+a)^(4+m)/b^4/(4+m)

Rubi [A] time = 0.13, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {142}

$$\frac{(a+bx)^{m+2}\left(3a^2dfh-2ab(cfh+deh+dfg)+b^2(ceh+cfg+deg)\right)}{b^4(m+2)}+\frac{(bc-ad)(be-af)(bg-ah)(a+bx)^{m+1}}{b^4(m+1)}-$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^m*(c + d*x)*(e + f*x)*(g + h*x),x]

[Out] ((b*c - a*d)*(b*e - a*f)*(b*g - a*h)*(a + b*x)^(1 + m))/(b^4*(1 + m)) + ((3*a^2*d*f*h + b^2*(d*e*g + c*f*g + c*e*h) - 2*a*b*(d*f*g + d*e*h + c*f*h))*(a + b*x)^(2 + m))/(b^4*(2 + m)) - ((3*a*d*f*h - b*(d*f*g + d*e*h + c*f*h))*(a + b*x)^(3 + m))/(b^4*(3 + m)) + (d*f*h*(a + b*x)^(4 + m))/(b^4*(4 + m))

Rule 142

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_))*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && (IGtQ[m, 0] || IntegersQ[m, n])
```

Rubi steps

$$\int (a+bx)^m(c+dx)(e+fx)(g+hx)\,dx = \int \left(\frac{(bc-ad)(be-af)(bg-ah)(a+bx)^m}{b^3}+\frac{\left(3a^2dfh+b^2(deg+cfg+\right.}{}\right.$$

$$=\frac{(bc-ad)(be-af)(bg-ah)(a+bx)^{1+m}}{b^4(1+m)}+\frac{\left(3a^2dfh+b^2(deg+cfg+c\right.}{}$$

Mathematica [A] time = 0.23, size = 149, normalized size = 0.89

$$\frac{(a+bx)^{m+1}\left(\frac{(a+bx)\left(3a^2dfh-2ab(cfh+deh+dfg)+b^2(ceh+cfg+deg)\right)}{m+2}+\frac{(a+bx)^2(b(cfh+deh+dfg)-3adfh)}{m+3}+\frac{(bc-ad)(be-af)(bg-ah)}{m+1}+\frac{dfh(a+}{m+}\right.}{b^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^m*(c + d*x)*(e + f*x)*(g + h*x),x]

[Out] ((a + b*x)^(1 + m)*(((b*c - a*d)*(b*e - a*f)*(b*g - a*h))/(1 + m) + ((3*a^2*d*f*h + b^2*(d*e*g + c*f*g + c*e*h) - 2*a*b*(d*f*g + d*e*h + c*f*h))*(a + b*x))/(2 + m) + ((-3*a*d*f*h + b*(d*f*g + d*e*h + c*f*h))*(a + b*x)^2)/(3 + m) + (d*f*h*(a + b*x)^3)/(4 + m)))/b^4

fricas [B] time = 0.92, size = 877, normalized size = 5.25

$$\frac{\left(ab^3cegm^3 + \left(b^4dfhm^3 + 6\,b^4dfhm^2 + 11\,b^4dfhm + 6\,b^4dfh\right)x^4 + \left(8\,b^4dfg + \left(b^4dfg + \left(b^4de + \left(b^4c + ab^3d\right)\right.\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)*(f*x+e)*(h*x+g),x, algorithm="fricas")
```

```
[Out] (a*b^3*c*e*g*m^3 + (b^4*d*f*h*m^3 + 6*b^4*d*f*h*m^2 + 11*b^4*d*f*h*m + 6*b^
4*d*f*h)*x^4 + (8*b^4*d*f*g + (b^4*d*f*g + (b^4*d*e + (b^4*c + a*b^3*d)*f)*
h)*m^3 + (7*b^4*d*f*g + (7*b^4*d*e + (7*b^4*c + 3*a*b^3*d)*f)*h)*m^2 + 8*(b
^4*d*e + b^4*c*f)*h + 2*(7*b^4*d*f*g + (7*b^4*d*e + (7*b^4*c + a*b^3*d)*f)*
h)*m)*x^3 - (a^2*b^2*c*e*h + (a^2*b^2*c*f - (9*a*b^3*c - a^2*b^2*d)*e)*g)*m
^2 + (12*b^4*c*e*h + ((b^4*d*e + (b^4*c + a*b^3*d)*f)*g + (a*b^3*c*f + (b^4
*c + a*b^3*d)*e)*h)*m^3 + ((8*b^4*d*e + (8*b^4*c + 5*a*b^3*d)*f)*g + ((8*b^
4*c + 5*a*b^3*d)*e + (5*a*b^3*c - 3*a^2*b^2*d)*f)*h)*m^2 + 12*(b^4*d*e + b^
4*c*f)*g + ((19*b^4*d*e + (19*b^4*c + 4*a*b^3*d)*f)*g + ((19*b^4*c + 4*a*b^
3*d)*e + (4*a*b^3*c - 3*a^2*b^2*d)*f)*h)*m)*x^2 + 4*(3*(2*a*b^3*c - a^2*b^2
*d)*e - (3*a^2*b^2*c - 2*a^3*b*d)*f)*g - 2*(2*(3*a^2*b^2*c - 2*a^3*b*d)*e -
 (4*a^3*b*c - 3*a^4*d)*f)*h + (((26*a*b^3*c - 7*a^2*b^2*d)*e - (7*a^2*b^2*c
 - 2*a^3*b*d)*f)*g + (2*a^3*b*c*f - (7*a^2*b^2*c - 2*a^3*b*d)*e)*h)*m + (24
*b^4*c*e*g + (a*b^3*c*e*h + (a*b^3*c*f + (b^4*c + a*b^3*d)*e)*g)*m^3 + (((9
*b^4*c + 7*a*b^3*d)*e + (7*a*b^3*c - 2*a^2*b^2*d)*f)*g - (2*a^2*b^2*c*f - (
7*a*b^3*c - 2*a^2*b^2*d)*e)*h)*m^2 + 2*(((13*b^4*c + 6*a*b^3*d)*e + 2*(3*a*
b^3*c - 2*a^2*b^2*d)*f)*g + (2*(3*a*b^3*c - 2*a^2*b^2*d)*e - (4*a^2*b^2*c -
 3*a^3*b*d)*f)*h)*m)*x)*(b*x + a)^m/(b^4*m^4 + 10*b^4*m^3 + 35*b^4*m^2 + 50
*b^4*m + 24*b^4)
```

giac [B] time = 1.03, size = 1665, normalized size = 9.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)*(f*x+e)*(h*x+g),x, algorithm="giac")
```

```
[Out] ((b*x + a)^m*b^4*d*f*h*m^3*x^4 + (b*x + a)^m*b^4*d*f*g*m^3*x^3 + (b*x + a)^
m*b^4*c*f*h*m^3*x^3 + (b*x + a)^m*a*b^3*d*f*h*m^3*x^3 + 6*(b*x + a)^m*b^4*d
*f*h*m^2*x^4 + (b*x + a)^m*b^4*d*h*m^3*x^3*e + (b*x + a)^m*b^4*c*f*g*m^3*x^
2 + (b*x + a)^m*a*b^3*d*f*g*m^3*x^2 + (b*x + a)^m*a*b^3*c*f*h*m^3*x^2 + 7*(
b*x + a)^m*b^4*d*f*g*m^2*x^3 + 7*(b*x + a)^m*b^4*c*f*h*m^2*x^3 + 3*(b*x + a
)^m*a*b^3*d*f*h*m^2*x^3 + 11*(b*x + a)^m*b^4*d*f*h*m*x^4 + (b*x + a)^m*b^4*
d*g*m^3*x^2*e + (b*x + a)^m*b^4*c*h*m^3*x^2*e + (b*x + a)^m*a*b^3*d*h*m^3*x
^2*e + 7*(b*x + a)^m*b^4*d*h*m^2*x^3*e + (b*x + a)^m*a*b^3*c*f*g*m^3*x + 8*
(b*x + a)^m*b^4*c*f*g*m^2*x^2 + 5*(b*x + a)^m*a*b^3*d*f*g*m^2*x^2 + 5*(b*x
+ a)^m*a*b^3*c*f*h*m^2*x^2 - 3*(b*x + a)^m*a^2*b^2*d*f*h*m^2*x^2 + 14*(b*x
+ a)^m*b^4*d*f*g*m*x^3 + 14*(b*x + a)^m*b^4*c*f*h*m*x^3 + 2*(b*x + a)^m*a*b
^3*d*f*h*m*x^3 + 6*(b*x + a)^m*b^4*d*f*h*x^4 + (b*x + a)^m*b^4*c*g*m^3*x*e
+ (b*x + a)^m*a*b^3*d*g*m^3*x*e + (b*x + a)^m*a*b^3*c*h*m^3*x*e + 8*(b*x +
a)^m*b^4*d*g*m^2*x^2*e + 8*(b*x + a)^m*b^4*c*h*m^2*x^2*e + 5*(b*x + a)^m*a*
b^3*d*h*m^2*x^2*e + 14*(b*x + a)^m*b^4*d*h*m*x^3*e + 7*(b*x + a)^m*a*b^3*c*
f*g*m^2*x - 2*(b*x + a)^m*a^2*b^2*d*f*g*m^2*x - 2*(b*x + a)^m*a^2*b^2*c*f*h
*m^2*x + 19*(b*x + a)^m*b^4*c*f*g*m*x^2 + 4*(b*x + a)^m*a*b^3*d*f*g*m*x^2 +
 4*(b*x + a)^m*a*b^3*c*f*h*m*x^2 - 3*(b*x + a)^m*a^2*b^2*d*f*h*m*x^2 + 8*(b
*x + a)^m*b^4*d*f*g*x^3 + 8*(b*x + a)^m*b^4*c*f*h*x^3 + (b*x + a)^m*a*b^3*c
*g*m^3*e + 9*(b*x + a)^m*b^4*c*g*m^2*x*e + 7*(b*x + a)^m*a*b^3*d*g*m^2*x*e
+ 7*(b*x + a)^m*a*b^3*c*h*m^2*x*e - 2*(b*x + a)^m*a^2*b^2*d*h*m^2*x*e + 19*
(b*x + a)^m*b^4*d*g*m*x^2*e + 19*(b*x + a)^m*b^4*c*h*m*x^2*e + 4*(b*x + a)^
m*a*b^3*d*h*m*x^2*e + 8*(b*x + a)^m*b^4*d*h*x^3*e - (b*x + a)^m*a^2*b^2*c*f
*g*m^2 + 12*(b*x + a)^m*a*b^3*c*f*g*m*x - 8*(b*x + a)^m*a^2*b^2*d*f*g*m*x -
```

```
8*(b*x + a)^m*a^2*b^2*c*f*h*m*x + 6*(b*x + a)^m*a^3*b*d*f*h*m*x + 12*(b*x
+ a)^m*b^4*c*f*g*x^2 + 9*(b*x + a)^m*a*b^3*c*g*m^2*e - (b*x + a)^m*a^2*b^2*
d*g*m^2*e - (b*x + a)^m*a^2*b^2*c*h*m^2*e + 26*(b*x + a)^m*b^4*c*g*m*x*e +
12*(b*x + a)^m*a*b^3*d*g*m*x*e + 12*(b*x + a)^m*a*b^3*c*h*m*x*e - 8*(b*x +
a)^m*a^2*b^2*d*h*m*x*e + 12*(b*x + a)^m*b^4*d*g*x^2*e + 12*(b*x + a)^m*b^4*
c*h*x^2*e - 7*(b*x + a)^m*a^2*b^2*c*f*g*m + 2*(b*x + a)^m*a^3*b*d*f*g*m + 2
*(b*x + a)^m*a^3*b*c*f*h*m + 26*(b*x + a)^m*a*b^3*c*g*m*e - 7*(b*x + a)^m*a
^2*b^2*d*g*m*e - 7*(b*x + a)^m*a^2*b^2*c*h*m*e + 2*(b*x + a)^m*a^3*b*d*h*m*
e + 24*(b*x + a)^m*b^4*c*g*x*e - 12*(b*x + a)^m*a^2*b^2*c*f*g + 8*(b*x + a)
^m*a^3*b*d*f*g + 8*(b*x + a)^m*a^3*b*c*f*h - 6*(b*x + a)^m*a^4*d*f*h + 24*(
b*x + a)^m*a*b^3*c*g*e - 12*(b*x + a)^m*a^2*b^2*d*g*e - 12*(b*x + a)^m*a^2*
b^2*c*h*e + 8*(b*x + a)^m*a^3*b*d*h*e)/(b^4*m^4 + 10*b^4*m^3 + 35*b^4*m^2 +
 50*b^4*m + 24*b^4)
```

maple [B] time = 0.01, size = 726, normalized size = 4.35

$$-\frac{\left(-b^3 dfh\, m^3 x^3 - b^3 cfh\, m^3 x^2 - b^3 deh\, m^3 x^2 - b^3 dfg\, m^3 x^2 - 6b^3 dfh\, m^2 x^3 + 3a\, b^2 dfh\, m^2 x^2 - b^3 ceh\, m^3 x - b^3 cfg\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)*(f*x+e)*(h*x+g),x)
```

```
[Out] -(b*x+a)^(m+1)*(-b^3*d*f*h*m^3*x^3-b^3*c*f*h*m^3*x^2-b^3*d*e*h*m^3*x^2-b^3*
d*f*g*m^3*x^2-6*b^3*d*f*h*m^2*x^3+3*a*b^2*d*f*h*m^2*x^2-b^3*c*e*h*m^3*x-b^3
*c*f*g*m^3*x-7*b^3*c*f*h*m^2*x^2-b^3*d*e*g*m^3*x-7*b^3*d*e*h*m^2*x^2-7*b^3*
d*f*g*m^2*x^2-11*b^3*d*f*h*m*x^3+2*a*b^2*c*f*h*m^2*x+2*a*b^2*d*e*h*m^2*x+2*
a*b^2*d*f*g*m^2*x+9*a*b^2*d*f*h*m*x^2-b^3*c*e*g*m^3-8*b^3*c*e*h*m^2*x-8*b^3
*c*f*g*m^2*x-14*b^3*c*f*h*m*x^2-8*b^3*d*e*g*m^2*x-14*b^3*d*e*h*m*x^2-14*b^3
*d*f*g*m*x^2-6*b^3*d*f*h*x^3-6*a^2*b*d*f*h*m*x+a*b^2*c*e*h*m^2+a*b^2*c*f*g*
m^2+10*a*b^2*c*f*h*m*x+a*b^2*d*e*g*m^2+10*a*b^2*d*e*h*m*x+10*a*b^2*d*f*g*m*
x+6*a*b^2*d*f*h*x^2-9*b^3*c*e*g*m^2-19*b^3*c*e*h*m*x-19*b^3*c*f*g*m*x-8*b^3
*c*f*h*x^2-19*b^3*d*e*g*m*x-8*b^3*d*e*h*x^2-8*b^3*d*f*g*x^2-2*a^2*b*c*f*h*m
-2*a^2*b*d*e*h*m-2*a^2*b*d*f*g*m-6*a^2*b*d*f*h*x+7*a*b^2*c*e*h*m+7*a*b^2*c*
f*g*m+8*a*b^2*c*f*h*x+7*a*b^2*d*e*g*m+8*a*b^2*d*e*h*x+8*a*b^2*d*f*g*x-26*b^
3*c*e*g*m-12*b^3*c*e*h*x-12*b^3*c*f*g*x-12*b^3*d*e*g*x+6*a^3*d*f*h-8*a^2*b*
c*f*h-8*a^2*b*d*e*h-8*a^2*b*d*f*g+12*a*b^2*c*e*h+12*a*b^2*c*f*g+12*a*b^2*d*
e*g-24*b^3*c*e*g)/b^4/(m^4+10*m^3+35*m^2+50*m+24)
```

maxima [B] time = 0.54, size = 474, normalized size = 2.84

$$\frac{\left(b^2(m+1)x^2 + abmx - a^2\right)(bx+a)^m deg}{\left(m^2+3\,m+2\right)b^2} + \frac{\left(b^2(m+1)x^2 + abmx - a^2\right)(bx+a)^m cfg}{\left(m^2+3\,m+2\right)b^2} + \frac{\left(b^2(m+1)x^2 + abmx - a^2\right)(}{\left(m^2+3\,m+2\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)*(f*x+e)*(h*x+g),x, algorithm="maxima")
```

```
[Out] (b^2*(m + 1)*x^2 + a*b*m*x - a^2)*(b*x + a)^m*d*e*g/((m^2 + 3*m + 2)*b^2) +
 (b^2*(m + 1)*x^2 + a*b*m*x - a^2)*(b*x + a)^m*c*f*g/((m^2 + 3*m + 2)*b^2)
+ (b^2*(m + 1)*x^2 + a*b*m*x - a^2)*(b*x + a)^m*c*e*h/((m^2 + 3*m + 2)*b^2)
 + (b*x + a)^(m + 1)*c*e*g/(b*(m + 1)) + ((m^2 + 3*m + 2)*b^3*x^3 + (m^2 +
m)*a*b^2*x^2 - 2*a^2*b*m*x + 2*a^3)*(b*x + a)^m*d*f*g/((m^3 + 6*m^2 + 11*m
+ 6)*b^3) + ((m^2 + 3*m + 2)*b^3*x^3 + (m^2 + m)*a*b^2*x^2 - 2*a^2*b*m*x +
2*a^3)*(b*x + a)^m*d*e*h/((m^3 + 6*m^2 + 11*m + 6)*b^3) + ((m^2 + 3*m + 2)*
b^3*x^3 + (m^2 + m)*a*b^2*x^2 - 2*a^2*b*m*x + 2*a^3)*(b*x + a)^m*c*f*h/((m^
3 + 6*m^2 + 11*m + 6)*b^3) + ((m^3 + 6*m^2 + 11*m + 6)*b^4*x^4 + (m^3 + 3*m
^2 + 2*m)*a*b^3*x^3 - 3*(m^2 + m)*a^2*b^2*x^2 + 6*a^3*b*m*x - 6*a^4)*(b*x +
 a)^m*d*f*h/((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*b^4)
```

mupad [B] time = 2.95, size = 819, normalized size = 4.90

$$\frac{x\,(a+b\,x)^m\,\left(24\,b^4\,c\,e\,g+9\,b^4\,c\,e\,g\,m^2+b^4\,c\,e\,g\,m^3+26\,b^4\,c\,e\,g\,m+12\,a\,b^3\,c\,e\,h\,m+12\,a\,b^3\,c\,f\,g\,m+12\,a\,b^3\,d\,e}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)*(g + h*x)*(a + b*x)^m*(c + d*x),x)
```

```
[Out] (x*(a + b*x)^m*(24*b^4*c*e*g + 9*b^4*c*e*g*m^2 + b^4*c*e*g*m^3 + 26*b^4*c*e
*g*m + 12*a*b^3*c*e*h*m + 12*a*b^3*c*f*g*m + 12*a*b^3*d*e*g*m + 6*a^3*b*d*f
*h*m + 7*a*b^3*c*e*h*m^2 + 7*a*b^3*c*f*g*m^2 + 7*a*b^3*d*e*g*m^2 + a*b^3*c*
e*h*m^3 + a*b^3*c*f*g*m^3 + a*b^3*d*e*g*m^3 - 8*a^2*b^2*c*f*h*m - 8*a^2*b^2
*d*e*h*m - 8*a^2*b^2*d*f*g*m - 2*a^2*b^2*c*f*h*m^2 - 2*a^2*b^2*d*e*h*m^2 -
2*a^2*b^2*d*f*g*m^2))/(b^4*(50*m + 35*m^2 + 10*m^3 + m^4 + 24)) - ((a + b*x
)^m*(6*a^4*d*f*h + 12*a^2*b^2*c*e*h + 12*a^2*b^2*c*f*g + 12*a^2*b^2*d*e*g -
 24*a*b^3*c*e*g - 8*a^3*b*c*f*h - 8*a^3*b*d*e*h - 8*a^3*b*d*f*g - 26*a*b^3*
c*e*g*m - 2*a^3*b*c*f*h*m - 2*a^3*b*d*e*h*m - 2*a^3*b*d*f*g*m - 9*a*b^3*c*e
*g*m^2 - a*b^3*c*e*g*m^3 + 7*a^2*b^2*c*e*h*m + 7*a^2*b^2*c*f*g*m + 7*a^2*b^
2*d*e*g*m + a^2*b^2*c*e*h*m^2 + a^2*b^2*c*f*g*m^2 + a^2*b^2*d*e*g*m^2))/(b^
4*(50*m + 35*m^2 + 10*m^3 + m^4 + 24)) + (x^3*(a + b*x)^m*(3*m + m^2 + 2)*(
4*b*c*f*h + 4*b*d*e*h + 4*b*d*f*g + a*d*f*h*m + b*c*f*h*m + b*d*e*h*m + b*d
*f*g*m))/(b*(50*m + 35*m^2 + 10*m^3 + m^4 + 24)) + (x^2*(m + 1)*(a + b*x)^m
*(12*b^2*c*e*h + 12*b^2*c*f*g + 12*b^2*d*e*g + b^2*c*e*h*m^2 + b^2*c*f*g*m^
2 + b^2*d*e*g*m^2 + 7*b^2*c*e*h*m + 7*b^2*c*f*g*m + 7*b^2*d*e*g*m - 3*a^2*d
*f*h*m + a*b*c*f*h*m^2 + a*b*d*e*h*m^2 + a*b*d*f*g*m^2 + 4*a*b*c*f*h*m + 4*
a*b*d*e*h*m + 4*a*b*d*f*g*m))/(b^2*(50*m + 35*m^2 + 10*m^3 + m^4 + 24)) + (
d*f*h*x^4*(a + b*x)^m*(11*m + 6*m^2 + m^3 + 6))/(50*m + 35*m^2 + 10*m^3 + m
^4 + 24)
```

sympy [A] time = 8.12, size = 8221, normalized size = 49.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)*(f*x+e)*(h*x+g),x)
```

```
[Out] Piecewise((a**m*(c*e*g*x + c*e*h*x**2/2 + c*f*g*x**2/2 + c*f*h*x**3/3 + d*e
*g*x**2/2 + d*e*h*x**3/3 + d*f*g*x**3/3 + d*f*h*x**4/4), Eq(b, 0)), (6*a**3
*d*f*h*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7
*x**3) + 11*a**3*d*f*h/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b
**7*x**3) - 2*a**2*b*c*f*h/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 +
 6*b**7*x**3) - 2*a**2*b*d*e*h/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x*
*2 + 6*b**7*x**3) - 2*a**2*b*d*f*g/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**
6*x**2 + 6*b**7*x**3) + 18*a**2*b*d*f*h*x*log(a/b + x)/(6*a**3*b**4 + 18*a*
*2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 27*a**2*b*d*f*h*x/(6*a**3*b**4
+ 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - a*b**2*c*e*h/(6*a**3*b**
4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - a*b**2*c*f*g/(6*a**3*b
**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 6*a*b**2*c*f*h*x/(6*
a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - a*b**2*d*e*g/(
6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 6*a*b**2*d*e
*h*x/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 6*a*b*
*2*d*f*g*x/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) +
18*a*b**2*d*f*h*x**2*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6
*x**2 + 6*b**7*x**3) + 18*a*b**2*d*f*h*x**2/(6*a**3*b**4 + 18*a**2*b**5*x +
 18*a*b**6*x**2 + 6*b**7*x**3) - 2*b**3*c*e*g/(6*a**3*b**4 + 18*a**2*b**5*x
 + 18*a*b**6*x**2 + 6*b**7*x**3) - 3*b**3*c*e*h*x/(6*a**3*b**4 + 18*a**2*b*
*5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 3*b**3*c*f*g*x/(6*a**3*b**4 + 18*a**
2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 6*b**3*c*f*h*x**2/(6*a**3*b**4 +
```

```
18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 3*b**3*d*e*g*x/(6*a**3*b*
*4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 6*b**3*d*e*h*x**2/(6*
a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 6*b**3*d*f*g*x
**2/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 6*b**3*
d*f*h*x**3*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*
b**7*x**3), Eq(m, -4)), (-6*a**3*d*f*h*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5
*x + 2*b**6*x**2) - 9*a**3*d*f*h/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) +
 2*a**2*b*c*f*h*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 3*a
**2*b*c*f*h/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 2*a**2*b*d*e*h*log(a
/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 3*a**2*b*d*e*h/(2*a**2*b
**4 + 4*a*b**5*x + 2*b**6*x**2) + 2*a**2*b*d*f*g*log(a/b + x)/(2*a**2*b**4
+ 4*a*b**5*x + 2*b**6*x**2) + 3*a**2*b*d*f*g/(2*a**2*b**4 + 4*a*b**5*x + 2*
b**6*x**2) - 12*a**2*b*d*f*h*x*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b
**6*x**2) - 12*a**2*b*d*f*h*x/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - a*
b**2*c*e*h/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - a*b**2*c*f*g/(2*a**2*
b**4 + 4*a*b**5*x + 2*b**6*x**2) + 4*a*b**2*c*f*h*x*log(a/b + x)/(2*a**2*b*
*4 + 4*a*b**5*x + 2*b**6*x**2) + 4*a*b**2*c*f*h*x/(2*a**2*b**4 + 4*a*b**5*x
 + 2*b**6*x**2) - a*b**2*d*e*g/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 4
*a*b**2*d*e*h*x*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 4*a
*b**2*d*e*h*x/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 4*a*b**2*d*f*g*x*l
og(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 4*a*b**2*d*f*g*x/(2*
a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 6*a*b**2*d*f*h*x**2*log(a/b + x)/(2
*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - b**3*c*e*g/(2*a**2*b**4 + 4*a*b**5
*x + 2*b**6*x**2) - 2*b**3*c*e*h*x/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2)
 - 2*b**3*c*f*g*x/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 2*b**3*c*f*h*x
**2*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 2*b**3*d*e*g*x/
(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 2*b**3*d*e*h*x**2*log(a/b + x)/(
2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 2*b**3*d*f*g*x**2*log(a/b + x)/(2
*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 2*b**3*d*f*h*x**3/(2*a**2*b**4 + 4
*a*b**5*x + 2*b**6*x**2), Eq(m, -3)), (6*a**3*d*f*h*log(a/b + x)/(2*a*b**4
+ 2*b**5*x) + 6*a**3*d*f*h/(2*a*b**4 + 2*b**5*x) - 4*a**2*b*c*f*h*log(a/b +
 x)/(2*a*b**4 + 2*b**5*x) - 4*a**2*b*c*f*h/(2*a*b**4 + 2*b**5*x) - 4*a**2*b
*d*e*h*log(a/b + x)/(2*a*b**4 + 2*b**5*x) - 4*a**2*b*d*e*h/(2*a*b**4 + 2*b*
*5*x) - 4*a**2*b*d*f*g*log(a/b + x)/(2*a*b**4 + 2*b**5*x) - 4*a**2*b*d*f*g/
(2*a*b**4 + 2*b**5*x) + 6*a**2*b*d*f*h*x*log(a/b + x)/(2*a*b**4 + 2*b**5*x)
 + 2*a*b**2*c*e*h*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 2*a*b**2*c*e*h/(2*a*
b**4 + 2*b**5*x) + 2*a*b**2*c*f*g*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 2*a*
b**2*c*f*g/(2*a*b**4 + 2*b**5*x) - 4*a*b**2*c*f*h*x*log(a/b + x)/(2*a*b**4
+ 2*b**5*x) + 2*a*b**2*d*e*g*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 2*a*b**2*
d*e*g/(2*a*b**4 + 2*b**5*x) - 4*a*b**2*d*e*h*x*log(a/b + x)/(2*a*b**4 + 2*b
**5*x) - 4*a*b**2*d*f*g*x*log(a/b + x)/(2*a*b**4 + 2*b**5*x) - 3*a*b**2*d*f
*h*x**2/(2*a*b**4 + 2*b**5*x) - 2*b**3*c*e*g/(2*a*b**4 + 2*b**5*x) + 2*b**3
*c*e*h*x*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 2*b**3*c*f*g*x*log(a/b + x)/(
2*a*b**4 + 2*b**5*x) + 2*b**3*c*f*h*x**2/(2*a*b**4 + 2*b**5*x) + 2*b**3*d*e
*g*x*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 2*b**3*d*e*h*x**2/(2*a*b**4 + 2*b
**5*x) + 2*b**3*d*f*g*x**2/(2*a*b**4 + 2*b**5*x) + b**3*d*f*h*x**3/(2*a*b**
4 + 2*b**5*x), Eq(m, -2)), (-a**3*d*f*h*log(a/b + x)/b**4 + a**2*c*f*h*log(
a/b + x)/b**3 + a**2*d*e*h*log(a/b + x)/b**3 + a**2*d*f*g*log(a/b + x)/b**3
 + a**2*d*f*h*x/b**3 - a*c*e*h*log(a/b + x)/b**2 - a*c*f*g*log(a/b + x)/b**
2 - a*c*f*h*x/b**2 - a*d*e*g*log(a/b + x)/b**2 - a*d*e*h*x/b**2 - a*d*f*g*x
/b**2 - a*d*f*h*x**2/(2*b**2) + c*e*g*log(a/b + x)/b + c*e*h*x/b + c*f*g*x/
b + c*f*h*x**2/(2*b) + d*e*g*x/b + d*e*h*x**2/(2*b) + d*f*g*x**2/(2*b) + d*
f*h*x**3/(3*b), Eq(m, -1)), (-6*a**4*d*f*h*(a + b*x)**m/(b**4*m**4 + 10*b**
4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 2*a**3*b*c*f*h*m*(a + b*x)**
m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 8*a**3*
b*c*f*h*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m +
 24*b**4) + 2*a**3*b*d*e*h*m*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b*
*4*m**2 + 50*b**4*m + 24*b**4) + 8*a**3*b*d*e*h*(a + b*x)**m/(b**4*m**4 + 1
0*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 2*a**3*b*d*f*g*m*(a + b
```

```
*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 8*
a**3*b*d*f*g*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**
4*m + 24*b**4) + 6*a**3*b*d*f*h*m*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3
 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) - a**2*b**2*c*e*h*m**2*(a + b*x)**m/(
b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 7*a**2*b**
2*c*e*h*m*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m
 + 24*b**4) - 12*a**2*b**2*c*e*h*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 3
5*b**4*m**2 + 50*b**4*m + 24*b**4) - a**2*b**2*c*f*g*m**2*(a + b*x)**m/(b**
4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 7*a**2*b**2*c
*f*g*m*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m +
24*b**4) - 12*a**2*b**2*c*f*g*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b
**4*m**2 + 50*b**4*m + 24*b**4) - 2*a**2*b**2*c*f*h*m**2*x*(a + b*x)**m/(b*
*4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 8*a**2*b**2*
c*f*h*m*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m
 + 24*b**4) - a**2*b**2*d*e*g*m**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 +
 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 7*a**2*b**2*d*e*g*m*(a + b*x)**m/(b*
*4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 12*a**2*b**2
*d*e*g*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m +
24*b**4) - 2*a**2*b**2*d*e*h*m**2*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3
 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 8*a**2*b**2*d*e*h*m*x*(a + b*x)**m/
(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 2*a**2*b*
*2*d*f*g*m**2*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*
b**4*m + 24*b**4) - 8*a**2*b**2*d*f*g*m*x*(a + b*x)**m/(b**4*m**4 + 10*b**4
*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 3*a**2*b**2*d*f*h*m**2*x**2*(
a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4)
 - 3*a**2*b**2*d*f*h*m*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**
4*m**2 + 50*b**4*m + 24*b**4) + a*b**3*c*e*g*m**3*(a + b*x)**m/(b**4*m**4 +
 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 9*a*b**3*c*e*g*m**2*(
a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4)
 + 26*a*b**3*c*e*g*m*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2
 + 50*b**4*m + 24*b**4) + 24*a*b**3*c*e*g*(a + b*x)**m/(b**4*m**4 + 10*b**4*
m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + a*b**3*c*e*h*m**3*x*(a + b*x)*
*m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 7*a*b*
*3*c*e*h*m**2*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*
b**4*m + 24*b**4) + 12*a*b**3*c*e*h*m*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m
**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + a*b**3*c*f*g*m**3*x*(a + b*x)**
m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 7*a*b**
3*c*f*g*m**2*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b
**4*m + 24*b**4) + 12*a*b**3*c*f*g*m*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m*
*3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + a*b**3*c*f*h*m**3*x**2*(a + b*x)
**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 5*a*b
**3*c*f*h*m**2*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 +
 50*b**4*m + 24*b**4) + 4*a*b**3*c*f*h*m*x**2*(a + b*x)**m/(b**4*m**4 + 10*
b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + a*b**3*d*e*g*m**3*x*(a +
b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 7
*a*b**3*d*e*g*m**2*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2
 + 50*b**4*m + 24*b**4) + 12*a*b**3*d*e*g*m*x*(a + b*x)**m/(b**4*m**4 + 10*b
**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + a*b**3*d*e*h*m**3*x**2*(a
 + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) +
 5*a*b**3*d*e*h*m**2*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*
m**2 + 50*b**4*m + 24*b**4) + 4*a*b**3*d*e*h*m*x**2*(a + b*x)**m/(b**4*m**4
 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + a*b**3*d*f*g*m**3*x
**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*
b**4) + 5*a*b**3*d*f*g*m**2*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 3
5*b**4*m**2 + 50*b**4*m + 24*b**4) + 4*a*b**3*d*f*g*m*x**2*(a + b*x)**m/(b*
*4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + a*b**3*d*f*h
*m**3*x**3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*
m + 24*b**4) + 3*a*b**3*d*f*h*m**2*x**3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m
**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 2*a*b**3*d*f*h*m*x**3*(a + b*x)
```

```
**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + b**4*
c*e*g*m**3*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**
4*m + 24*b**4) + 9*b**4*c*e*g*m**2*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3
 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 26*b**4*c*e*g*m*x*(a + b*x)**m/(b*
*4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 24*b**4*c*e*
g*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*
b**4) + b**4*c*e*h*m**3*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b*
*4*m**2 + 50*b**4*m + 24*b**4) + 8*b**4*c*e*h*m**2*x**2*(a + b*x)**m/(b**4*
m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 19*b**4*c*e*h*m
*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 2
4*b**4) + 12*b**4*c*e*h*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b*
*4*m**2 + 50*b**4*m + 24*b**4) + b**4*c*f*g*m**3*x**2*(a + b*x)**m/(b**4*m*
*4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 8*b**4*c*f*g*m**2
*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 2
4*b**4) + 19*b**4*c*f*g*m*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*
b**4*m**2 + 50*b**4*m + 24*b**4) + 12*b**4*c*f*g*x**2*(a + b*x)**m/(b**4*m*
*4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + b**4*c*f*h*m**3*x
**3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*
b**4) + 7*b**4*c*f*h*m**2*x**3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*
b**4*m**2 + 50*b**4*m + 24*b**4) + 14*b**4*c*f*h*m*x**3*(a + b*x)**m/(b**4*
m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 8*b**4*c*f*h*x*
*3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b
**4) + b**4*d*e*g*m**3*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**
4*m**2 + 50*b**4*m + 24*b**4) + 8*b**4*d*e*g*m**2*x**2*(a + b*x)**m/(b**4*m
**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 19*b**4*d*e*g*m*
x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24
*b**4) + 12*b**4*d*e*g*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**
4*m**2 + 50*b**4*m + 24*b**4) + b**4*d*e*h*m**3*x**3*(a + b*x)**m/(b**4*m**
4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 7*b**4*d*e*h*m**2*
x**3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24
*b**4) + 14*b**4*d*e*h*m*x**3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b
**4*m**2 + 50*b**4*m + 24*b**4) + 8*b**4*d*e*h*x**3*(a + b*x)**m/(b**4*m**4
 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + b**4*d*f*g*m**3*x**
3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b*
*4) + 7*b**4*d*f*g*m**2*x**3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b*
*4*m**2 + 50*b**4*m + 24*b**4) + 14*b**4*d*f*g*m*x**3*(a + b*x)**m/(b**4*m*
*4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 8*b**4*d*f*g*x**3
*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**
4) + b**4*d*f*h*m**3*x**4*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*
m**2 + 50*b**4*m + 24*b**4) + 6*b**4*d*f*h*m**2*x**4*(a + b*x)**m/(b**4*m**
4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 11*b**4*d*f*h*m*x*
*4*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b
**4) + 6*b**4*d*f*h*x**4*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m
**2 + 50*b**4*m + 24*b**4), True))
```

3.120 $\int \frac{(a+bx)^m(c+dx)(e+fx)}{g+hx}\,dx$

Optimal. Leaf size=134

$$\frac{(a+bx)^{m+1}(dg-ch)(fg-eh)\,{}_2F_1\left(1,m+1;m+2;-\frac{h(a+bx)}{bg-ah}\right)}{h^2(m+1)(bg-ah)}-\frac{(a+bx)^{m+1}(adfh+b(m+2)(-cfh-deh+dfg$$

```
[Out] -(b*x+a)^(1+m)*(a*d*f*h+b*(-c*f*h-d*e*h+d*f*g)*(2+m)-b*d*f*h*(1+m)*x)/b^2/h
^2/(1+m)/(2+m)+(-c*h+d*g)*(-e*h+f*g)*(b*x+a)^(1+m)*hypergeom([1, 1+m],[2+m]
,-h*(b*x+a)/(-a*h+b*g))/h^2/(-a*h+b*g)/(1+m)
```

Rubi [A] time = 0.09, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {147, 68}

$$\frac{(a+bx)^{m+1}(dg-ch)(fg-eh)\,{}_2F_1\left(1,m+1;m+2;-\frac{h(a+bx)}{bg-ah}\right)}{h^2(m+1)(bg-ah)}-\frac{(a+bx)^{m+1}(adfh+b(m+2)(-cfh-deh+dfg$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^m*(c + d*x)*(e + f*x))/(g + h*x),x]
```

```
[Out] -(((a + b*x)^(1 + m)*(a*d*f*h + b*(d*f*g - d*e*h - c*f*h)*(2 + m) - b*d*f*h
*(1 + m)*x))/(b^2*h^2*(1 + m)*(2 + m))) + ((d*g - c*h)*(f*g - e*h)*(a + b*x
)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((h*(a + b*x))/(b*g - a*h))])
/(h^2*(b*g - a*h)*(1 + m))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rubi steps

$$\int \frac{(a+bx)^m(c+dx)(e+fx)}{g+hx}\,dx = -\frac{(a+bx)^{1+m}(adfh+b(dfg-deh-cfh)(2+m)-bdfh(1+m)x)}{b^2h^2(1+m)(2+m)}+\frac{((dg-$$

$$= -\frac{(a+bx)^{1+m}(adfh+b(dfg-deh-cfh)(2+m)-bdfh(1+m)x)}{b^2h^2(1+m)(2+m)}+\frac{(dg-$$

Mathematica [A] time = 0.20, size = 120, normalized size = 0.90

$$\frac{(a+bx)^{m+1}\left(\frac{b(cfh+deh-dfg)-adfh}{b^2(m+1)}+\frac{dfh(a+bx)}{b^2(m+2)}+\frac{(dg-ch)(fg-eh)\,{}_2F_1\left(1,m+1;m+2;\frac{h(a+bx)}{ah-bg}\right)}{(m+1)(bg-ah)}\right)}{h^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^m*(c + d*x)*(e + f*x))/(g + h*x),x]
```

```
[Out] ((a + b*x)^(1 + m)*((-(a*d*f*h) + b*(-(d*f*g) + d*e*h + c*f*h))/(b^2*(1 + m
)) + (d*f*h*(a + b*x))/(b^2*(2 + m)) + ((d*g - c*h)*(f*g - e*h)*Hypergeomet
ric2F1[1, 1 + m, 2 + m, (h*(a + b*x))/(-(b*g) + a*h)])/((b*g - a*h)*(1 + m)
)))/h^2
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(dfx^2+ce+\left(de+cf\right)x\right)(bx+a)^m}{hx+g},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)*(f*x+e)/(h*x+g),x, algorithm="fricas")
```

```
[Out] integral((d*f*x^2 + c*e + (d*e + c*f)*x)*(b*x + a)^m/(h*x + g), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)\left(fx+e\right)(bx+a)^m}{hx+g}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)*(f*x+e)/(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*(f*x + e)*(b*x + a)^m/(h*x + g), x)
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)\left(fx+e\right)(bx+a)^m}{hx+g}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)*(f*x+e)/(h*x+g),x)
```

```
[Out] int((b*x+a)^m*(d*x+c)*(f*x+e)/(h*x+g),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)\left(fx+e\right)(bx+a)^m}{hx+g}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)*(f*x+e)/(h*x+g),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)*(f*x + e)*(b*x + a)^m/(h*x + g), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(e+f\,x\right)\,{\left(a+b\,x\right)}^m\,\left(c+d\,x\right)}{g+h\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(a + b*x)^m*(c + d*x))/(g + h*x),x)
```

```
[Out] int(((e + f*x)*(a + b*x)^m*(c + d*x))/(g + h*x), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^m (c+dx) \left(e+fx\right)}{g+hx} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)*(f*x+e)/(h*x+g),x)
```

```
[Out] Integral((a + b*x)**m*(c + d*x)*(e + f*x)/(g + h*x), x)
```

3.121 $\int \frac{(a+bx)^m(c+dx)}{(e+fx)(g+hx)} dx$

Optimal. Leaf size=140

$$\frac{(a+bx)^{m+1}(dg-ch)\,{}_2F_1\left(1,m+1;m+2;-\frac{h(a+bx)}{bg-ah}\right)}{(m+1)(bg-ah)(fg-eh)}-\frac{(a+bx)^{m+1}(de-cf)\,{}_2F_1\left(1,m+1;m+2;-\frac{f(a+bx)}{be-af}\right)}{(m+1)(be-af)(fg-eh)}$$

```
[Out] -(-c*f+d*e)*(b*x+a)^(1+m)*hypergeom([1, 1+m],[2+m],-f*(b*x+a)/(-a*f+b*e))/(
-a*f+b*e)/(-e*h+f*g)/(1+m)+(-c*h+d*g)*(b*x+a)^(1+m)*hypergeom([1, 1+m],[2+m
],-h*(b*x+a)/(-a*h+b*g))/(-a*h+b*g)/(-e*h+f*g)/(1+m)
```

Rubi [A] time = 0.06, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {156, 68}

$$\frac{(a+bx)^{m+1}(dg-ch)\,{}_2F_1\left(1,m+1;m+2;-\frac{h(a+bx)}{bg-ah}\right)}{(m+1)(bg-ah)(fg-eh)}-\frac{(a+bx)^{m+1}(de-cf)\,{}_2F_1\left(1,m+1;m+2;-\frac{f(a+bx)}{be-af}\right)}{(m+1)(be-af)(fg-eh)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^m*(c + d*x))/((e + f*x)*(g + h*x)),x]
```

```
[Out] -(((d*e - c*f)*(a + b*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((f*(a
 + b*x))/(b*e - a*f))])/((b*e - a*f)*(f*g - e*h)*(1 + m))) + ((d*g - c*h)*(
a + b*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((h*(a + b*x))/(b*g -
a*h))])/((b*g - a*h)*(f*g - e*h)*(1 + m))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^m(c+dx)}{(e+fx)(g+hx)}\,dx &= -\frac{(de-cf)\int \frac{(a+bx)^m}{e+fx}\,dx}{fg-eh}+\frac{(dg-ch)\int \frac{(a+bx)^m}{g+hx}\,dx}{fg-eh}\\ &= -\frac{(de-cf)(a+bx)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{f(a+bx)}{be-af}\right)}{(be-af)(fg-eh)(1+m)}+\frac{(dg-ch)(a+bx)^{1+m}\,{}_2F_1\left(1,1\right.}{(bg-ah)(fg-eh}\end{aligned}$$

Mathematica [A] time = 0.08, size = 115, normalized size = 0.82

$$\frac{(a+bx)^{m+1}\left(\frac{(dg-ch)\,{}_2F_1\left(1,m+1;m+2;\frac{h(a+bx)}{ah-bg}\right)}{bg-ah}-\frac{(de-cf)\,{}_2F_1\left(1,m+1;m+2;\frac{f(a+bx)}{af-be}\right)}{be-af}\right)}{(m+1)(fg-eh)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^m*(c + d*x))/((e + f*x)*(g + h*x)),x]
```

```
[Out] ((a + b*x)^(1 + m)*(-(((d*e - c*f)*Hypergeometric2F1[1, 1 + m, 2 + m, (f*(a
 + b*x))/(-(b*e) + a*f)])/(b*e - a*f)) + ((d*g - c*h)*Hypergeometric2F1[1,
1 + m, 2 + m, (h*(a + b*x))/(-(b*g) + a*h)])/(b*g - a*h)))/((f*g - e*h)*(1
+ m))
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(dx+c)(bx+a)^m}{fhx^2+eg+\left(fg+eh\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)*(b*x + a)^m/(f*h*x^2 + e*g + (f*g + e*h)*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)(bx+a)^m}{\left(fx+e\right)\left(hx+g\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*(b*x + a)^m/((f*x + e)*(h*x + g)), x)
```

maple [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)\,(bx+a)^m}{\left(fx+e\right)\,\left(hx+g\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)/(f*x+e)/(h*x+g),x)
```

```
[Out] int((b*x+a)^m*(d*x+c)/(f*x+e)/(h*x+g),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)(bx+a)^m}{\left(fx+e\right)\left(hx+g\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)*(b*x + a)^m/((f*x + e)*(h*x + g)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^m\,\left(c+d\,x\right)}{\left(e+f\,x\right)\,\left(g+h\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)^m*(c + d*x))/((e + f*x)*(g + h*x)),x)
```

```
[Out] int(((a + b*x)^m*(c + d*x))/((e + f*x)*(g + h*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^m (c+dx)}{(e+fx)(g+hx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)/(f*x+e)/(h*x+g),x)
```

```
[Out] Integral((a + b*x)**m*(c + d*x)/((e + f*x)*(g + h*x)), x)
```

3.122 $\int \frac{(a+bx)^m}{(c+dx)(e+fx)(g+hx)} dx$

Optimal. Leaf size=224

$$\frac{d^2(a+bx)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{d(a+bx)}{bc-ad}\right)}{(m+1)(bc-ad)(de-cf)(dg-ch)}-\frac{f^2(a+bx)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{f(a+bx)}{be-af}\right)}{(m+1)(be-af)(de-cf)(fg-eh)}+\frac{h^2(a+bx)^{m+1}\,{}_2F}{(m+1)(bg}$$

```
[Out] d^2*(b*x+a)^(1+m)*hypergeom([1, 1+m],[2+m],-d*(b*x+a)/(-a*d+b*c))/(-a*d+b*c
)/(-c*f+d*e)/(-c*h+d*g)/(1+m)-f^2*(b*x+a)^(1+m)*hypergeom([1, 1+m],[2+m],-f
*(b*x+a)/(-a*f+b*e))/(-a*f+b*e)/(-c*f+d*e)/(-e*h+f*g)/(1+m)+h^2*(b*x+a)^(1+
m)*hypergeom([1, 1+m],[2+m],-h*(b*x+a)/(-a*h+b*g))/(-a*h+b*g)/(-c*h+d*g)/(-
e*h+f*g)/(1+m)
```

Rubi [A] time = 0.19, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {180, 68}

$$\frac{d^2(a+bx)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{d(a+bx)}{bc-ad}\right)}{(m+1)(bc-ad)(de-cf)(dg-ch)}-\frac{f^2(a+bx)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{f(a+bx)}{be-af}\right)}{(m+1)(be-af)(de-cf)(fg-eh)}+\frac{h^2(a+bx)^{m+1}\,{}_2F}{(m+1)(bg}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m/((c + d*x)*(e + f*x)*(g + h*x)),x]
```

```
[Out] (d^2*(a + b*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((d*(a + b*x))/(
b*c - a*d))])/((b*c - a*d)*(d*e - c*f)*(d*g - c*h)*(1 + m)) - (f^2*(a + b*x
)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((f*(a + b*x))/(b*e - a*f))])
/((b*e - a*f)*(d*e - c*f)*(f*g - e*h)*(1 + m)) + (h^2*(a + b*x)^(1 + m)*Hyp
ergeometric2F1[1, 1 + m, 2 + m, -((h*(a + b*x))/(b*g - a*h))])/((b*g - a*h)
*(d*g - c*h)*(f*g - e*h)*(1 + m))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^m}{(c+dx)(e+fx)(g+hx)}\,dx &= \int \left(\frac{d^2(a+bx)^m}{(de-cf)(dg-ch)(c+dx)}+\frac{f^2(a+bx)^m}{(de-cf)(-fg+eh)(e+fx)}+\frac{h^2(a}{(dg-ch)(f}\right.\\
&= \frac{d^2\int \frac{(a+bx)^m}{c+dx}\,dx}{(de-cf)(dg-ch)}-\frac{f^2\int \frac{(a+bx)^m}{e+fx}\,dx}{(de-cf)(fg-eh)}+\frac{h^2\int \frac{(a+bx)^m}{g+hx}\,dx}{(dg-ch)(fg-eh)}\\
&= \frac{d^2(a+bx)^{1+m}\,{}_2F_1\left(1,1+m;2+m;-\frac{d(a+bx)}{bc-ad}\right)}{(bc-ad)(de-cf)(dg-ch)(1+m)}-\frac{f^2(a+bx)^{1+m}\,{}_2F_1\left(1,1+m;\right.}{(be-af)(de-cf)(fg-}
\end{aligned}$$

Mathematica [A] time = 0.37, size = 193, normalized size = 0.86

$$\frac{(a+bx)^{m+1}\left(\frac{d^2\,{}_2F_1\left(1,m+1;m+2;\frac{d(a+bx)}{ad-bc}\right)}{(bc-ad)(cf-de)(ch-dg)}+\frac{f^2\,{}_2F_1\left(1,m+1;m+2;\frac{f(a+bx)}{af-be}\right)}{(be-af)(de-cf)(eh-fg)}+\frac{h^2\,{}_2F_1\left(1,m+1;m+2;\frac{h(a+bx)}{ah-bg}\right)}{(bg-ah)(dg-ch)(fg-eh)}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m/((c + d*x)*(e + f*x)*(g + h*x)),x]
```

```
[Out] ((a + b*x)^(1 + m)*((d^2*Hypergeometric2F1[1, 1 + m, 2 + m, (d*(a + b*x))/(
-(b*c) + a*d)])/((b*c - a*d)*(-(d*e) + c*f)*(-(d*g) + c*h)) + (f^2*Hypergeo
metric2F1[1, 1 + m, 2 + m, (f*(a + b*x))/(-(b*e) + a*f)])/((b*e - a*f)*(d*e
 - c*f)*(-(f*g) + e*h)) + (h^2*Hypergeometric2F1[1, 1 + m, 2 + m, (h*(a + b
*x))/(-(b*g) + a*h)])/((b*g - a*h)*(d*g - c*h)*(f*g - e*h))))/(1 + m)
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(bx+a)^m}{dfhx^3+ceg+\left(dfg+\left(de+cf\right)h\right)x^2+\left(ceh+\left(de+cf\right)g\right)x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^m/(d*f*h*x^3 + c*e*g + (d*f*g + (d*e + c*f)*h)*x^2 + (c*
e*h + (d*e + c*f)*g)*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^m}{(dx+c)\left(fx+e\right)\left(hx+g\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^m/((d*x + c)*(f*x + e)*(h*x + g)), x)
```

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^m}{(dx+c)\left(fx+e\right)\left(hx+g\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m/(d*x+c)/(f*x+e)/(h*x+g),x)
```

```
[Out] int((b*x+a)^m/(d*x+c)/(f*x+e)/(h*x+g),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^m}{(dx+c)\left(fx+e\right)\left(hx+g\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^m/((d*x + c)*(f*x + e)*(h*x + g)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^m}{\left(e+f\,x\right)\,\left(g+h\,x\right)\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^m/((e + f*x)*(g + h*x)*(c + d*x)),x)
[Out] int((a + b*x)^m/((e + f*x)*(g + h*x)*(c + d*x)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m/(d*x+c)/(f*x+e)/(h*x+g),x)
[Out] Exception raised: HeuristicGCDFailed
```

3.123 $\int \frac{x^m(e+fx)^n}{(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=140

$$\frac{bx^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}F_1\left(m+1;-n,1;m+2;-\frac{fx}{e},-\frac{bx}{a}\right)}{a(m+1)(bc-ad)}-\frac{dx^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}F_1\left(m+1;-n,1;m+2;\right.}{c(m+1)(bc-ad)}$$

```
[Out] b*x^(1+m)*(f*x+e)^n*AppellF1(1+m,1,-n,2+m,-b*x/a,-f*x/e)/a/(-a*d+b*c)/(1+m)
/((1+f*x/e)^n)-d*x^(1+m)*(f*x+e)^n*AppellF1(1+m,1,-n,2+m,-d*x/c,-f*x/e)/c/(
-a*d+b*c)/(1+m)/((1+f*x/e)^n)
```

Rubi [A] time = 0.12, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {180, 135, 133}

$$\frac{bx^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}F_1\left(m+1;-n,1;m+2;-\frac{fx}{e},-\frac{bx}{a}\right)}{a(m+1)(bc-ad)}-\frac{dx^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}F_1\left(m+1;-n,1;m+2;\right.}{c(m+1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
```

```
[Out] (b*x^(1 + m)*(e + f*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((f*x)/e), -((b*x)/
a)])/(a*(b*c - a*d)*(1 + m)*(1 + (f*x)/e)^n) - (d*x^(1 + m)*(e + f*x)^n*App
ellF1[1 + m, -n, 1, 2 + m, -((f*x)/e), -((d*x)/c)])/(c*(b*c - a*d)*(1 + m)*
(1 + (f*x)/e)^n)
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rubi steps

$$\int \frac{x^m(e+fx)^n}{(a+bx)(c+dx)}\,dx = \int \left(\frac{bx^m(e+fx)^n}{(bc-ad)(a+bx)} - \frac{dx^m(e+fx)^n}{(bc-ad)(c+dx)}\right)dx$$
$$= \frac{b\int \frac{x^m(e+fx)^n}{a+bx}\,dx}{bc-ad} - \frac{d\int \frac{x^m(e+fx)^n}{c+dx}\,dx}{bc-ad}$$
$$= \frac{\left(b(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}\right)\int \frac{x^m\left(1+\frac{fx}{e}\right)^n}{a+bx}\,dx}{bc-ad} - \frac{\left(d(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}\right)\int \frac{x^m\left(1+\frac{fx}{e}\right)^n}{c+dx}\,dx}{bc-ad}$$
$$= \frac{bx^{1+m}(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}F_1\left(1+m;-n,1;2+m;-\frac{fx}{e},-\frac{bx}{a}\right)}{a(bc-ad)(1+m)} - \frac{dx^{1+m}(e+fx)^n\left(1+\right.}{}$$

Mathematica [A] time = 0.23, size = 104, normalized size = 0.74

$$\frac{x^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}\left(adF_1\left(m+1;-n,1;m+2;-\frac{fx}{e},-\frac{dx}{c}\right)-bcF_1\left(m+1;-n,1;m+2;-\frac{fx}{e},-\frac{bx}{a}\right)\right)}{ac(m+1)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(e + f*x)^n)/((a + b*x)*(c + d*x)),x]
```

```
[Out] (x^(1 + m)*(e + f*x)^n*(-(b*c*AppellF1[1 + m, -n, 1, 2 + m, -((f*x)/e), -((
b*x)/a)]) + a*d*AppellF1[1 + m, -n, 1, 2 + m, -((f*x)/e), -((d*x)/c)]))/(a*
c*(-(b*c) + a*d)*(1 + m)*(1 + (f*x)/e)^n)
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(fx+e\right)^n x^m}{bdx^2+ac+(bc+ad)x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n*x^m/(b*d*x^2 + a*c + (b*c + a*d)*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx+e\right)^n x^m}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n*x^m/((b*x + a)*(d*x + c)), x)
```

maple [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{x^m\left(fx+e\right)^n}{(bx+a)\,(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

```
[Out] int(x^m*(f*x+e)^n/(b*x+a)/(d*x+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx+e\right)^{n} x^{m}}{(bx+a)(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(f*x+e)^n/(b*x+a)/(d*x+c),x, algorithm="maxima")
[Out] integrate((f*x + e)^n*x^m/((b*x + a)*(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{m} \left(e+f\,x\right)^{n}}{(a+b\,x)\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(e + f*x)^n)/((a + b*x)*(c + d*x)),x)
[Out] int((x^m*(e + f*x)^n)/((a + b*x)*(c + d*x)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(f*x+e)**n/(b*x+a)/(d*x+c),x)
[Out] Exception raised: HeuristicGCDFailed
```

3.124 $\int (a+bx)^m(c+dx)^n(e+fx)(g+hx)\,dx$

Optimal. Leaf size=266

$$\frac{(a+bx)^{m+1}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\,{}_2F_1\left(m+1,-n;m+2;-\frac{d(a+bx)}{bc-ad}\right)\left(a^2d^2fh(n+1)(n+2)+abd(n+1)(2cfh(m\right.}{b^3d^2(m+1)}$$

```
[Out] -(b*x+a)^(1+m)*(d*x+c)^(1+n)*(b*c*f*h*(2+m)+a*d*f*h*(2+n)-b*d*(e*h+f*g)*(3+
m+n)-b*d*f*h*(2+m+n)*x)/b^2/d^2/(2+m+n)/(3+m+n)+(a^2*d^2*f*h*(1+n)*(2+n)+a*
b*d*(1+n)*(2*c*f*h*(1+m)-d*(e*h+f*g)*(3+m+n))+b^2*(c^2*f*h*(1+m)*(2+m)-c*d*
(e*h+f*g)*(1+m)*(3+m+n)+d^2*e*g*(2+m+n)*(3+m+n)))*(b*x+a)^(1+m)*(d*x+c)^n*h
ypergeom([-n, 1+m],[2+m],-d*(b*x+a)/(-a*d+b*c))/b^3/d^2/(1+m)/(2+m+n)/(3+m+
n)/((b*(d*x+c)/(-a*d+b*c))^n)
```

Rubi [A] time = 0.17, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {147, 70, 69}

$$\frac{(a+bx)^{m+1}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\,{}_2F_1\left(m+1,-n;m+2;-\frac{d(a+bx)}{bc-ad}\right)\left(a^2d^2fh(n+1)(n+2)+abd(n+1)(2cfh(m\right.}{b^3d^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)*(g + h*x),x]
```

```
[Out] -(((a + b*x)^(1 + m)*(c + d*x)^(1 + n)*(b*c*f*h*(2 + m) + a*d*f*h*(2 + n) -
 b*d*(f*g + e*h)*(3 + m + n) - b*d*f*h*(2 + m + n)*x))/(b^2*d^2*(2 + m + n)
*(3 + m + n))) + ((a^2*d^2*f*h*(1 + n)*(2 + n) + a*b*d*(1 + n)*(2*c*f*h*(1
+ m) - d*(f*g + e*h)*(3 + m + n)) + b^2*(c^2*f*h*(1 + m)*(2 + m) - c*d*(f*g
 + e*h)*(1 + m)*(3 + m + n) + d^2*e*g*(2 + m + n)*(3 + m + n)))*(a + b*x)^(
1 + m)*(c + d*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((d*(a + b*x))/(b*c
 - a*d))])/(b^3*d^2*(1 + m)*(2 + m + n)*(3 + m + n)*((b*(c + d*x))/(b*c - a
*d))^n)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
```

```
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rubi steps

$$\int (a+bx)^m(c+dx)^n(e+fx)(g+hx)\,dx = -\frac{(a+bx)^{1+m}(c+dx)^{1+n}(bcfh(2+m)+adfh(2+n)-bd(fg+eh)(3}{b^2d^2(2+m+n)(3+m+n)}$$

$$= -\frac{(a+bx)^{1+m}(c+dx)^{1+n}(bcfh(2+m)+adfh(2+n)-bd(fg+eh)(3}{b^2d^2(2+m+n)(3+m+n)}$$

$$= -\frac{(a+bx)^{1+m}(c+dx)^{1+n}(bcfh(2+m)+adfh(2+n)-bd(fg+eh)(3}{b^2d^2(2+m+n)(3+m+n)}$$

Mathematica [A] time = 0.24, size = 195, normalized size = 0.73

$$\frac{(a+bx)^{m+1}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(b\left(b(de-cf)(dg-ch)\,{}_2F_1\left(m+1,-n;m+2;\frac{d(a+bx)}{ad-bc}\right)-(bc-ad)(2cfh-d(eh+\right.\right.}{b^3d^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^n*(e + f*x)*(g + h*x),x]
```

```
[Out] ((a + b*x)^(1 + m)*(c + d*x)^n*((b*c - a*d)^2*f*h*Hypergeometric2F1[1 + m,
-2 - n, 2 + m, (d*(a + b*x))/(-(b*c) + a*d)] + b*(-((b*c - a*d)*(2*c*f*h -
d*(f*g + e*h))*Hypergeometric2F1[1 + m, -1 - n, 2 + m, (d*(a + b*x))/(-(b*c
) + a*d)]) + b*(d*e - c*f)*(d*g - c*h)*Hypergeometric2F1[1 + m, -n, 2 + m,
(d*(a + b*x))/(-(b*c) + a*d)])))/(b^3*d^2*(1 + m)*((b*(c + d*x))/(b*c - a*d
))^n)
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(fhx^2+eg+\left(fg+eh\right)x\right)(bx+a)^m(dx+c)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)*(h*x+g),x, algorithm="fricas")
```

```
[Out] integral((f*h*x^2 + e*g + (f*g + e*h)*x)*(b*x + a)^m*(d*x + c)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx+e\right)\left(hx+g\right)(bx+a)^m(dx+c)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)*(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^n, x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \left(fx+e\right)\left(hx+g\right)(bx+a)^m\,(dx+c)^n\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^n*(f*x+e)*(h*x+g),x)
[Out] int((b*x+a)^m*(d*x+c)^n*(f*x+e)*(h*x+g),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx+e)(hx+g)(bx+a)^m(dx+c)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)*(h*x+g),x, algorithm="maxima")
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(e+f\,x\right)\,\left(g+h\,x\right)\,{\left(a+b\,x\right)}^m\,{\left(c+d\,x\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)*(g + h*x)*(a + b*x)^m*(c + d*x)^n,x)
[Out] int((e + f*x)*(g + h*x)*(a + b*x)^m*(c + d*x)^n, x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**n*(f*x+e)*(h*x+g),x)
[Out] Exception raised: HeuristicGCDFailed
```

3.125 $\int (a+bx)^m(c+dx)^{1-m}(e+fx)(g+hx)\,dx$

Optimal. Leaf size=245

$$\frac{(bc-ad)(a+bx)^{m+1}(c+dx)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^m {}_2F_1\left(m-1,m+1;m+2;-\frac{d(a+bx)}{bc-ad}\right)\left(a^2d^2fh\left(m^2-5m+6\right)-2abd(2-\right.}{12b^4d^2(m+1)}$$

```
[Out] 1/12*(b*x+a)^(1+m)*(d*x+c)^(2-m)*(4*b*d*(e*h+f*g)-a*d*f*h*(3-m)-b*c*f*h*(2+
m)+3*b*d*f*h*x)/b^2/d^2+1/12*(-a*d+b*c)*(a^2*d^2*f*h*(m^2-5*m+6)-2*a*b*d*(2
-m)*(2*d*(e*h+f*g)-c*f*h*(1+m))+b^2*(12*d^2*e*g-4*c*d*(e*h+f*g)*(1+m)+c^2*f
*h*(m^2+3*m+2)))*(b*x+a)^(1+m)*(b*(d*x+c)/(-a*d+b*c))^m*hypergeom([-1+m, 1+
m],[2+m],-d*(b*x+a)/(-a*d+b*c))/b^4/d^2/(1+m)/((d*x+c)^m)
```

Rubi [A] time = 0.15, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {147, 70, 69}

$$\frac{(bc-ad)(a+bx)^{m+1}(c+dx)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^m {}_2F_1\left(m-1,m+1;m+2;-\frac{d(a+bx)}{bc-ad}\right)\left(a^2d^2fh\left(m^2-5m+6\right)-2abd(2-\right.}{12b^4d^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^(1 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] ((a + b*x)^(1 + m)*(c + d*x)^(2 - m)*(4*b*d*(f*g + e*h) - a*d*f*h*(3 - m) -
 b*c*f*h*(2 + m) + 3*b*d*f*h*x))/(12*b^2*d^2) + ((b*c - a*d)*(a^2*d^2*f*h*(
6 - 5*m + m^2) - 2*a*b*d*(2 - m)*(2*d*(f*g + e*h) - c*f*h*(1 + m)) + b^2*(1
2*d^2*e*g - 4*c*d*(f*g + e*h)*(1 + m) + c^2*f*h*(2 + 3*m + m^2)))*(a + b*x)
^(1 + m)*((b*(c + d*x))/(b*c - a*d))^m*Hypergeometric2F1[-1 + m, 1 + m, 2 +
 m, -((d*(a + b*x))/(b*c - a*d))])/(12*b^4*d^2*(1 + m)*(c + d*x)^m)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rubi steps

$$\int (a+bx)^m(c+dx)^{1-m}(e+fx)(g+hx)\,dx = \frac{(a+bx)^{1+m}(c+dx)^{2-m}(4bd(fg+eh)-adfh(3-m)-bcfh(2+}{12b^2d^2}$$

$$= \frac{(a+bx)^{1+m}(c+dx)^{2-m}(4bd(fg+eh)-adfh(3-m)-bcfh(2+}{12b^2d^2}$$

$$= \frac{(a+bx)^{1+m}(c+dx)^{2-m}(4bd(fg+eh)-adfh(3-m)-bcfh(2+}{12b^2d^2}$$

Mathematica [A] time = 0.26, size = 195, normalized size = 0.80

$$\frac{(a+bx)^{m+1}(c+dx)^{1-m}\left(\frac{b(c+dx)}{bc-ad}\right)^{m-1}\left(b\left(b(de-cf)(dg-ch)\,{}_2F_1\left(m-1,m+1;m+2;\frac{d(a+bx)}{ad-bc}\right)-(bc-ad)(2cfh\right.\right.}{b^3d^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^(1 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] ((a + b*x)^(1 + m)*(c + d*x)^(1 - m)*((b*(c + d*x))/(b*c - a*d))^(-1 + m)*(
(b*c - a*d)^2*f*h*Hypergeometric2F1[-3 + m, 1 + m, 2 + m, (d*(a + b*x))/(-(
b*c) + a*d)] + b*(-((b*c - a*d)*(2*c*f*h - d*(f*g + e*h))*Hypergeometric2F1
[-2 + m, 1 + m, 2 + m, (d*(a + b*x))/(-(b*c) + a*d)]) + b*(d*e - c*f)*(d*g
- c*h)*Hypergeometric2F1[-1 + m, 1 + m, 2 + m, (d*(a + b*x))/(-(b*c) + a*d)
])))/(b^3*d^2*(1 + m))
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(fhx^2+eg+\left(fg+eh\right)x\right)(bx+a)^m(dx+c)^{-m+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(1-m)*(f*x+e)*(h*x+g),x, algorithm="fricas")
```

```
[Out] integral((f*h*x^2 + e*g + (f*g + e*h)*x)*(b*x + a)^m*(d*x + c)^(-m + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx+e\right)\left(hx+g\right)(bx+a)^m(dx+c)^{-m+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(1-m)*(f*x+e)*(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m + 1), x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \left(fx+e\right)\left(hx+g\right)(bx+a)^m\,(dx+c)^{-m+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^(-m+1)*(f*x+e)*(h*x+g),x)
```

```
[Out] int((b*x+a)^m*(d*x+c)^(-m+1)*(f*x+e)*(h*x+g),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx+e\right)\left(hx+g\right)(bx+a)^{m}(dx+c)^{-m+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(1-m)*(f*x+e)*(h*x+g),x, algorithm="maxima")
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(e+f\,x\right)\,\left(g+h\,x\right)\,{\left(a+b\,x\right)}^m\,{\left(c+d\,x\right)}^{1-m} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)*(g + h*x)*(a + b*x)^m*(c + d*x)^(1 - m),x)
[Out] int((e + f*x)*(g + h*x)*(a + b*x)^m*(c + d*x)^(1 - m), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**(1-m)*(f*x+e)*(h*x+g),x)
[Out] Exception raised: HeuristicGCDFailed
```

3.126 $\int (a+bx)^m (c+dx)^{-m} (e+fx)(g+hx)\,dx$

Optimal. Leaf size=235

$$\frac{(a+bx)^{m+1}(c+dx)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^m {}_2F_1\left(m,m+1;m+2;-\frac{d(a+bx)}{bc-ad}\right)\left(a^2d^2fh\left(m^2-3m+2\right)-abd(1-m)(3d(eh+\right.}{6b^3d^2(m+1)}$$

```
[Out] 1/6*(b*x+a)^(1+m)*(d*x+c)^(1-m)*(3*b*d*(e*h+f*g)-a*d*f*h*(2-m)-b*c*f*h*(2+m
)+2*b*d*f*h*x)/b^2/d^2+1/6*(a^2*d^2*f*h*(m^2-3*m+2)-a*b*d*(1-m)*(3*d*(e*h+f
*g)-2*c*f*h*(1+m))+b^2*(6*d^2*e*g-3*c*d*(e*h+f*g)*(1+m)+c^2*f*h*(m^2+3*m+2)
))*(b*x+a)^(1+m)*(b*(d*x+c)/(-a*d+b*c))^m*hypergeom([m, 1+m],[2+m],-d*(b*x+
a)/(-a*d+b*c))/b^3/d^2/(1+m)/((d*x+c)^m)
```

Rubi [A] time = 0.14, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {147, 70, 69}

$$\frac{(a+bx)^{m+1}(c+dx)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^m {}_2F_1\left(m,m+1;m+2;-\frac{d(a+bx)}{bc-ad}\right)\left(a^2d^2fh\left(m^2-3m+2\right)-abd(1-m)(3d(eh+\right.}{6b^3d^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^m*(e + f*x)*(g + h*x))/(c + d*x)^m,x]
```

```
[Out] ((a + b*x)^(1 + m)*(c + d*x)^(1 - m)*(3*b*d*(f*g + e*h) - a*d*f*h*(2 - m) -
 b*c*f*h*(2 + m) + 2*b*d*f*h*x))/(6*b^2*d^2) + ((a^2*d^2*f*h*(2 - 3*m + m^2
) - a*b*d*(1 - m)*(3*d*(f*g + e*h) - 2*c*f*h*(1 + m)) + b^2*(6*d^2*e*g - 3*
c*d*(f*g + e*h)*(1 + m) + c^2*f*h*(2 + 3*m + m^2)))*(a + b*x)^(1 + m)*((b*(
c + d*x))/(b*c - a*d))^m*Hypergeometric2F1[m, 1 + m, 2 + m, -((d*(a + b*x))
/(b*c - a*d))])/(6*b^3*d^2*(1 + m)*(c + d*x)^m)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rubi steps

$$\int (a+bx)^m(c+dx)^{-m}(e+fx)(g+hx)\,dx = \frac{(a+bx)^{1+m}(c+dx)^{1-m}(3bd(fg+eh)-adfh(2-m)-bcfh(2+m)}{6b^2d^2}$$

$$= \frac{(a+bx)^{1+m}(c+dx)^{1-m}(3bd(fg+eh)-adfh(2-m)-bcfh(2+m)}{6b^2d^2}$$

$$= \frac{(a+bx)^{1+m}(c+dx)^{1-m}(3bd(fg+eh)-adfh(2-m)-bcfh(2+m)}{6b^2d^2}$$

Mathematica [A] time = 0.22, size = 189, normalized size = 0.80

$$\frac{(a+bx)^{m+1}(c+dx)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^m\left(b\left(b(de-cf)(dg-ch)\,{}_2F_1\left(m,m+1;m+2;\frac{d(a+bx)}{ad-bc}\right)-(bc-ad)(2cfh-d(eh+\right.\right.}{b^3d^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^m*(e + f*x)*(g + h*x))/(c + d*x)^m,x]
```

```
[Out] ((a + b*x)^(1 + m)*((b*(c + d*x))/(b*c - a*d))^m*((b*c - a*d)^2*f*h*Hyperge
ometric2F1[-2 + m, 1 + m, 2 + m, (d*(a + b*x))/(-(b*c) + a*d)] + b*(-((b*c
- a*d)*(2*c*f*h - d*(f*g + e*h))*Hypergeometric2F1[-1 + m, 1 + m, 2 + m, (d
*(a + b*x))/(-(b*c) + a*d)]) + b*(d*e - c*f)*(d*g - c*h)*Hypergeometric2F1[
m, 1 + m, 2 + m, (d*(a + b*x))/(-(b*c) + a*d)])))/(b^3*d^2*(1 + m)*(c + d*x
)^m)
```

fricas [F] time = 1.10, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(fhx^2+eg+\left(fg+eh\right)x\right)(bx+a)^m}{(dx+c)^m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(f*x+e)*(h*x+g)/((d*x+c)^m),x, algorithm="fricas")
```

```
[Out] integral((f*h*x^2 + e*g + (f*g + e*h)*x)*(b*x + a)^m/(d*x + c)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx+e\right)\left(hx+g\right)(bx+a)^m}{(dx+c)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(f*x+e)*(h*x+g)/((d*x+c)^m),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m/(d*x + c)^m, x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \left(fx+e\right)\left(hx+g\right)(bx+a)^m\,(dx+c)^{-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(f*x+e)*(h*x+g)/((d*x+c)^m),x)
```

```
[Out] int((b*x+a)^m*(f*x+e)*(h*x+g)/((d*x+c)^m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)(hx+g)(bx+a)^m}{(dx+c)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(f*x+e)*(h*x+g)/((d*x+c)^m),x, algorithm="maxima")
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m/(d*x + c)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)\,(g+h\,x)\,(a+b\,x)^m}{(c+d\,x)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(g + h*x)*(a + b*x)^m)/(c + d*x)^m,x)
[Out] int(((e + f*x)*(g + h*x)*(a + b*x)^m)/(c + d*x)^m, x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(f*x+e)*(h*x+g)/((d*x+c)**m),x)
[Out] Exception raised: HeuristicGCDFailed
```

3.127 $\int (a+bx)^m (c+dx)^{-1-m} (e+fx)(g+hx)\,dx$

Optimal. Leaf size=261

$$\frac{(a+bx)^{m+1}(c+dx)^{-m}\left(-cd(afhm+2b(eh+fg))+dfhmx(bc-ad)+bc^2fh(m+2)+2bd^2eg\right)}{2bd^2m(bc-ad)}-\frac{(a+bx)^{m+1}(c+$$

```
[Out] 1/2*(b*x+a)^(1+m)*(2*b*d^2*e*g+b*c^2*f*h*(2+m)-c*d*(2*b*(e*h+f*g)+a*f*h*m)+
d*(-a*d+b*c)*f*h*m*x)/b/d^2/(-a*d+b*c)/m/((d*x+c)^m)-1/2*(b^2*c^2*f*h*(1+m)
*(2+m)-2*b*c*d*(1+m)*(a*f*h*m+b*e*h+b*f*g)+d^2*(2*b^2*e*g+2*a*b*(e*h+f*g)*m
-a^2*f*h*(1-m)*m))*(b*x+a)^(1+m)*(b*(d*x+c)/(-a*d+b*c))^m*hypergeom([m, 1+m
],[2+m],-d*(b*x+a)/(-a*d+b*c))/b^2/d^2/(-a*d+b*c)/m/(1+m)/((d*x+c)^m)
```

Rubi [A] time = 0.19, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {146, 70, 69}

$$\frac{(a+bx)^{m+1}(c+dx)^{-m}\left(-cd(afhm+2b(eh+fg))+dfhmx(bc-ad)+bc^2fh(m+2)+2bd^2eg\right)}{2bd^2m(bc-ad)}-\frac{(a+bx)^{m+1}(c+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^(-1 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] ((a + b*x)^(1 + m)*(2*b*d^2*e*g + b*c^2*f*h*(2 + m) - c*d*(2*b*(f*g + e*h)
+ a*f*h*m) + d*(b*c - a*d)*f*h*m*x))/(2*b*d^2*(b*c - a*d)*m*(c + d*x)^m) -
((b^2*c^2*f*h*(1 + m)*(2 + m) - 2*b*c*d*(1 + m)*(b*f*g + b*e*h + a*f*h*m) +
 d^2*(2*b^2*e*g + 2*a*b*(f*g + e*h)*m - a^2*f*h*(1 - m)*m))*(a + b*x)^(1 +
m)*((b*(c + d*x))/(b*c - a*d))^m*Hypergeometric2F1[m, 1 + m, 2 + m, -((d*(a
 + b*x))/(b*c - a*d))])/(2*b^2*d^2*(b*c - a*d)*m*(1 + m)*(c + d*x)^m)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 146

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m
 + 1)*(m + n + 3)), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
```

```
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rubi steps

$$\int (a+bx)^m (c+dx)^{-1-m} (e+fx)(g+hx)\,dx = \frac{(a+bx)^{1+m}(c+dx)^{-m}\left(2bd^2eg+bc^2fh(2+m)-cd(2b(fg+eh\right.}{2bd^2(bc-ad)m}$$

$$= \frac{(a+bx)^{1+m}(c+dx)^{-m}\left(2bd^2eg+bc^2fh(2+m)-cd(2b(fg+eh\right.}{2bd^2(bc-ad)m}$$

$$= \frac{(a+bx)^{1+m}(c+dx)^{-m}\left(2bd^2eg+bc^2fh(2+m)-cd(2b(fg+eh\right.}{2bd^2(bc-ad)m}$$

Mathematica [A] time = 0.21, size = 221, normalized size = 0.85

$$\frac{(a+bx)^{m+1}(c+dx)^{-m}\left(\frac{\left(\frac{b(c+dx)}{bc-ad}\right)^m {}_2F_1\left(m,m+1;m+2;\frac{d(a+bx)}{ad-bc}\right)\left(a^2d^2fh(m-1)m+2abdm(d(eh+fg)-cfh(m+1))+b^2\left(c^2fh\left(m^2+3m+2\right)-2cd(m-\right.\right.}{m+1}\right.}{2b^2d^2m(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^(-1 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] ((a + b*x)^(1 + m)*(b*(a*d*f*h*m*(c + d*x) - b*(2*d^2*e*g + c^2*f*h*(2 + m)
 + c*d*(-2*f*g - 2*e*h + f*h*m*x))) + ((a^2*d^2*f*h*(-1 + m)*m + 2*a*b*d*m*
(d*(f*g + e*h) - c*f*h*(1 + m)) + b^2*(2*d^2*e*g - 2*c*d*(f*g + e*h)*(1 + m
) + c^2*f*h*(2 + 3*m + m^2)))*((b*(c + d*x))/(b*c - a*d))^m*Hypergeometric2
F1[m, 1 + m, 2 + m, (d*(a + b*x))/(-(b*c) + a*d)])/(1 + m)))/(2*b^2*d^2*(-(
b*c) + a*d)*m*(c + d*x)^m)
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(fhx^2+eg+\left(fg+eh\right)x\right)(bx+a)^m(dx+c)^{-m-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-1-m)*(f*x+e)*(h*x+g),x, algorithm="fricas")
```

```
[Out] integral((f*h*x^2 + e*g + (f*g + e*h)*x)*(b*x + a)^m*(d*x + c)^(-m - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx+e\right)\left(hx+g\right)(bx+a)^m(dx+c)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-1-m)*(f*x+e)*(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 1), x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \left(fx+e\right)\left(hx+g\right)(bx+a)^m(dx+c)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^(-m-1)*(f*x+e)*(h*x+g),x)
```

```
[Out] int((b*x+a)^m*(d*x+c)^(-m-1)*(f*x+e)*(h*x+g),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx+e)(hx+g)(bx+a)^m(dx+c)^{-m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-1-m)*(f*x+e)*(h*x+g),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)\,(g+h\,x)\,(a+b\,x)^m}{(c+d\,x)^{m+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(g + h*x)*(a + b*x)^m)/(c + d*x)^(m + 1),x)
```

```
[Out] int(((e + f*x)*(g + h*x)*(a + b*x)^m)/(c + d*x)^(m + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**(-1-m)*(f*x+e)*(h*x+g),x)
```

```
[Out] Timed out
```

3.128 $\int (a+bx)^m (c+dx)^{-2-m} (e+fx)(g+hx)\,dx$

Optimal. Leaf size=203

$$\frac{(a+bx)^{m+1}(c+dx)^{-m-1}\left(-cd(afh(m+1)+b(eh+fg))+dfh(m+1)x(bc-ad)+bc^2fh(m+2)+bd^2eg\right)}{bd^2(m+1)(bc-ad)}-\frac{(a}{}$$

```
[Out] (b*x+a)^(1+m)*(d*x+c)^(-1-m)*(b*d^2*e*g+b*c^2*f*h*(2+m)-c*d*(b*(e*h+f*g)+a*
f*h*(1+m))+d*(-a*d+b*c)*f*h*(1+m)*x)/b/d^2/(-a*d+b*c)/(1+m)-(a*d*f*h*m+b*(d
*(e*h+f*g)-c*f*h*(2+m)))*(b*x+a)^m*hypergeom([-m, -m],[1-m],b*(d*x+c)/(-a*d
+b*c))/b/d^3/m/((-d*(b*x+a)/(-a*d+b*c))^m)/((d*x+c)^m)
```

Rubi [A] time = 0.11, antiderivative size = 205, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {143, 70, 69}

$$-\frac{(a+bx)^{m+1}(c+dx)^{-m-1}\left(-dfh(m+1)x(bc-ad)+acdfh(m+1)-b\left(c^2fh(m+2)-cd(eh+fg)+d^2eg\right)\right)}{bd^2(m+1)(bc-ad)}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^(-2 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] -(((a + b*x)^(1 + m)*(c + d*x)^(-1 - m)*(a*c*d*f*h*(1 + m) - b*(d^2*e*g - c
*d*(f*g + e*h) + c^2*f*h*(2 + m)) - d*(b*c - a*d)*f*h*(1 + m)*x))/(b*d^2*(b
*c - a*d)*(1 + m))) - ((b*d*(f*g + e*h) + a*d*f*h*m - b*c*f*h*(2 + m))*(a +
 b*x)^m*Hypergeometric2F1[-m, -m, 1 - m, (b*(c + d*x))/(b*c - a*d)])/(b*d^3
*m*(-((d*(a + b*x))/(b*c - a*d)))^m*(c + d*x)^m)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 143

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*d*e*g - a^2*d*f*h*m - a*b*(
d*(f*g + e*h) - c*f*h*(m + 1)) + b*f*h*(b*c - a*d)*(m + 1)*x)*(a + b*x)^(m
+ 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m + 1)), x] + Dist[(a*d*f*h*m +
 b*(d*(f*g + e*h) - c*f*h*(m + 2)))/(b^2*d), Int[(a + b*x)^(m + 1)*(c + d*x
)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && EqQ[m + n + 2, 0
] && NeQ[m, -1] &&  !(SumSimplerQ[n, 1] &&  !SumSimplerQ[m, 1])
```

Rubi steps

$$\int (a+bx)^m (c+dx)^{-2-m} (e+fx)(g+hx)\,dx = -\frac{(a+bx)^{1+m}(c+dx)^{-1-m}\left(acdfh(1+m)-b\left(d^2eg-cd(fg+eh)\cdots\right.\right.}{bd^2(bc-ad)(1+m)}$$

$$= -\frac{(a+bx)^{1+m}(c+dx)^{-1-m}\left(acdfh(1+m)-b\left(d^2eg-cd(fg+eh)\cdots\right.\right.}{bd^2(bc-ad)(1+m)}$$

$$= -\frac{(a+bx)^{1+m}(c+dx)^{-1-m}\left(acdfh(1+m)-b\left(d^2eg-cd(fg+eh)\cdots\right.\right.}{bd^2(bc-ad)(1+m)}$$

Mathematica [A] time = 0.23, size = 198, normalized size = 0.98

$$\frac{(a+bx)^m(c+dx)^{-m}\left(\frac{(m+1)(bc-ad)\left(\frac{d(a+bx)}{ad-bc}\right)^{-m}{}_2F_1\left(-m,-m;1-m;\frac{b(c+dx)}{bc-ad}\right)(-adfhm+bcfh(m+2)-bd(eh+fg))}{m}-\frac{d(a+bx)\left(adfh(m+1)(c+dx)-b\left(\cdots\right.\right.}{\cdots}\right.}{bd^3(m+1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^(-2 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] ((a + b*x)^m*(-((d*(a + b*x)*(a*d*f*h*(1 + m)*(c + d*x) - b*(d^2*e*g + c^2*
f*h*(2 + m) + c*d*(-(f*g) - e*h + f*h*(1 + m)*x))))/(c + d*x)) + ((b*c - a*
d)*(1 + m)*(-(b*d*(f*g + e*h)) - a*d*f*h*m + b*c*f*h*(2 + m))*Hypergeometri
c2F1[-m, -m, 1 - m, (b*(c + d*x))/(b*c - a*d)])/(m*((d*(a + b*x))/(-(b*c) +
 a*d))^m)))/(b*d^3*(b*c - a*d)*(1 + m)*(c + d*x)^m)
```

fricas [F] time = 1.00, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(fhx^2+eg+\left(fg+eh\right)x\right)(bx+a)^m(dx+c)^{-m-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-2-m)*(f*x+e)*(h*x+g),x, algorithm="fricas")
```

```
[Out] integral((f*h*x^2 + e*g + (f*g + e*h)*x)*(b*x + a)^m*(d*x + c)^(-m - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx+e\right)\left(hx+g\right)(bx+a)^m(dx+c)^{-m-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-2-m)*(f*x+e)*(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 2), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \left(fx+e\right)\left(hx+g\right)\left(bx+a\right)^{m}\left(dx+c\right)^{-m-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^(-m-2)*(f*x+e)*(h*x+g),x)
```

```
[Out] int((b*x+a)^m*(d*x+c)^(-m-2)*(f*x+e)*(h*x+g),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx+e\right)\left(hx+g\right)(bx+a)^m(dx+c)^{-m-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-2-m)*(f*x+e)*(h*x+g),x, algorithm="maxima")
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e+f\,x\right)\,\left(g+h\,x\right)\,{\left(a+b\,x\right)}^m}{{\left(c+d\,x\right)}^{m+2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(g + h*x)*(a + b*x)^m)/(c + d*x)^(m + 2),x)
[Out] int(((e + f*x)*(g + h*x)*(a + b*x)^m)/(c + d*x)^(m + 2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**(-2-m)*(f*x+e)*(h*x+g),x)
[Out] Timed out
```

3.129 $\int (a+bx)^m (c+dx)^{-3-m} (e+fx)(g+hx)\,dx$

Optimal. Leaf size=246

$$\frac{fh(a+bx)^{m+3}(c+dx)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^m {}_2F_1\left(m+3,m+3;m+4;-\frac{d(a+bx)}{bc-ad}\right)}{(m+3)(bc-ad)^3}-\frac{(a+bx)^{m+1}(c+dx)^{-m-2}\left(a^3(-d)fh(m+\right.}{}$$

```
[Out] -(b*x+a)^(1+m)*(d*x+c)^(-2-m)*(a^2*b*c*f*h*m-a^3*d*f*h*(1+m)-b^3*c*e*g*(2+m
)+a*b^2*(c*(e*h+f*g)+d*e*g*(1+m))-b*(a^2*d*f*h*(3+2*m)+b^2*(d*e*g+c*(e*h+f*
g)*(1+m))-a*b*(2*c*f*h*(1+m)+d*(e*h+f*g)*(2+m)))*x)/b^2/(-a*d+b*c)^2/(1+m)/
(2+m)+f*h*(b*x+a)^(3+m)*(b*(d*x+c)/(-a*d+b*c))^m*hypergeom([3+m, 3+m],[4+m]
,-d*(b*x+a)/(-a*d+b*c))/(-a*d+b*c)^3/(3+m)/((d*x+c)^m)
```

Rubi [A] time = 0.16, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {145, 70, 69}

$$\frac{fh(a+bx)^{m+3}(c+dx)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^m {}_2F_1\left(m+3,m+3;m+4;-\frac{d(a+bx)}{bc-ad}\right)}{(m+3)(bc-ad)^3}-\frac{(a+bx)^{m+1}(c+dx)^{-m-2}\left(-bx\left(a^2dfh(2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^(-3 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] -(((a + b*x)^(1 + m)*(c + d*x)^(-2 - m)*(a^2*b*c*f*h*m - a^3*d*f*h*(1 + m)
- b^3*c*e*g*(2 + m) + a*b^2*(c*(f*g + e*h) + d*e*g*(1 + m)) - b*(a^2*d*f*h*
(3 + 2*m) + b^2*(d*e*g + c*(f*g + e*h)*(1 + m)) - a*b*(2*c*f*h*(1 + m) + d*
(f*g + e*h)*(2 + m)))*x))/(b^2*(b*c - a*d)^2*(1 + m)*(2 + m))) + (f*h*(a +
b*x)^(3 + m)*((b*(c + d*x))/(b*c - a*d))^m*Hypergeometric2F1[3 + m, 3 + m,
4 + m, -((d*(a + b*x))/(b*c - a*d))])/((b*c - a*d)^3*(3 + m)*(c + d*x)^m)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 145

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(
n + 2) - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h)
 + d*e*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(
f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)*(
a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), x]
 + Dist[(f*h)/b^2 - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1
) - d*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)
)))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), Int[(a + b*x)^(m + 2)*(c + d*x)^n,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[
```

```
m + n + 3, 0] &&  !LtQ[n, -2]))
```

Rubi steps

$$\int (a+bx)^m(c+dx)^{-3-m}(e+fx)(g+hx)\,dx = -\frac{(a+bx)^{1+m}(c+dx)^{-2-m}\left(a^2bcfhm - a^3dfh(1+m) - b^3ceg(2\right.}{}$$

$$= -\frac{(a+bx)^{1+m}(c+dx)^{-2-m}\left(a^2bcfhm - a^3dfh(1+m) - b^3ceg(2\right.}{}$$

$$= -\frac{(a+bx)^{1+m}(c+dx)^{-2-m}\left(a^2bcfhm - a^3dfh(1+m) - b^3ceg(2\right.}{}$$

Mathematica [A] time = 0.32, size = 237, normalized size = 0.96

$$-\frac{(a+bx)^m(c+dx)^{-m-2}\left(d^3(a+bx)\left(a^3(-d)fh(m+1) + a^2bfh(cm - d(2m+3)x) + ab^2(ceh + cf(g + 2h(m+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^(-3 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] -(((a + b*x)^m*(c + d*x)^(-2 - m)*(d^3*(a + b*x)*(-(a^3*d*f*h*(1 + m)) + a^
2*b*f*h*(c*m - d*(3 + 2*m)*x) + a*b^2*(c*e*h + d*e*g*(1 + m) + d*f*g*(2 + m
)*x + d*e*h*(2 + m)*x + c*f*(g + 2*h*(1 + m)*x)) - b^3*(d*e*g*x + c*(e*g*(2
 + m) + f*g*(1 + m)*x + e*h*(1 + m)*x))) + ((b*c - a*d)^4*f*h*(1 + m)*Hyper
geometric2F1[-2 - m, -2 - m, -1 - m, (b*(c + d*x))/(b*c - a*d)])/((d*(a + b
*x))/(-(b*c) + a*d))^m))/(b^2*d^3*(b*c - a*d)^2*(1 + m)*(2 + m)))
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(fhx^2 + eg + \left(fg + eh\right)x\right)(bx+a)^m(dx+c)^{-m-3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-3-m)*(f*x+e)*(h*x+g),x, algorithm="fricas")
```

```
[Out] integral((f*h*x^2 + e*g + (f*g + e*h)*x)*(b*x + a)^m*(d*x + c)^(-m - 3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx+e\right)\left(hx+g\right)(bx+a)^m(dx+c)^{-m-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-3-m)*(f*x+e)*(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 3), x)
```

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \left(fx+e\right)\left(hx+g\right)\left(bx+a\right)^m\left(dx+c\right)^{-m-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^(-m-3)*(f*x+e)*(h*x+g),x)
```

```
[Out] int((b*x+a)^m*(d*x+c)^(-m-3)*(f*x+e)*(h*x+g),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx+e)(hx+g)(bx+a)^m(dx+c)^{-m-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-3-m)*(f*x+e)*(h*x+g),x, algorithm="maxima")
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)\,(g+h\,x)\,(a+b\,x)^m}{(c+d\,x)^{m+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(g + h*x)*(a + b*x)^m)/(c + d*x)^(m + 3),x)
[Out] int(((e + f*x)*(g + h*x)*(a + b*x)^m)/(c + d*x)^(m + 3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**(-3-m)*(f*x+e)*(h*x+g),x)
[Out] Timed out
```

3.130 $\int (a+bx)^m(c+dx)^{-4-m}(e+fx)(g+hx)\,dx$

Optimal. Leaf size=362

$$\frac{(a+bx)^{m+1}(c+dx)^{-m-2}\left(a^2d^2fh\left(m^2+5m+6\right)-abd(m+3)(2cfh(m+1)+d(eh+fg))+b^2\left(c^2fh\left(m^2+3m\right.\right.\right.}{bd^2(m+2)(m+3)(bc-ad)^2}$$

```
[Out] (a^2*d^2*f*h*(m^2+5*m+6)-a*b*d*(3+m)*(d*(e*h+f*g)+2*c*f*h*(1+m))+b^2*(2*d^2
*e*g+c*d*(e*h+f*g)*(1+m)+c^2*f*h*(m^2+3*m+2)))*(b*x+a)^(1+m)*(d*x+c)^(-2-m)
/b/d^2/(-a*d+b*c)^2/(2+m)/(3+m)+(a^2*d^2*f*h*(m^2+5*m+6)-a*b*d*(3+m)*(d*(e*
h+f*g)+2*c*f*h*(1+m))+b^2*(2*d^2*e*g+c*d*(e*h+f*g)*(1+m)+c^2*f*h*(m^2+3*m+2
)))*(b*x+a)^(1+m)*(d*x+c)^(-1-m)/d^2/(-a*d+b*c)^3/(1+m)/(2+m)/(3+m)+(b*x+a)
^(1+m)*(d*x+c)^(-3-m)*(a*c*d*f*h*(3+m)+b*(d^2*e*g-c*d*(e*h+f*g)-c^2*f*h*(2+
m))-d*(-a*d+b*c)*f*h*(3+m)*x)/b/d^2/(-a*d+b*c)/(3+m)
```

Rubi [A] time = 0.36, antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {146, 45, 37}

$$\frac{(a+bx)^{m+1}(c+dx)^{-m-2}\left(a^2d^2fh\left(m^2+5m+6\right)-abd(m+3)(2cfh(m+1)+d(eh+fg))+b^2\left(c^2fh\left(m^2+3m\right.\right.\right.}{bd^2(m+2)(m+3)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^(-4 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] ((a^2*d^2*f*h*(6 + 5*m + m^2) - a*b*d*(3 + m)*(d*(f*g + e*h) + 2*c*f*h*(1 +
 m)) + b^2*(2*d^2*e*g + c*d*(f*g + e*h)*(1 + m) + c^2*f*h*(2 + 3*m + m^2)))
*(a + b*x)^(1 + m)*(c + d*x)^(-2 - m))/(b*d^2*(b*c - a*d)^2*(2 + m)*(3 + m)
) + ((a^2*d^2*f*h*(6 + 5*m + m^2) - a*b*d*(3 + m)*(d*(f*g + e*h) + 2*c*f*h*
(1 + m)) + b^2*(2*d^2*e*g + c*d*(f*g + e*h)*(1 + m) + c^2*f*h*(2 + 3*m + m^
2)))*(a + b*x)^(1 + m)*(c + d*x)^(-1 - m))/(d^2*(b*c - a*d)^3*(1 + m)*(2 +
m)*(3 + m)) + ((a + b*x)^(1 + m)*(c + d*x)^(-3 - m)*(a*c*d*f*h*(3 + m) + b*
(d^2*e*g - c*d*(f*g + e*h) - c^2*f*h*(2 + m)) - d*(b*c - a*d)*f*h*(3 + m)*x
))/(b*d^2*(b*c - a*d)*(3 + m))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 146

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m
 + 1)*(m + n + 3)), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
```

```
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rubi steps

$$\int (a+bx)^m(c+dx)^{-4-m}(e+fx)(g+hx)\,dx = \frac{(a+bx)^{1+m}(c+dx)^{-3-m}\left(acdfh(3+m)+b\left(d^2eg-cd(fg+eh)-\right.\right.}{bd^2(bc-ad)(3+m)}$$

$$= \frac{\left(a^2d^2fh\left(6+5m+m^2\right)-abd(3+m)(d(fg+eh)+2cfh(1+m))\right.}{bd^2(bc}$$

$$= \frac{\left(a^2d^2fh\left(6+5m+m^2\right)-abd(3+m)(d(fg+eh)+2cfh(1+m))\right.}{bd^2(bc}$$

Mathematica [A] time = 0.48, size = 220, normalized size = 0.61

$$\frac{(a+bx)^{m+1}(c+dx)^{-m-3}\left(\frac{(c+dx)(-ad(m+1)+bc(m+2)+bdx)\left(a^2d^2fh\left(m^2+5m+6\right)-abd(m+3)(2cfh(m+1)+d(eh+fg))+b^2\left(c^2fh\left(m^2+3m+2\right)+cd\right.\right.}{(m+1)(m+2)(bc-ad)^2}\right.}{bd^2(m+3)(bc-}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^(-4 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] ((a + b*x)^(1 + m)*(c + d*x)^(-3 - m)*(a*d*f*h*(3 + m)*(c + d*x) + ((a^2*d^
2*f*h*(6 + 5*m + m^2) - a*b*d*(3 + m)*(d*(f*g + e*h) + 2*c*f*h*(1 + m)) + b
^2*(2*d^2*e*g + c*d*(f*g + e*h)*(1 + m) + c^2*f*h*(2 + 3*m + m^2)))*(c + d*
x)*(-(a*d*(1 + m)) + b*c*(2 + m) + b*d*x))/((b*c - a*d)^2*(1 + m)*(2 + m))
+ b*(d^2*e*g - c^2*f*h*(2 + m) - c*d*(e*h + f*(g + h*(3 + m)*x)))))/(b*d^2*
(b*c - a*d)*(3 + m))
```

fricas [B] time = 1.08, size = 1659, normalized size = 4.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-4-m)*(f*x+e)*(h*x+g),x, algorithm="fricas")
```

```
[Out] ((a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*e*g*m^2 + ((b^3*c^2*d - 2*a*b^2*c*
d^2 + a^2*b*d^3)*f*h*m^2 + (2*b^3*d^3*e + (b^3*c*d^2 - 3*a*b^2*d^3)*f)*g +
((b^3*c*d^2 - 3*a*b^2*d^3)*e + 2*(b^3*c^2*d - 3*a*b^2*c*d^2 + 3*a^2*b*d^3)*
f)*h + ((b^3*c*d^2 - a*b^2*d^3)*f*g + ((b^3*c*d^2 - a*b^2*d^3)*e + (3*b^3*c
^2*d - 8*a*b^2*c*d^2 + 5*a^2*b*d^3)*f)*h)*m)*x^4 + (((b^3*c^2*d - 2*a*b^2*c
*d^2 + a^2*b*d^3)*f*g + ((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*e + (b^3*c
^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*f)*h)*m^2 + 4*(2*b^3*c*d^2*e + (b
^3*c^2*d - 3*a*b^2*c*d^2)*f)*g + 2*(2*(b^3*c^2*d - 3*a*b^2*c*d^2)*e + (b^3*
c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 + 3*a^3*d^3)*f)*h + ((2*(b^3*c*d^2 - a*
b^2*d^3)*e + (5*b^3*c^2*d - 8*a*b^2*c*d^2 + 3*a^2*b*d^3)*f)*g + ((5*b^3*c^2
*d - 8*a*b^2*c*d^2 + 3*a^2*b*d^3)*e + (3*b^3*c^3 - 7*a*b^2*c^2*d - a^2*b*c*
d^2 + 5*a^3*d^3)*f)*h)*m)*x^3 + ((((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*
e + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*f)*g + ((b^3*c^3 - a*b^
2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*e + (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2
)*f)*h)*m^2 + 3*(4*b^3*c^2*d*e + (b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 +
 a^3*d^3)*f)*g + 3*(4*a^3*c*d^2*f + (b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^
```

```
2 + a^3*d^3)*e)*h + (((7*b^3*c^2*d - 8*a*b^2*c*d^2 + a^2*b*d^3)*e + 4*(b^3*
c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*f)*g + (4*(b^3*c^3 - a*b^2*c^2*d
 - a^2*b*c*d^2 + a^3*d^3)*e + (a*b^2*c^3 - 8*a^2*b*c^2*d + 7*a^3*c*d^2)*f)*
h)*m)*x^2 + (2*(3*a*b^2*c^3 - 3*a^2*b*c^2*d + a^3*c*d^2)*e - (3*a^2*b*c^3 -
 a^3*c^2*d)*f)*g + (2*a^3*c^3*f - (3*a^2*b*c^3 - a^3*c^2*d)*e)*h - ((a^2*b*
c^3 - a^3*c^2*d)*e*h - ((5*a*b^2*c^3 - 8*a^2*b*c^2*d + 3*a^3*c*d^2)*e - (a^
2*b*c^3 - a^3*c^2*d)*f)*g)*m + (((a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*e*
h + ((b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*e + (a*b^2*c^3 - 2*a^2
*b*c^2*d + a^3*c*d^2)*f)*g)*m^2 + 2*((3*b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c
*d^2 + a^3*d^3)*e - 2*(3*a^2*b*c^2*d - a^3*c*d^2)*f)*g + 4*(2*a^3*c^2*d*f -
 (3*a^2*b*c^2*d - a^3*c*d^2)*e)*h + (((5*b^3*c^3 - a*b^2*c^2*d - 7*a^2*b*c*
d^2 + 3*a^3*d^3)*e + (3*a*b^2*c^3 - 8*a^2*b*c^2*d + 5*a^3*c*d^2)*f)*g + ((3
*a*b^2*c^3 - 8*a^2*b*c^2*d + 5*a^3*c*d^2)*e - 2*(a^2*b*c^3 - a^3*c^2*d)*f)*
h)*m)*x)*(b*x + a)^m*(d*x + c)^(-m - 4)/(6*b^3*c^3 - 18*a*b^2*c^2*d + 18*a^
2*b*c*d^2 - 6*a^3*d^3 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)
*m^3 + 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*m^2 + 11*(b^3*
c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*m)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx+e\right)\left(hx+g\right)(bx+a)^m(dx+c)^{-m-4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-4-m)*(f*x+e)*(h*x+g),x, algorithm="giac")
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 4), x)
```

maple [B] time = 0.01, size = 894, normalized size = 2.47

$$-\frac{\left(a^2d^2fh\,m^2x^2-2abcdfh\,m^2x^2+b^2c^2fh\,m^2x^2+a^2d^2eh\,m^2x+a^2d^2fg\,m^2x+5a^2d^2fhm\,x^2-2abcdeh\,m^2x-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^(-m-4)*(f*x+e)*(h*x+g),x)
[Out] -(b*x+a)^(m+1)*(d*x+c)^(-m-3)*(a^2*d^2*f*h*m^2*x^2-2*a*b*c*d*f*h*m^2*x^2+b^
2*c^2*f*h*m^2*x^2+a^2*d^2*e*h*m^2*x+a^2*d^2*f*g*m^2*x+5*a^2*d^2*f*h*m*x^2-2
*a*b*c*d*e*h*m^2*x-2*a*b*c*d*f*g*m^2*x-8*a*b*c*d*f*h*m*x^2-a*b*d^2*e*h*m*x^
2-a*b*d^2*f*g*m*x^2+b^2*c^2*e*h*m^2*x+b^2*c^2*f*g*m^2*x+3*b^2*c^2*f*h*m*x^2
+b^2*c*d*e*h*m*x^2+b^2*c*d*f*g*m*x^2+2*a^2*c*d*f*h*m*x+a^2*d^2*e*g*m^2+4*a^
2*d^2*e*h*m*x+4*a^2*d^2*f*g*m*x+6*a^2*d^2*f*h*x^2-2*a*b*c^2*f*h*m*x-2*a*b*c
*d*e*g*m^2-8*a*b*c*d*e*h*m*x-8*a*b*c*d*f*g*m*x-6*a*b*c*d*f*h*x^2-2*a*b*d^2*
e*g*m*x-3*a*b*d^2*e*h*x^2-3*a*b*d^2*f*g*x^2+b^2*c^2*e*g*m^2+4*b^2*c^2*e*h*m
*x+4*b^2*c^2*f*g*m*x+2*b^2*c^2*f*h*x^2+2*b^2*c*d*e*g*m*x+b^2*c*d*e*h*x^2+b^
2*c*d*f*g*x^2+2*b^2*d^2*e*g*x^2+a^2*c*d*e*h*m+a^2*c*d*f*g*m+6*a^2*c*d*f*h*x
+3*a^2*d^2*e*g*m+3*a^2*d^2*e*h*x+3*a^2*d^2*f*g*x-a*b*c^2*e*h*m-a*b*c^2*f*g*
m-2*a*b*c^2*f*h*x-8*a*b*c*d*e*g*m-10*a*b*c*d*e*h*x-10*a*b*c*d*f*g*x-2*a*b*d
^2*e*g*x+5*b^2*c^2*e*g*m+3*b^2*c^2*e*h*x+3*b^2*c^2*f*g*x+6*b^2*c*d*e*g*x+2*
a^2*c^2*f*h+a^2*c*d*e*h+a^2*c*d*f*g+2*a^2*d^2*e*g-3*a*b*c^2*e*h-3*a*b*c^2*f
*g-6*a*b*c*d*e*g+6*b^2*c^2*e*g)/(a^3*d^3*m^3-3*a^2*b*c*d^2*m^3+3*a*b^2*c^2*
d*m^3-b^3*c^3*m^3+6*a^3*d^3*m^2-18*a^2*b*c*d^2*m^2+18*a*b^2*c^2*d*m^2-6*b^3
*c^3*m^2+11*a^3*d^3*m-33*a^2*b*c*d^2*m+33*a*b^2*c^2*d*m-11*b^3*c^3*m+6*a^3*
d^3-18*a^2*b*c*d^2+18*a*b^2*c^2*d-6*b^3*c^3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx+e\right)\left(hx+g\right)(bx+a)^m(dx+c)^{-m-4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-4-m)*(f*x+e)*(h*x+g),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 4), x)
```

mupad [B] time = 4.49, size = 1895, normalized size = 5.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(g + h*x)*(a + b*x)^m)/(c + d*x)^(m + 4),x)
```

```
[Out] - ((a + b*x)^m*(2*a^3*c^3*f*h + 6*a*b^2*c^3*e*g - 3*a^2*b*c^3*e*h - 3*a^2*b
*c^3*f*g + 2*a^3*c*d^2*e*g + a^3*c^2*d*e*h + a^3*c^2*d*f*g - 6*a^2*b*c^2*d*
e*g + 5*a*b^2*c^3*e*g*m - a^2*b*c^3*e*h*m - a^2*b*c^3*f*g*m + 3*a^3*c*d^2*e
*g*m + a^3*c^2*d*e*h*m + a^3*c^2*d*f*g*m + a*b^2*c^3*e*g*m^2 + a^3*c*d^2*e*
g*m^2 - 2*a^2*b*c^2*d*e*g*m^2 - 8*a^2*b*c^2*d*e*g*m))/((a*d - b*c)^3*(c + d
*x)^(m + 4)*(11*m + 6*m^2 + m^3 + 6)) - (x^3*(a + b*x)^m*(6*a^3*d^3*f*h + 2
*b^3*c^3*f*h + 8*b^3*c*d^2*e*g + 4*b^3*c^2*d*e*h + 4*b^3*c^2*d*f*g + 5*a^3*
d^3*f*h*m + 3*b^3*c^3*f*h*m + a^3*d^3*f*h*m^2 + b^3*c^3*f*h*m^2 - 12*a*b^2*
c*d^2*e*h - 12*a*b^2*c*d^2*f*g - 6*a*b^2*c^2*d*f*h + 6*a^2*b*c*d^2*f*h - 2*
a*b^2*d^3*e*g*m + 3*a^2*b*d^3*e*h*m + 3*a^2*b*d^3*f*g*m + 2*b^3*c*d^2*e*g*m
 + 5*b^3*c^2*d*e*h*m + 5*b^3*c^2*d*f*g*m + a^2*b*d^3*e*h*m^2 + a^2*b*d^3*f*
g*m^2 + b^3*c^2*d*e*h*m^2 + b^3*c^2*d*f*g*m^2 - 2*a*b^2*c*d^2*e*h*m^2 - 2*a
*b^2*c*d^2*f*g*m^2 - a*b^2*c^2*d*f*h*m^2 - a^2*b*c*d^2*f*h*m^2 - 8*a*b^2*c*
d^2*e*h*m - 8*a*b^2*c*d^2*f*g*m - 7*a*b^2*c^2*d*f*h*m - a^2*b*c*d^2*f*h*m))
/((a*d - b*c)^3*(c + d*x)^(m + 4)*(11*m + 6*m^2 + m^3 + 6)) - (x*(a + b*x)^
m*(2*a^3*d^3*e*g + 6*b^3*c^3*e*g + 4*a^3*c*d^2*e*h + 4*a^3*c*d^2*f*g + 8*a^
3*c^2*d*f*h + 3*a^3*d^3*e*g*m + 5*b^3*c^3*e*g*m + a^3*d^3*e*g*m^2 + b^3*c^3
*e*g*m^2 + 6*a*b^2*c^2*d*e*g - 6*a^2*b*c*d^2*e*g - 12*a^2*b*c^2*d*e*h - 12*
a^2*b*c^2*d*f*g + 3*a*b^2*c^3*e*h*m + 3*a*b^2*c^3*f*g*m - 2*a^2*b*c^3*f*h*m
 + 5*a^3*c*d^2*e*h*m + 5*a^3*c*d^2*f*g*m + 2*a^3*c^2*d*f*h*m + a*b^2*c^3*e*
h*m^2 + a*b^2*c^3*f*g*m^2 + a^3*c*d^2*e*h*m^2 + a^3*c*d^2*f*g*m^2 - a*b^2*c
^2*d*e*g*m^2 - a^2*b*c*d^2*e*g*m^2 - 2*a^2*b*c^2*d*e*h*m^2 - 2*a^2*b*c^2*d*
f*g*m^2 - a*b^2*c^2*d*e*g*m - 7*a^2*b*c*d^2*e*g*m - 8*a^2*b*c^2*d*e*h*m - 8
*a^2*b*c^2*d*f*g*m))/((a*d - b*c)^3*(c + d*x)^(m + 4)*(11*m + 6*m^2 + m^3 +
 6)) - (x^2*(a + b*x)^m*(3*a^3*d^3*e*h + 3*a^3*d^3*f*g + 3*b^3*c^3*e*h + 3*
b^3*c^3*f*g + 12*b^3*c^2*d*e*g + 12*a^3*c*d^2*f*h + 4*a^3*d^3*e*h*m + 4*a^3
*d^3*f*g*m + 4*b^3*c^3*e*h*m + 4*b^3*c^3*f*g*m + a^3*d^3*e*h*m^2 + a^3*d^3*
f*g*m^2 + b^3*c^3*e*h*m^2 + b^3*c^3*f*g*m^2 - 9*a*b^2*c^2*d*e*h - 9*a*b^2*c
^2*d*f*g - 9*a^2*b*c*d^2*e*h - 9*a^2*b*c*d^2*f*g + a^2*b*d^3*e*g*m + a*b^2*
c^3*f*h*m + 7*b^3*c^2*d*e*g*m + 7*a^3*c*d^2*f*h*m + a^2*b*d^3*e*g*m^2 + a*b
^2*c^3*f*h*m^2 + b^3*c^2*d*e*g*m^2 + a^3*c*d^2*f*h*m^2 - 2*a*b^2*c*d^2*e*g*
m^2 - a*b^2*c^2*d*e*h*m^2 - a*b^2*c^2*d*f*g*m^2 - a^2*b*c*d^2*e*h*m^2 - a^2
*b*c*d^2*f*g*m^2 - 2*a^2*b*c^2*d*f*h*m^2 - 8*a*b^2*c*d^2*e*g*m - 4*a*b^2*c^
2*d*e*h*m - 4*a*b^2*c^2*d*f*g*m - 4*a^2*b*c*d^2*e*h*m - 4*a^2*b*c*d^2*f*g*m
 - 8*a^2*b*c^2*d*f*h*m))/((a*d - b*c)^3*(c + d*x)^(m + 4)*(11*m + 6*m^2 + m
^3 + 6)) - (x^4*(a + b*x)^m*(2*b^3*d^3*e*g - 3*a*b^2*d^3*e*h - 3*a*b^2*d^3*
f*g + 6*a^2*b*d^3*f*h + b^3*c*d^2*e*h + b^3*c*d^2*f*g + 2*b^3*c^2*d*f*h - 6
*a*b^2*c*d^2*f*h - a*b^2*d^3*e*h*m - a*b^2*d^3*f*g*m + 5*a^2*b*d^3*f*h*m +
b^3*c*d^2*e*h*m + b^3*c*d^2*f*g*m + 3*b^3*c^2*d*f*h*m + a^2*b*d^3*f*h*m^2 +
 b^3*c^2*d*f*h*m^2 - 2*a*b^2*c*d^2*f*h*m^2 - 8*a*b^2*c*d^2*f*h*m))/((a*d -
b*c)^3*(c + d*x)^(m + 4)*(11*m + 6*m^2 + m^3 + 6))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**(-4-m)*(f*x+e)*(h*x+g),x)
```

```
[Out] Timed out
```

3.131 $\int (a+bx)^m (c+dx)^{-5-m} (e+fx)(g+hx)\,dx$

Optimal. Leaf size=507

$$\frac{(a+bx)^{m+1}(c+dx)^{-m-3}\left(a^2d^2fh\left(m^2+7m+12\right)-2abd(m+4)(cfh(m+1)+d(eh+fg))+b^2\left(c^2fh\left(m^2+3\right.\right.\right.}{2bd^2(m+3)(m+4)(bc-ad)^2}$$

```
[Out] 1/2*(a^2*d^2*f*h*(m^2+7*m+12)-2*a*b*d*(4+m)*(d*(e*h+f*g)+c*f*h*(1+m))+b^2*(
6*d^2*e*g+2*c*d*(e*h+f*g)*(1+m)+c^2*f*h*(m^2+3*m+2)))*(b*x+a)^(1+m)*(d*x+c)
^(-3-m)/b/d^2/(-a*d+b*c)^2/(3+m)/(4+m)+(a^2*d^2*f*h*(m^2+7*m+12)-2*a*b*d*(4
+m)*(d*(e*h+f*g)+c*f*h*(1+m))+b^2*(6*d^2*e*g+2*c*d*(e*h+f*g)*(1+m)+c^2*f*h*
(m^2+3*m+2)))*(b*x+a)^(1+m)*(d*x+c)^(-2-m)/d^2/(-a*d+b*c)^3/(2+m)/(3+m)/(4+
m)+b*(a^2*d^2*f*h*(m^2+7*m+12)-2*a*b*d*(4+m)*(d*(e*h+f*g)+c*f*h*(1+m))+b^2*
(6*d^2*e*g+2*c*d*(e*h+f*g)*(1+m)+c^2*f*h*(m^2+3*m+2)))*(b*x+a)^(1+m)*(d*x+c
)^(-1-m)/d^2/(-a*d+b*c)^4/(1+m)/(2+m)/(3+m)/(4+m)+1/2*(b*x+a)^(1+m)*(d*x+c)
^(-4-m)*(a*c*d*f*h*(4+m)+b*(2*d^2*e*g-2*c*d*(e*h+f*g)-c^2*f*h*(2+m))-d*(-a*
d+b*c)*f*h*(4+m)*x)/b/d^2/(-a*d+b*c)/(4+m)
```

Rubi [A] time = 0.59, antiderivative size = 507, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {146, 45, 37}

$$\frac{(a+bx)^{m+1}(c+dx)^{-m-3}\left(a^2d^2fh\left(m^2+7m+12\right)-2abd(m+4)(cfh(m+1)+d(eh+fg))+b^2\left(c^2fh\left(m^2+3\right.\right.\right.}{2bd^2(m+3)(m+4)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^(-5 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] ((a^2*d^2*f*h*(12 + 7*m + m^2) - 2*a*b*d*(4 + m)*(d*(f*g + e*h) + c*f*h*(1
+ m)) + b^2*(6*d^2*e*g + 2*c*d*(f*g + e*h)*(1 + m) + c^2*f*h*(2 + 3*m + m^2
)))*(a + b*x)^(1 + m)*(c + d*x)^(-3 - m))/(2*b*d^2*(b*c - a*d)^2*(3 + m)*(4
 + m)) + ((a^2*d^2*f*h*(12 + 7*m + m^2) - 2*a*b*d*(4 + m)*(d*(f*g + e*h) +
c*f*h*(1 + m)) + b^2*(6*d^2*e*g + 2*c*d*(f*g + e*h)*(1 + m) + c^2*f*h*(2 +
3*m + m^2)))*(a + b*x)^(1 + m)*(c + d*x)^(-2 - m))/(d^2*(b*c - a*d)^3*(2 +
m)*(3 + m)*(4 + m)) + (b*(a^2*d^2*f*h*(12 + 7*m + m^2) - 2*a*b*d*(4 + m)*(d
*(f*g + e*h) + c*f*h*(1 + m)) + b^2*(6*d^2*e*g + 2*c*d*(f*g + e*h)*(1 + m)
+ c^2*f*h*(2 + 3*m + m^2)))*(a + b*x)^(1 + m)*(c + d*x)^(-1 - m))/(d^2*(b*c
 - a*d)^4*(1 + m)*(2 + m)*(3 + m)*(4 + m)) + ((a + b*x)^(1 + m)*(c + d*x)^(
-4 - m)*(a*c*d*f*h*(4 + m) + b*(2*d^2*e*g - 2*c*d*(f*g + e*h) - c^2*f*h*(2
+ m)) - d*(b*c - a*d)*f*h*(4 + m)*x))/(2*b*d^2*(b*c - a*d)*(4 + m))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 146

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m
 + 1)*(m + n + 3)), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rubi steps

$$\int (a+bx)^m (c+dx)^{-5-m} (e+fx)(g+hx)\,dx = \frac{(a+bx)^{1+m}(c+dx)^{-4-m}\left(acdfh(4+m)+b\left(2d^2eg-2cd(fg+eh)\right.\right.}{2bd^2(bc-ad)(4+m)}$$

$$= \frac{\left(a^2d^2fh\left(12+7m+m^2\right)-2abd(4+m)(d(fg+eh)+cfh(1+m))\right.}{2bd^2(}$$

$$= \frac{\left(a^2d^2fh\left(12+7m+m^2\right)-2abd(4+m)(d(fg+eh)+cfh(1+m))\right.}{2bd^2(}$$

$$= \frac{\left(a^2d^2fh\left(12+7m+m^2\right)-2abd(4+m)(d(fg+eh)+cfh(1+m))\right.}{2bd^2(}$$

Mathematica [A] time = 0.70, size = 279, normalized size = 0.55

$$\frac{(a+bx)^{m+1}(c+dx)^{-m-4}\left(\frac{(c+dx)\left(a^2d^2\left(m^2+3m+2\right)-2abd(m+1)(c(m+3)+dx)+b^2\left(c^2\left(m^2+5m+6\right)+2cd(m+3)x+2d^2x^2\right)\right)\left(a^2d^2fh\left(m^2+7m+12\right)-\right.}{(m+1)(m+2)(m+3)(bc-ad)^3}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^(-5 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] ((a + b*x)^(1 + m)*(c + d*x)^(-4 - m)*(a*d*f*h*(4 + m)*(c + d*x) + b*(2*d^2
*e*g - c^2*f*h*(2 + m) - c*d*(2*f*g + 2*e*h + f*h*(4 + m)*x)) + ((a^2*d^2*f
*h*(12 + 7*m + m^2) - 2*a*b*d*(4 + m)*(d*(f*g + e*h) + c*f*h*(1 + m)) + b^2
*(6*d^2*e*g + 2*c*d*(f*g + e*h)*(1 + m) + c^2*f*h*(2 + 3*m + m^2)))*(c + d*
x)*(a^2*d^2*(2 + 3*m + m^2) - 2*a*b*d*(1 + m)*(c*(3 + m) + d*x) + b^2*(c^2*
(6 + 5*m + m^2) + 2*c*d*(3 + m)*x + 2*d^2*x^2)))/((b*c - a*d)^3*(1 + m)*(2
+ m)*(3 + m))))/(2*b*d^2*(b*c - a*d)*(4 + m))
```

fricas [B] time = 1.18, size = 3441, normalized size = 6.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-5-m)*(f*x+e)*(h*x+g),x, algorithm="fricas")
```

```
[Out] ((a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3)*e*g*m^3 + ((b^
4*c^2*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)*f*h*m^2 + 2*(3*b^4*d^4*e + (b^4*c*
d^3 - 4*a*b^3*d^4)*f)*g + 2*((b^4*c*d^3 - 4*a*b^3*d^4)*e + (b^4*c^2*d^2 - 4
*a*b^3*c*d^3 + 6*a^2*b^2*d^4)*f)*h + (2*(b^4*c*d^3 - a*b^3*d^4)*f*g + (2*(b
^4*c*d^3 - a*b^3*d^4)*e + (3*b^4*c^2*d^2 - 10*a*b^3*c*d^3 + 7*a^2*b^2*d^4)*
f)*h)*m)*x^5 + ((b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)
*f*h*m^3 + (2*(b^4*c^2*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)*f*g + (2*(b^4*c^2
*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)*e + (8*b^4*c^3*d - 23*a*b^3*c^2*d^2 + 2
```

```
2*a^2*b^2*c*d^3 - 7*a^3*b*d^4)*f)*h)*m^2 + 10*(3*b^4*c*d^3*e + (b^4*c^2*d^2
 - 4*a*b^3*c*d^3)*f)*g + 10*((b^4*c^2*d^2 - 4*a*b^3*c*d^3)*e + (b^4*c^3*d -
 4*a*b^3*c^2*d^2 + 6*a^2*b^2*c*d^3)*f)*h + (2*(3*(b^4*c*d^3 - a*b^3*d^4)*e
+ 2*(3*b^4*c^2*d^2 - 5*a*b^3*c*d^3 + 2*a^2*b^2*d^4)*f)*g + (4*(3*b^4*c^2*d^
2 - 5*a*b^3*c*d^3 + 2*a^2*b^2*d^4)*e + (17*b^4*c^3*d - 60*a*b^3*c^2*d^2 + 5
5*a^2*b^2*c*d^3 - 12*a^3*b*d^4)*f)*h)*m)*x^4 + (((b^4*c^3*d - 3*a*b^3*c^2*d
^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*f*g + ((b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a
^2*b^2*c*d^3 - a^3*b*d^4)*e + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^
4*d^4)*f)*h)*m^3 + ((3*(b^4*c^2*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)*e + 5*(2
*b^4*c^3*d - 5*a*b^3*c^2*d^2 + 4*a^2*b^2*c*d^3 - a^3*b*d^4)*f)*g + (5*(2*b^
4*c^3*d - 5*a*b^3*c^2*d^2 + 4*a^2*b^2*c*d^3 - a^3*b*d^4)*e + (7*b^4*c^4 - 1
6*a*b^3*c^3*d + 3*a^2*b^2*c^2*d^2 + 14*a^3*b*c*d^3 - 8*a^4*d^4)*f)*h)*m^2 +
 20*(3*b^4*c^2*d^2*e + (b^4*c^3*d - 4*a*b^3*c^2*d^2)*f)*g + 4*(5*(b^4*c^3*d
 - 4*a*b^3*c^2*d^2)*e + (2*b^4*c^4 - 8*a*b^3*c^3*d + 12*a^2*b^2*c^2*d^2 + 1
2*a^3*b*c*d^3 - 3*a^4*d^4)*f)*h + ((3*(9*b^4*c^2*d^2 - 10*a*b^3*c*d^3 + a^2
*b^2*d^4)*e + (29*b^4*c^3*d - 66*a*b^3*c^2*d^2 + 41*a^2*b^2*c*d^3 - 4*a^3*b
*d^4)*f)*g + ((29*b^4*c^3*d - 66*a*b^3*c^2*d^2 + 41*a^2*b^2*c*d^3 - 4*a^3*b
*d^4)*e + (14*b^4*c^4 - 46*a*b^3*c^3*d + 15*a^2*b^2*c^2*d^2 + 36*a^3*b*c*d^
3 - 19*a^4*d^4)*f)*h)*m)*x^3 - ((a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)
*e*h - (3*(3*a*b^3*c^4 - 8*a^2*b^2*c^3*d + 7*a^3*b*c^2*d^2 - 2*a^4*c*d^3)*e
 - (a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*f)*g)*m^2 + ((((b^4*c^3*d -
3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*e + (b^4*c^4 - 2*a*b^3*c^3*d
 + 2*a^3*b*c*d^3 - a^4*d^4)*f)*g + ((b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^
3 - a^4*d^4)*e + (a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3
)*f)*h)*m^3 + ((3*(4*b^4*c^3*d - 9*a*b^3*c^2*d^2 + 6*a^2*b^2*c*d^3 - a^3*b*
d^4)*e + (8*b^4*c^4 - 14*a*b^3*c^3*d - 3*a^2*b^2*c^2*d^2 + 16*a^3*b*c*d^3 -
 7*a^4*d^4)*f)*g + ((8*b^4*c^4 - 14*a*b^3*c^3*d - 3*a^2*b^2*c^2*d^2 + 16*a^
3*b*c*d^3 - 7*a^4*d^4)*e + 5*(a*b^3*c^4 - 4*a^2*b^2*c^3*d + 5*a^3*b*c^2*d^2
 - 2*a^4*c*d^3)*f)*h)*m^2 + 4*(15*b^4*c^3*d*e + (3*b^4*c^4 - 12*a*b^3*c^3*d
 - 12*a^2*b^2*c^2*d^2 + 8*a^3*b*c*d^3 - 2*a^4*d^4)*f)*g + 4*((3*b^4*c^4 - 1
2*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 8*a^3*b*c*d^3 - 2*a^4*d^4)*e + 5*(4*a^
3*b*c^2*d^2 - a^4*c*d^3)*f)*h + (((47*b^4*c^3*d - 60*a*b^3*c^2*d^2 + 15*a^2
*b^2*c*d^3 - 2*a^3*b*d^4)*e + (19*b^4*c^4 - 36*a*b^3*c^3*d - 15*a^2*b^2*c^2
*d^2 + 46*a^3*b*c*d^3 - 14*a^4*d^4)*f)*g + ((19*b^4*c^4 - 36*a*b^3*c^3*d -
15*a^2*b^2*c^2*d^2 + 46*a^3*b*c*d^3 - 14*a^4*d^4)*e + (4*a*b^3*c^4 - 41*a^2
*b^2*c^3*d + 66*a^3*b*c^2*d^2 - 29*a^4*c*d^3)*f)*h)*m)*x^2 + 2*(3*(4*a*b^3*
c^4 - 6*a^2*b^2*c^3*d + 4*a^3*b*c^2*d^2 - a^4*c*d^3)*e - (6*a^2*b^2*c^4 - 4
*a^3*b*c^3*d + a^4*c^2*d^2)*f)*g - 2*((6*a^2*b^2*c^4 - 4*a^3*b*c^3*d + a^4*
c^2*d^2)*e - (4*a^3*b*c^4 - a^4*c^3*d)*f)*h + (((26*a*b^3*c^4 - 57*a^2*b^2*
c^3*d + 42*a^3*b*c^2*d^2 - 11*a^4*c*d^3)*e - (7*a^2*b^2*c^4 - 10*a^3*b*c^3*
d + 3*a^4*c^2*d^2)*f)*g - ((7*a^2*b^2*c^4 - 10*a^3*b*c^3*d + 3*a^4*c^2*d^2)
*e - 2*(a^3*b*c^4 - a^4*c^3*d)*f)*h)*m + (((a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3
*a^3*b*c^2*d^2 - a^4*c*d^3)*e*h + ((b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3
 - a^4*d^4)*e + (a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3)
*f)*g)*m^3 + ((3*(3*b^4*c^4 - 4*a*b^3*c^3*d - 3*a^2*b^2*c^2*d^2 + 6*a^3*b*c
*d^3 - 2*a^4*d^4)*e + (7*a*b^3*c^4 - 22*a^2*b^2*c^3*d + 23*a^3*b*c^2*d^2 -
8*a^4*c*d^3)*f)*g + ((7*a*b^3*c^4 - 22*a^2*b^2*c^3*d + 23*a^3*b*c^2*d^2 - 8
*a^4*c*d^3)*e - 2*(a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*f)*h)*m^2 + 2
*(3*(4*b^4*c^4 + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^
4)*e - 5*(6*a^2*b^2*c^3*d - 4*a^3*b*c^2*d^2 + a^4*c*d^3)*f)*g - 10*((6*a^2*
b^2*c^3*d - 4*a^3*b*c^2*d^2 + a^4*c*d^3)*e - (4*a^3*b*c^3*d - a^4*c^2*d^2)*
f)*h + (((26*b^4*c^4 - 10*a*b^3*c^3*d - 45*a^2*b^2*c^2*d^2 + 40*a^3*b*c*d^3
 - 11*a^4*d^4)*e + (12*a*b^3*c^4 - 55*a^2*b^2*c^3*d + 60*a^3*b*c^2*d^2 - 17
*a^4*c*d^3)*f)*g + ((12*a*b^3*c^4 - 55*a^2*b^2*c^3*d + 60*a^3*b*c^2*d^2 - 1
7*a^4*c*d^3)*e - 4*(2*a^2*b^2*c^4 - 5*a^3*b*c^3*d + 3*a^4*c^2*d^2)*f)*h)*m)
*x)*(b*x + a)^m*(d*x + c)^(-m - 5)/(24*b^4*c^4 - 96*a*b^3*c^3*d + 144*a^2*b
^2*c^2*d^2 - 96*a^3*b*c*d^3 + 24*a^4*d^4 + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2
*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*m^4 + 10*(b^4*c^4 - 4*a*b^3*c^3*d +
 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*m^3 + 35*(b^4*c^4 - 4*a*b^3*c
```

```
^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*m^2 + 50*(b^4*c^4 - 4*a
*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*m)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx+e\right)\left(hx+g\right)(bx+a)^m(dx+c)^{-m-5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-5-m)*(f*x+e)*(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 5), x)
```

maple [B] time = 0.02, size = 2343, normalized size = 4.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^(-m-5)*(f*x+e)*(h*x+g),x)
```

```
[Out] -(b*x+a)^(m+1)*(d*x+c)^(-m-4)*(a^3*d^3*f*h*m^3*x^2-3*a^2*b*c*d^2*f*h*m^3*x^
2-a^2*b*d^3*f*h*m^2*x^3+3*a*b^2*c^2*d*f*h*m^3*x^2+2*a*b^2*c*d^2*f*h*m^2*x^3
-b^3*c^3*f*h*m^3*x^2-b^3*c^2*d*f*h*m^2*x^3+a^3*d^3*e*h*m^3*x+a^3*d^3*f*g*m^
3*x+8*a^3*d^3*f*h*m^2*x^2-3*a^2*b*c*d^2*e*h*m^3*x-3*a^2*b*c*d^2*f*g*m^3*x-2
3*a^2*b*c*d^2*f*h*m^2*x^2-2*a^2*b*d^3*e*h*m^2*x^2-2*a^2*b*d^3*f*g*m^2*x^2-7
*a^2*b*d^3*f*h*m*x^3+3*a*b^2*c^2*d*e*h*m^3*x+3*a*b^2*c^2*d*f*g*m^3*x+22*a*b
^2*c^2*d*f*h*m^2*x^2+4*a*b^2*c*d^2*e*h*m^2*x^2+4*a*b^2*c*d^2*f*g*m^2*x^2+10
*a*b^2*c*d^2*f*h*m*x^3+2*a*b^2*d^3*e*h*m*x^3+2*a*b^2*d^3*f*g*m*x^3-b^3*c^3*
e*h*m^3*x-b^3*c^3*f*g*m^3*x-7*b^3*c^3*f*h*m^2*x^2-2*b^3*c^2*d*e*h*m^2*x^2-2
*b^3*c^2*d*f*g*m^2*x^2-3*b^3*c^2*d*f*h*m*x^3-2*b^3*c*d^2*e*h*m*x^3-2*b^3*c*
d^2*f*g*m*x^3+2*a^3*c*d^2*f*h*m^2*x+a^3*d^3*e*g*m^3+7*a^3*d^3*e*h*m^2*x+7*a
^3*d^3*f*g*m^2*x+19*a^3*d^3*f*h*m*x^2-4*a^2*b*c^2*d*f*h*m^2*x-3*a^2*b*c*d^2
*e*g*m^3-22*a^2*b*c*d^2*e*h*m^2*x-22*a^2*b*c*d^2*f*g*m^2*x-58*a^2*b*c*d^2*f
*h*m*x^2-3*a^2*b*d^3*e*g*m^2*x-10*a^2*b*d^3*e*h*m*x^2-10*a^2*b*d^3*f*g*m*x^
2-12*a^2*b*d^3*f*h*x^3+2*a*b^2*c^3*f*h*m^2*x+3*a*b^2*c^2*d*e*g*m^3+23*a*b^2
*c^2*d*e*h*m^2*x+23*a*b^2*c^2*d*f*g*m^2*x+53*a*b^2*c^2*d*f*h*m*x^2+6*a*b^2*
c*d^2*e*g*m^2*x+20*a*b^2*c*d^2*e*h*m*x^2+20*a*b^2*c*d^2*f*g*m*x^2+8*a*b^2*c
*d^2*f*h*x^3+6*a*b^2*d^3*e*g*m*x^2+8*a*b^2*d^3*e*h*x^3+8*a*b^2*d^3*f*g*x^3-
b^3*c^3*e*g*m^3-8*b^3*c^3*e*h*m^2*x-8*b^3*c^3*f*g*m^2*x-14*b^3*c^3*f*h*m*x^
2-3*b^3*c^2*d*e*g*m^2*x-10*b^3*c^2*d*e*h*m*x^2-10*b^3*c^2*d*f*g*m*x^2-2*b^3
*c^2*d*f*h*x^3-6*b^3*c*d^2*e*g*m*x^2-2*b^3*c*d^2*e*h*x^3-2*b^3*c*d^2*f*g*x^
3-6*b^3*d^3*e*g*x^3+a^3*c*d^2*e*h*m^2+a^3*c*d^2*f*g*m^2+10*a^3*c*d^2*f*h*m*
x+6*a^3*d^3*e*g*m^2+14*a^3*d^3*e*h*m*x+14*a^3*d^3*f*g*m*x+12*a^3*d^3*f*h*x^
2-2*a^2*b*c^2*d*e*h*m^2-2*a^2*b*c^2*d*f*g*m^2-20*a^2*b*c^2*d*f*h*m*x-21*a^2
*b*c*d^2*e*g*m^2-53*a^2*b*c*d^2*e*h*m*x-53*a^2*b*c*d^2*f*g*m*x-56*a^2*b*c*d
^2*f*h*x^2-9*a^2*b*d^3*e*g*m*x-8*a^2*b*d^3*e*h*x^2-8*a^2*b*d^3*f*g*x^2+a*b^
2*c^3*e*h*m^2+a*b^2*c^3*f*g*m^2+10*a*b^2*c^3*f*h*m*x+24*a*b^2*c^2*d*e*g*m^2
+58*a*b^2*c^2*d*e*h*m*x+58*a*b^2*c^2*d*f*g*m*x+34*a*b^2*c^2*d*f*h*x^2+30*a*
b^2*c*d^2*e*g*m*x+34*a*b^2*c*d^2*e*h*x^2+34*a*b^2*c*d^2*f*g*x^2+6*a*b^2*d^3
*e*g*x^2-9*b^3*c^3*e*g*m^2-19*b^3*c^3*e*h*m*x-19*b^3*c^3*f*g*m*x-8*b^3*c^3*
f*h*x^2-21*b^3*c^2*d*e*g*m*x-8*b^3*c^2*d*e*h*x^2-8*b^3*c^2*d*f*g*x^2-24*b^3
*c*d^2*e*g*x^2+2*a^3*c^2*d*f*h*m+3*a^3*c*d^2*e*h*m+3*a^3*c*d^2*f*g*m+8*a^3*
c*d^2*f*h*x+11*a^3*d^3*e*g*m+8*a^3*d^3*e*h*x+8*a^3*d^3*f*g*x-2*a^2*b*c^3*f*
h*m-10*a^2*b*c^2*d*e*h*m-10*a^2*b*c^2*d*f*g*m-34*a^2*b*c^2*d*f*h*x-42*a^2*b
*c*d^2*e*g*m-34*a^2*b*c*d^2*e*h*x-34*a^2*b*c*d^2*f*g*x-6*a^2*b*d^3*e*g*x+7*
a*b^2*c^3*e*h*m+7*a*b^2*c^3*f*g*m+8*a*b^2*c^3*f*h*x+57*a*b^2*c^2*d*e*g*m+56
*a*b^2*c^2*d*e*h*x+56*a*b^2*c^2*d*f*g*x+24*a*b^2*c*d^2*e*g*x-26*b^3*c^3*e*g
*m-12*b^3*c^3*e*h*x-12*b^3*c^3*f*g*x-36*b^3*c^2*d*e*g*x+2*a^3*c^2*d*f*h+2*a
^3*c*d^2*e*h+2*a^3*c*d^2*f*g+6*a^3*d^3*e*g-8*a^2*b*c^3*f*h-8*a^2*b*c^2*d*e*
h-8*a^2*b*c^2*d*f*g-24*a^2*b*c*d^2*e*g+12*a*b^2*c^3*e*h+12*a*b^2*c^3*f*g+36
```

```
*a*b^2*c^2*d*e*g-24*b^3*c^3*e*g)/(a^4*d^4*m^4-4*a^3*b*c*d^3*m^4+6*a^2*b^2*c
^2*d^2*m^4-4*a*b^3*c^3*d*m^4+b^4*c^4*m^4+10*a^4*d^4*m^3-40*a^3*b*c*d^3*m^3+
60*a^2*b^2*c^2*d^2*m^3-40*a*b^3*c^3*d*m^3+10*b^4*c^4*m^3+35*a^4*d^4*m^2-140
*a^3*b*c*d^3*m^2+210*a^2*b^2*c^2*d^2*m^2-140*a*b^3*c^3*d*m^2+35*b^4*c^4*m^2
+50*a^4*d^4*m-200*a^3*b*c*d^3*m+300*a^2*b^2*c^2*d^2*m-200*a*b^3*c^3*d*m+50*
b^4*c^4*m+24*a^4*d^4-96*a^3*b*c*d^3+144*a^2*b^2*c^2*d^2-96*a*b^3*c^3*d+24*b
^4*c^4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx+e\right)\left(hx+g\right)(bx+a)^{m}(dx+c)^{-m-5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-5-m)*(f*x+e)*(h*x+g),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 5), x)
```

mupad [B] time = 6.75, size = 3720, normalized size = 7.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(g + h*x)*(a + b*x)^m)/(c + d*x)^(m + 5),x)
```

```
[Out] (x^5*(a + b*x)^m*(6*b^4*d^4*e*g - 8*a*b^3*d^4*e*h - 8*a*b^3*d^4*f*g + 2*b^4
*c*d^3*e*h + 2*b^4*c*d^3*f*g + 12*a^2*b^2*d^4*f*h + 2*b^4*c^2*d^2*f*h + a^2
*b^2*d^4*f*h*m^2 + b^4*c^2*d^2*f*h*m^2 - 8*a*b^3*c*d^3*f*h - 2*a*b^3*d^4*e*
h*m - 2*a*b^3*d^4*f*g*m + 2*b^4*c*d^3*e*h*m + 2*b^4*c*d^3*f*g*m + 7*a^2*b^2
*d^4*f*h*m + 3*b^4*c^2*d^2*f*h*m - 2*a*b^3*c*d^3*f*h*m^2 - 10*a*b^3*c*d^3*f
*h*m))/((a*d - b*c)^4*(c + d*x)^(m + 5)*(50*m + 35*m^2 + 10*m^3 + m^4 + 24)
) - (x*(a + b*x)^m*(6*a^4*d^4*e*g - 24*b^4*c^4*e*g + 10*a^4*c*d^3*e*h + 10*
a^4*c*d^3*f*g + 11*a^4*d^4*e*g*m - 26*b^4*c^4*e*g*m + 10*a^4*c^2*d^2*f*h +
6*a^4*d^4*e*g*m^2 - 9*b^4*c^4*e*g*m^2 + a^4*d^4*e*g*m^3 - b^4*c^4*e*g*m^3 +
 36*a^2*b^2*c^2*d^2*e*g + 2*a^2*b^2*c^4*f*h*m^2 + 2*a^4*c^2*d^2*f*h*m^2 - 2
4*a*b^3*c^3*d*e*g - 24*a^3*b*c*d^3*e*g - 40*a^3*b*c^3*d*f*h - 12*a*b^3*c^4*
e*h*m - 12*a*b^3*c^4*f*g*m + 17*a^4*c*d^3*e*h*m + 17*a^4*c*d^3*f*g*m + 60*a
^2*b^2*c^3*d*e*h + 60*a^2*b^2*c^3*d*f*g - 40*a^3*b*c^2*d^2*e*h - 40*a^3*b*c
^2*d^2*f*g - 7*a*b^3*c^4*e*h*m^2 - 7*a*b^3*c^4*f*g*m^2 - a*b^3*c^4*e*h*m^3
- a*b^3*c^4*f*g*m^3 + 8*a^2*b^2*c^4*f*h*m + 8*a^4*c*d^3*e*h*m^2 + 8*a^4*c*d
^3*f*g*m^2 + a^4*c*d^3*e*h*m^3 + a^4*c*d^3*f*g*m^3 + 12*a^4*c^2*d^2*f*h*m +
 12*a*b^3*c^3*d*e*g*m^2 - 18*a^3*b*c*d^3*e*g*m^2 + 2*a*b^3*c^3*d*e*g*m^3 -
2*a^3*b*c*d^3*e*g*m^3 + 55*a^2*b^2*c^3*d*e*h*m + 55*a^2*b^2*c^3*d*f*g*m - 6
0*a^3*b*c^2*d^2*e*h*m - 60*a^3*b*c^2*d^2*f*g*m - 4*a^3*b*c^3*d*f*h*m^2 + 45
*a^2*b^2*c^2*d^2*e*g*m + 22*a^2*b^2*c^3*d*e*h*m^2 + 22*a^2*b^2*c^3*d*f*g*m^
2 - 23*a^3*b*c^2*d^2*e*h*m^2 - 23*a^3*b*c^2*d^2*f*g*m^2 + 3*a^2*b^2*c^3*d*e
*h*m^3 + 3*a^2*b^2*c^3*d*f*g*m^3 - 3*a^3*b*c^2*d^2*e*h*m^3 - 3*a^3*b*c^2*d^
2*f*g*m^3 + 10*a*b^3*c^3*d*e*g*m - 40*a^3*b*c*d^3*e*g*m - 20*a^3*b*c^3*d*f*
h*m + 9*a^2*b^2*c^2*d^2*e*g*m^2))/((a*d - b*c)^4*(c + d*x)^(m + 5)*(50*m +
35*m^2 + 10*m^3 + m^4 + 24)) - ((a + b*x)^m*(6*a^4*c*d^3*e*g - 8*a^3*b*c^4*
f*h - 24*a*b^3*c^4*e*g + 2*a^4*c^3*d*f*h + 12*a^2*b^2*c^4*e*h + 12*a^2*b^2*
c^4*f*g + 2*a^4*c^2*d^2*e*h + 2*a^4*c^2*d^2*f*g + a^2*b^2*c^4*e*h*m^2 + a^2
*b^2*c^4*f*g*m^2 + a^4*c^2*d^2*e*h*m^2 + a^4*c^2*d^2*f*g*m^2 - 8*a^3*b*c^3*
d*e*h - 8*a^3*b*c^3*d*f*g - 26*a*b^3*c^4*e*g*m - 2*a^3*b*c^4*f*h*m + 11*a^4
*c*d^3*e*g*m + 2*a^4*c^3*d*f*h*m + 36*a^2*b^2*c^3*d*e*g - 24*a^3*b*c^2*d^2*
e*g - 9*a*b^3*c^4*e*g*m^2 - a*b^3*c^4*e*g*m^3 + 7*a^2*b^2*c^4*e*h*m + 7*a^2
*b^2*c^4*f*g*m + 6*a^4*c*d^3*e*g*m^2 + a^4*c*d^3*e*g*m^3 + 3*a^4*c^2*d^2*e*
h*m + 3*a^4*c^2*d^2*f*g*m + 57*a^2*b^2*c^3*d*e*g*m - 42*a^3*b*c^2*d^2*e*g*m
 - 2*a^3*b*c^3*d*e*h*m^2 - 2*a^3*b*c^3*d*f*g*m^2 + 24*a^2*b^2*c^3*d*e*g*m^2
 - 21*a^3*b*c^2*d^2*e*g*m^2 + 3*a^2*b^2*c^3*d*e*g*m^3 - 3*a^3*b*c^2*d^2*e*g
```

```
*m^3 - 10*a^3*b*c^3*d*e*h*m - 10*a^3*b*c^3*d*f*g*m))/((a*d - b*c)^4*(c + d*
x)^(m + 5)*(50*m + 35*m^2 + 10*m^3 + m^4 + 24)) + (x^3*(a + b*x)^m*(8*b^4*c
^4*f*h - 12*a^4*d^4*f*h + 20*b^4*c^3*d*e*h + 20*b^4*c^3*d*f*g - 19*a^4*d^4*
f*h*m + 14*b^4*c^4*f*h*m + 60*b^4*c^2*d^2*e*g - 8*a^4*d^4*f*h*m^2 + 7*b^4*c
^4*f*h*m^2 - a^4*d^4*f*h*m^3 + b^4*c^4*f*h*m^3 + 48*a^2*b^2*c^2*d^2*f*h + 3
*a^2*b^2*d^4*e*g*m^2 + 3*b^4*c^2*d^2*e*g*m^2 - 32*a*b^3*c^3*d*f*h + 48*a^3*
b*c*d^3*f*h - 4*a^3*b*d^4*e*h*m - 4*a^3*b*d^4*f*g*m + 29*b^4*c^3*d*e*h*m +
29*b^4*c^3*d*f*g*m - 80*a*b^3*c^2*d^2*e*h - 80*a*b^3*c^2*d^2*f*g + 3*a^2*b^
2*d^4*e*g*m - 5*a^3*b*d^4*e*h*m^2 - 5*a^3*b*d^4*f*g*m^2 - a^3*b*d^4*e*h*m^3
 - a^3*b*d^4*f*g*m^3 + 27*b^4*c^2*d^2*e*g*m + 10*b^4*c^3*d*e*h*m^2 + 10*b^4
*c^3*d*f*g*m^2 + b^4*c^3*d*e*h*m^3 + b^4*c^3*d*f*g*m^3 + 3*a^2*b^2*c^2*d^2*
f*h*m^2 - 6*a*b^3*c*d^3*e*g*m^2 - 66*a*b^3*c^2*d^2*e*h*m - 66*a*b^3*c^2*d^2
*f*g*m + 41*a^2*b^2*c*d^3*e*h*m + 41*a^2*b^2*c*d^3*f*g*m - 16*a*b^3*c^3*d*f
*h*m^2 + 14*a^3*b*c*d^3*f*h*m^2 - 2*a*b^3*c^3*d*f*h*m^3 + 2*a^3*b*c*d^3*f*h
*m^3 - 25*a*b^3*c^2*d^2*e*h*m^2 - 25*a*b^3*c^2*d^2*f*g*m^2 + 20*a^2*b^2*c*d
^3*e*h*m^2 + 20*a^2*b^2*c*d^3*f*g*m^2 - 3*a*b^3*c^2*d^2*e*h*m^3 - 3*a*b^3*c
^2*d^2*f*g*m^3 + 3*a^2*b^2*c*d^3*e*h*m^3 + 3*a^2*b^2*c*d^3*f*g*m^3 + 15*a^2
*b^2*c^2*d^2*f*h*m - 30*a*b^3*c*d^3*e*g*m - 46*a*b^3*c^3*d*f*h*m + 36*a^3*b
*c*d^3*f*h*m))/((a*d - b*c)^4*(c + d*x)^(m + 5)*(50*m + 35*m^2 + 10*m^3 + m
^4 + 24)) - (x^2*(a + b*x)^m*(8*a^4*d^4*e*h + 8*a^4*d^4*f*g - 12*b^4*c^4*e*
h - 12*b^4*c^4*f*g - 60*b^4*c^3*d*e*g + 20*a^4*c*d^3*f*h + 14*a^4*d^4*e*h*m
 + 14*a^4*d^4*f*g*m - 19*b^4*c^4*e*h*m - 19*b^4*c^4*f*g*m + 7*a^4*d^4*e*h*m
^2 + 7*a^4*d^4*f*g*m^2 - 8*b^4*c^4*e*h*m^2 - 8*b^4*c^4*f*g*m^2 + a^4*d^4*e*
h*m^3 + a^4*d^4*f*g*m^3 - b^4*c^4*e*h*m^3 - b^4*c^4*f*g*m^3 + 48*a^2*b^2*c^
2*d^2*e*h + 48*a^2*b^2*c^2*d^2*f*g + 48*a*b^3*c^3*d*e*h + 48*a*b^3*c^3*d*f*
g - 32*a^3*b*c*d^3*e*h - 32*a^3*b*c*d^3*f*g + 2*a^3*b*d^4*e*g*m - 4*a*b^3*c
^4*f*h*m - 47*b^4*c^3*d*e*g*m + 29*a^4*c*d^3*f*h*m - 80*a^3*b*c^2*d^2*f*h +
 3*a^3*b*d^4*e*g*m^2 + a^3*b*d^4*e*g*m^3 - 5*a*b^3*c^4*f*h*m^2 - a*b^3*c^4*
f*h*m^3 - 12*b^4*c^3*d*e*g*m^2 - b^4*c^3*d*e*g*m^3 + 10*a^4*c*d^3*f*h*m^2 +
 a^4*c*d^3*f*h*m^3 + 3*a^2*b^2*c^2*d^2*e*h*m^2 + 3*a^2*b^2*c^2*d^2*f*g*m^2
+ 60*a*b^3*c^2*d^2*e*g*m - 15*a^2*b^2*c*d^3*e*g*m + 14*a*b^3*c^3*d*e*h*m^2
+ 14*a*b^3*c^3*d*f*g*m^2 - 16*a^3*b*c*d^3*e*h*m^2 - 16*a^3*b*c*d^3*f*g*m^2
+ 2*a*b^3*c^3*d*e*h*m^3 + 2*a*b^3*c^3*d*f*g*m^3 - 2*a^3*b*c*d^3*e*h*m^3 - 2
*a^3*b*c*d^3*f*g*m^3 + 41*a^2*b^2*c^3*d*f*h*m - 66*a^3*b*c^2*d^2*f*h*m + 27
*a*b^3*c^2*d^2*e*g*m^2 - 18*a^2*b^2*c*d^3*e*g*m^2 + 3*a*b^3*c^2*d^2*e*g*m^3
 - 3*a^2*b^2*c*d^3*e*g*m^3 + 15*a^2*b^2*c^2*d^2*e*h*m + 15*a^2*b^2*c^2*d^2*
f*g*m + 20*a^2*b^2*c^3*d*f*h*m^2 - 25*a^3*b*c^2*d^2*f*h*m^2 + 3*a^2*b^2*c^3
*d*f*h*m^3 - 3*a^3*b*c^2*d^2*f*h*m^3 + 36*a*b^3*c^3*d*e*h*m + 36*a*b^3*c^3*
d*f*g*m - 46*a^3*b*c*d^3*e*h*m - 46*a^3*b*c*d^3*f*g*m))/((a*d - b*c)^4*(c +
 d*x)^(m + 5)*(50*m + 35*m^2 + 10*m^3 + m^4 + 24)) + (b*d*x^4*(a + b*x)^m*(
5*b*c - a*d*m + b*c*m)*(6*b^2*d^2*e*g + 12*a^2*d^2*f*h + 2*b^2*c^2*f*h + 7*
a^2*d^2*f*h*m + 3*b^2*c^2*f*h*m + a^2*d^2*f*h*m^2 + b^2*c^2*f*h*m^2 - 8*a*b
*d^2*e*h - 8*a*b*d^2*f*g + 2*b^2*c*d*e*h + 2*b^2*c*d*f*g - 2*a*b*d^2*e*h*m
- 2*a*b*d^2*f*g*m + 2*b^2*c*d*e*h*m + 2*b^2*c*d*f*g*m - 8*a*b*c*d*f*h - 10*
a*b*c*d*f*h*m - 2*a*b*c*d*f*h*m^2))/((a*d - b*c)^4*(c + d*x)^(m + 5)*(50*m
+ 35*m^2 + 10*m^3 + m^4 + 24))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**(-5-m)*(f*x+e)*(h*x+g),x)
```

```
[Out] Timed out
```

3.132 $\int (a+bx)^3(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx$

Optimal. Leaf size=815

$$\frac{h(a+bx)^3(e+fx)^{m+1}(c+dx)^{-m-3}}{df}+\frac{(bc-ad)^2(adf+b(cf(m+2)-de(m+3)))(cfh(m+4)-d(fg+eh(m+}{d^4f^2(de-cf)(m+3)}$$

```
[Out] (-a*d+b*c)^2*(a*d*f+b*(c*f*(2+m)-d*e*(3+m)))*(c*f*h*(4+m)-d*(f*g+e*h*(3+m))
)*(d*x+c)^(-3-m)*(f*x+e)^(1+m)/d^4/f^2/(-c*f+d*e)/(3+m)-b*(-a*d+b*c)*(c*f*h
*(4+m)-d*(f*g+e*h*(3+m)))*(b*x+a)*(d*x+c)^(-3-m)*(f*x+e)^(1+m)/d^3/f^2+h*(b
*x+a)^3*(d*x+c)^(-3-m)*(f*x+e)^(1+m)/d/f-(-a*d+b*c)^2*(3*a*d*f*h-b*(c*f*h*(
4+m)-d*(e*h*m+f*g)))*(d*x+c)^(-2-m)*(f*x+e)^(1+m)/d^4/f/(-c*f+d*e)/(2+m)+(-
a*d+b*c)*(c*f*h*(4+m)-d*(f*g+e*h*(3+m)))*(2*a^2*d^2*f^2+2*a*b*d*f*(c*f*(1+m
)-d*e*(3+m))+b^2*(c^2*f^2*(m^2+3*m+2)-2*c*d*e*f*(m^2+4*m+3)+d^2*e^2*(m^2+5*
m+6)))*(d*x+c)^(-2-m)*(f*x+e)^(1+m)/d^4/f^2/(-c*f+d*e)^2/(2+m)/(3+m)-(-a*d+
b*c)*(a*d*f-b*(2*d*e*(2+m)-c*f*(3+2*m)))*(3*a*d*f*h-b*(c*f*h*(4+m)-d*(e*h*m
+f*g)))*(d*x+c)^(-1-m)*(f*x+e)^(1+m)/d^4/f/(-c*f+d*e)^2/(1+m)/(2+m)-(-a*d+b
*c)*(c*f*h*(4+m)-d*(f*g+e*h*(3+m)))*(2*a^2*d^2*f^2+2*a*b*d*f*(c*f*(1+m)-d*e
*(3+m))+b^2*(c^2*f^2*(m^2+3*m+2)-2*c*d*e*f*(m^2+4*m+3)+d^2*e^2*(m^2+5*m+6))
)*(d*x+c)^(-1-m)*(f*x+e)^(1+m)/d^4/f/(-c*f+d*e)^3/(1+m)/(2+m)/(3+m)-b^2*(3*
a*d*f*h-b*(c*f*h*(4+m)-d*(e*h*m+f*g)))*(f*x+e)^m*hypergeom([-m, -m],[1-m],-
f*(d*x+c)/(-c*f+d*e))/d^5/f/m/((d*x+c)^m)/((d*(f*x+e)/(-c*f+d*e))^m)
```

Rubi [A] time = 1.43, antiderivative size = 803, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {153, 159, 89, 79, 70, 69, 90, 45, 37}

$$\frac{h(a+bx)^3(e+fx)^{m+1}(c+dx)^{-m-3}}{df}+\frac{(bc-ad)^2(adf+bc(m+2)f-bde(m+3))(cfh(m+4)-d(fg+eh(m+}{d^4f^2(de-cf)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(c + d*x)^(-4 - m)*(e + f*x)^m*(g + h*x),x]
```

```
[Out] ((b*c - a*d)^2*(a*d*f + b*c*f*(2 + m) - b*d*e*(3 + m))*(c*f*h*(4 + m) - d*(
f*g + e*h*(3 + m)))*(c + d*x)^(-3 - m)*(e + f*x)^(1 + m))/(d^4*f^2*(d*e - c
*f)*(3 + m)) - (b*(b*c - a*d)*(c*f*h*(4 + m) - d*(f*g + e*h*(3 + m)))*(a +
b*x)*(c + d*x)^(-3 - m)*(e + f*x)^(1 + m))/(d^3*f^2) + (h*(a + b*x)^3*(c +
d*x)^(-3 - m)*(e + f*x)^(1 + m))/(d*f) - ((b*c - a*d)^2*(b*d*f*g + 3*a*d*f*
h + b*d*e*h*m - b*c*f*h*(4 + m))*(c + d*x)^(-2 - m)*(e + f*x)^(1 + m))/(d^4
*f*(d*e - c*f)*(2 + m)) - ((b*c - a*d)*(d*f*g + d*e*h*(3 + m) - c*f*h*(4 +
m))*(2*a^2*d^2*f^2 + 2*a*b*d*f*(c*f*(1 + m) - d*e*(3 + m)) + b^2*(c^2*f^2*(
2 + 3*m + m^2) - 2*c*d*e*f*(3 + 4*m + m^2) + d^2*e^2*(6 + 5*m + m^2)))*(c +
 d*x)^(-2 - m)*(e + f*x)^(1 + m))/(d^4*f^2*(d*e - c*f)^2*(2 + m)*(3 + m)) -
 ((b*c - a*d)*(b*d*f*g + 3*a*d*f*h + b*d*e*h*m - b*c*f*h*(4 + m))*(a*d*f -
2*b*d*e*(2 + m) + b*c*f*(3 + 2*m))*(c + d*x)^(-1 - m)*(e + f*x)^(1 + m))/(d
^4*f*(d*e - c*f)^2*(1 + m)*(2 + m)) + ((b*c - a*d)*(d*f*g + d*e*h*(3 + m) -
 c*f*h*(4 + m))*(2*a^2*d^2*f^2 + 2*a*b*d*f*(c*f*(1 + m) - d*e*(3 + m)) + b^
2*(c^2*f^2*(2 + 3*m + m^2) - 2*c*d*e*f*(3 + 4*m + m^2) + d^2*e^2*(6 + 5*m +
 m^2)))*(c + d*x)^(-1 - m)*(e + f*x)^(1 + m))/(d^4*f*(d*e - c*f)^3*(1 + m)*
(2 + m)*(3 + m)) - (b^2*(b*d*f*g + 3*a*d*f*h + b*d*e*h*m - b*c*f*h*(4 + m))
*(e + f*x)^m*Hypergeometric2F1[-m, -m, 1 - m, -((f*(c + d*x))/(d*e - c*f))]
)/(d^5*f*m*(c + d*x)^m*((d*(e + f*x))/(d*e - c*f))^m)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
```

```
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m + 1)*(
c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[(a + b*x)^m*(c + d
*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] &&
 (SumSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))
```

Rubi steps

$$\int (a+bx)^3(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx = \frac{h(a+bx)^3(c+dx)^{-3-m}(e+fx)^{1+m}}{df} + \frac{\int (a+bx)^2(c+dx)^{-4-m}(\ldots}{\ldots}$$

$$= \frac{h(a+bx)^3(c+dx)^{-3-m}(e+fx)^{1+m}}{df} - \frac{((bc-ad)(dfg+deh(3\ldots}{\ldots}$$

$$= \frac{b(bc-ad)(dfg+deh(3+m)-cfh(4+m))(a+bx)(c+dx)^{-3\ldots}}{d^3f^2}$$

$$= -\frac{(bc-ad)^2(adf+bcf(2+m)-bde(3+m))(dfg+deh(3+m\ldots}{d^4f^2(de-cf)(3+m\ldots}$$

$$= -\frac{(bc-ad)^2(adf+bcf(2+m)-bde(3+m))(dfg+deh(3+m\ldots}{d^4f^2(de-cf)(3+m\ldots}$$

$$= -\frac{(bc-ad)^2(adf+bcf(2+m)-bde(3+m))(dfg+deh(3+m\ldots}{d^4f^2(de-cf)(3+m\ldots}$$

Mathematica [C] time = 44.24, size = 3579, normalized size = 4.39

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x)^3*(c + d*x)^(-4 - m)*(e + f*x)^m*(g + h*x),x]
```

```
[Out] (3*a*b^2*c^3*g*(c + d*x)^(-7 - m)*((c + d*x)/c)^(4 + m)*(e + f*x)^(3 + m)*(
-2*c^3*e^3 - 6*c^2*d*e^3*x - 2*c^2*d*e^3*m*x + 2*c^3*e^2*f*m*x - 6*c*d^2*e^
3*x^2 - 5*c*d^2*e^3*m*x^2 + 6*c^2*d*e^2*f*m*x^2 - c^3*e*f^2*m*x^2 - c*d^2*e
^3*m^2*x^2 + 2*c^2*d*e^2*f*m^2*x^2 - c^3*e*f^2*m^2*x^2 - 6*c*d^2*e^2*f*x^3
+ 6*c^2*d*e*f^2*x^3 - 2*c^3*f^3*x^3 - 5*c*d^2*e^2*f*m*x^3 + 8*c^2*d*e*f^2*m
*x^3 - 3*c^3*f^3*m*x^3 - c*d^2*e^2*f*m^2*x^3 + 2*c^2*d*e*f^2*m^2*x^3 - c^3*
f^3*m^2*x^3 + 2*c^3*e^3*((c*e + d*e*x)/(c*(e + f*x)))^m + 6*c^2*d*e^3*x*((c
*e + d*e*x)/(c*(e + f*x)))^m + 6*c*d^2*e^3*x^2*((c*e + d*e*x)/(c*(e + f*x))
)^m + 2*d^3*e^3*x^3*((c*e + d*e*x)/(c*(e + f*x)))^m))/(e^3*(d*e - c*f)^3*(1
 + m)*(2 + m)*(3 + m)*((c*e + d*e*x)/(c*(e + f*x)))^m*((e + f*x)/e)^m*(1 +
(f*x)/e)^3) + (3*a^2*b*c^3*h*(c + d*x)^(-7 - m)*((c + d*x)/c)^(4 + m)*(e +
```

```
f*x)^(3 + m)*(-2*c^3*e^3 - 6*c^2*d*e^3*x - 2*c^2*d*e^3*m*x + 2*c^3*e^2*f*m*
x - 6*c*d^2*e^3*x^2 - 5*c*d^2*e^3*m*x^2 + 6*c^2*d*e^2*f*m*x^2 - c^3*e*f^2*m
*x^2 - c*d^2*e^3*m^2*x^2 + 2*c^2*d*e^2*f*m^2*x^2 - c^3*e*f^2*m^2*x^2 - 6*c*
d^2*e^2*f*x^3 + 6*c^2*d*e*f^2*x^3 - 2*c^3*f^3*x^3 - 5*c*d^2*e^2*f*m*x^3 + 8
*c^2*d*e*f^2*m*x^3 - 3*c^3*f^3*m*x^3 - c*d^2*e^2*f*m^2*x^3 + 2*c^2*d*e*f^2*
m^2*x^3 - c^3*f^3*m^2*x^3 + 2*c^3*e^3*((c*e + d*e*x)/(c*(e + f*x)))^m + 6*c
^2*d*e^3*x*((c*e + d*e*x)/(c*(e + f*x)))^m + 6*c*d^2*e^3*x^2*((c*e + d*e*x)
/(c*(e + f*x)))^m + 2*d^3*e^3*x^3*((c*e + d*e*x)/(c*(e + f*x)))^m))/(e^3*(d
*e - c*f)^3*(1 + m)*(2 + m)*(3 + m)*((c*e + d*e*x)/(c*(e + f*x)))^m*((e + f
*x)/e)^m*(1 + (f*x)/e)^3) + (b^3*g*x^4*(c + d*x)^(-4 - m)*((c + d*x)/c)^(4
+ m)*(e + f*x)^m*AppellF1[4, 4 + m, -m, 5, -((d*x)/c), -((f*x)/e)])/(4*((e
+ f*x)/e)^m) + (3*a*b^2*h*x^4*(c + d*x)^(-4 - m)*((c + d*x)/c)^(4 + m)*(e +
 f*x)^m*AppellF1[4, 4 + m, -m, 5, -((d*x)/c), -((f*x)/e)])/(4*((e + f*x)/e)
^m) + (b^3*h*x^5*(c + d*x)^(-4 - m)*((c + d*x)/c)^(4 + m)*(e + f*x)^m*Appel
lF1[5, 4 + m, -m, 6, -((d*x)/c), -((f*x)/e)])/(5*((e + f*x)/e)^m) + (3*a^2*
b*g*(c + d*x)^(-4 - m)*((c + d*x)/c)^(4 + m)*(1 + (d*x)/c)^(-4 - m)*(e + f*
x)^m*((c*(e + f*x))/(e*(c + d*x)))^(-1 - m)*(1 + (f*x)/e)^(1 + m)*(c*(4 + m
)*(3*e + f*x)*(-2*d^3*e^3*x^3 + c^3*(-2*e^2*f*m*x*((c*(e + f*x))/(e*(c + d*
x)))^m + e*f^2*m*(1 + m)*x^2*((c*(e + f*x))/(e*(c + d*x)))^m + f^3*(2 + 3*m
 + m^2)*x^3*((c*(e + f*x))/(e*(c + d*x)))^m + 2*e^3*(-1 + ((c*(e + f*x))/(e
*(c + d*x)))^m)) - 2*c^2*d*e*x*(e*f*m*(3 + m)*x*((c*(e + f*x))/(e*(c + d*x)
))^m + f^2*(3 + 4*m + m^2)*x^2*((c*(e + f*x))/(e*(c + d*x)))^m - e^2*(-3 +
3*((c*(e + f*x))/(e*(c + d*x)))^m + m*((c*(e + f*x))/(e*(c + d*x)))^m)) + c
*d^2*e^2*x^2*(f*(6 + 5*m + m^2)*x*((c*(e + f*x))/(e*(c + d*x)))^m + e*(-6 +
 6*((c*(e + f*x))/(e*(c + d*x)))^m + 5*m*((c*(e + f*x))/(e*(c + d*x)))^m +
m^2*((c*(e + f*x))/(e*(c + d*x)))^m)))*Gamma[4 + m] - (2*d^4*e^4*(1 + m)*x^
4 - 2*c*d^3*e^3*x^3*(-3*e*m + f*(4 + m)*x) + c^4*(e^2*f^2*(-5 + m)*m*x^2*((
c*(e + f*x))/(e*(c + d*x)))^m + 2*e*f^3*m*(1 + m)*x^3*((c*(e + f*x))/(e*(c
+ d*x)))^m + f^4*(2 + 3*m + m^2)*x^4*((c*(e + f*x))/(e*(c + d*x)))^m + 6*e^
4*(-1 + ((c*(e + f*x))/(e*(c + d*x)))^m) - 2*e^3*f*x*(4 + m - 4*((c*(e + f*
x))/(e*(c + d*x)))^m + 2*m*((c*(e + f*x))/(e*(c + d*x)))^m)) - 2*c^3*d*e*x*
(2*e*f^2*m*(4 + m)*x^2*((c*(e + f*x))/(e*(c + d*x)))^m + f^3*(4 + 5*m + m^2
)*x^3*((c*(e + f*x))/(e*(c + d*x)))^m + e^2*f*(4 + m)*x*(3 - 3*((c*(e + f*x
))/(e*(c + d*x)))^m + m*((c*(e + f*x))/(e*(c + d*x)))^m) - e^3*(-8 + m + 8*
((c*(e + f*x))/(e*(c + d*x)))^m + 2*m*((c*(e + f*x))/(e*(c + d*x)))^m)) + c
^2*d^2*e^2*x^2*(f^2*(12 + 7*m + m^2)*x^2*((c*(e + f*x))/(e*(c + d*x)))^m +
2*e*f*(4 + m)*x*(-3 + 3*((c*(e + f*x))/(e*(c + d*x)))^m + m*((c*(e + f*x))/
(e*(c + d*x)))^m) + e^2*(m^2*((c*(e + f*x))/(e*(c + d*x)))^m + 12*(-1 + ((c
*(e + f*x))/(e*(c + d*x)))^m) + m*(6 + 7*((c*(e + f*x))/(e*(c + d*x)))^m)))
)*Gamma[5 + m]))/(2*c*e*(-(d*e) + c*f)^3*(1 + m)*(2 + m)*(3 + m)*(4 + m)*x*
((e + f*x)/e)^m*Gamma[4 + m]) + (a^3*h*(c + d*x)^(-4 - m)*((c + d*x)/c)^(4
+ m)*(1 + (d*x)/c)^(-4 - m)*(e + f*x)^m*((c*(e + f*x))/(e*(c + d*x)))^(-1 -
 m)*(1 + (f*x)/e)^(1 + m)*(c*(4 + m)*(3*e + f*x)*(-2*d^3*e^3*x^3 + c^3*(-2*
e^2*f*m*x*((c*(e + f*x))/(e*(c + d*x)))^m + e*f^2*m*(1 + m)*x^2*((c*(e + f*
x))/(e*(c + d*x)))^m + f^3*(2 + 3*m + m^2)*x^3*((c*(e + f*x))/(e*(c + d*x))
)^m + 2*e^3*(-1 + ((c*(e + f*x))/(e*(c + d*x)))^m)) - 2*c^2*d*e*x*(e*f*m*(3
 + m)*x*((c*(e + f*x))/(e*(c + d*x)))^m + f^2*(3 + 4*m + m^2)*x^2*((c*(e +
f*x))/(e*(c + d*x)))^m - e^2*(-3 + 3*((c*(e + f*x))/(e*(c + d*x)))^m + m*((
c*(e + f*x))/(e*(c + d*x)))^m)) + c*d^2*e^2*x^2*(f*(6 + 5*m + m^2)*x*((c*(e
 + f*x))/(e*(c + d*x)))^m + e*(-6 + 6*((c*(e + f*x))/(e*(c + d*x)))^m + 5*m
*((c*(e + f*x))/(e*(c + d*x)))^m + m^2*((c*(e + f*x))/(e*(c + d*x)))^m)))*G
amma[4 + m] - (2*d^4*e^4*(1 + m)*x^4 - 2*c*d^3*e^3*x^3*(-3*e*m + f*(4 + m)*
x) + c^4*(e^2*f^2*(-5 + m)*m*x^2*((c*(e + f*x))/(e*(c + d*x)))^m + 2*e*f^3*
m*(1 + m)*x^3*((c*(e + f*x))/(e*(c + d*x)))^m + f^4*(2 + 3*m + m^2)*x^4*((c
*(e + f*x))/(e*(c + d*x)))^m + 6*e^4*(-1 + ((c*(e + f*x))/(e*(c + d*x)))^m)
 - 2*e^3*f*x*(4 + m - 4*((c*(e + f*x))/(e*(c + d*x)))^m + 2*m*((c*(e + f*x)
)/(e*(c + d*x)))^m)) - 2*c^3*d*e*x*(2*e*f^2*m*(4 + m)*x^2*((c*(e + f*x))/(e
*(c + d*x)))^m + f^3*(4 + 5*m + m^2)*x^3*((c*(e + f*x))/(e*(c + d*x)))^m +
e^2*f*(4 + m)*x*(3 - 3*((c*(e + f*x))/(e*(c + d*x)))^m + m*((c*(e + f*x))/(
```

```
e*(c + d*x)))^m) - e^3*(-8 + m + 8*((c*(e + f*x))/(e*(c + d*x)))^m + 2*m*((
c*(e + f*x))/(e*(c + d*x)))^m)) + c^2*d^2*e^2*x^2*(f^2*(12 + 7*m + m^2)*x^2
*((c*(e + f*x))/(e*(c + d*x)))^m + 2*e*f*(4 + m)*x*(-3 + 3*((c*(e + f*x))/(
e*(c + d*x)))^m + m*((c*(e + f*x))/(e*(c + d*x)))^m) + e^2*(m^2*((c*(e + f*
x))/(e*(c + d*x)))^m + 12*(-1 + ((c*(e + f*x))/(e*(c + d*x)))^m) + m*(6 + 7
*((c*(e + f*x))/(e*(c + d*x)))^m))))*Gamma[5 + m]))/(2*c*e*(-(d*e) + c*f)^3
*(1 + m)*(2 + m)*(3 + m)*(4 + m)*x*((e + f*x)/e)^m*Gamma[4 + m]) + (a^3*f^3
*g*(e + f*x)^(1 + m)*(1 + (d*(e + f*x))/((c - (d*e)/f)*f))^m*Hypergeometric
2F1[1 + m, 4 + m, 2 + m, -((d*(e + f*x))/((c - (d*e)/f)*f))])/((-(d*e) + c*
f)^4*(1 + m)*(c - (d*e)/f + (d*(e + f*x))/f)^m)
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^3hx^4 + a^3g + \left(b^3g + 3\,ab^2h\right)x^3 + 3\left(ab^2g + a^2bh\right)x^2 + \left(3\,a^2bg + a^3h\right)x\right)(dx + c)^{-m-4}\left(fx + e\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="fricas")
```

```
[Out] integral((b^3*h*x^4 + a^3*g + (b^3*g + 3*a*b^2*h)*x^3 + 3*(a*b^2*g + a^2*b*
h)*x^2 + (3*a^2*b*g + a^3*h)*x)*(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx + a)^3\left(hx + g\right)(dx + c)^{-m-4}\left(fx + e\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^3*(h*x + g)*(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int (bx + a)^3\left(hx + g\right)(dx + c)^{-m-4}\left(fx + e\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(d*x+c)^(-m-4)*(f*x+e)^m*(h*x+g),x)
```

```
[Out] int((b*x+a)^3*(d*x+c)^(-m-4)*(f*x+e)^m*(h*x+g),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx + a)^3\left(hx + g\right)(dx + c)^{-m-4}\left(fx + e\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^3*(h*x + g)*(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e+f\,x\right)^m\,\left(g+h\,x\right)\,\left(a+b\,x\right)^3}{\left(c+d\,x\right)^{m+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)^m*(g + h*x)*(a + b*x)^3)/(c + d*x)^(m + 4),x)
```

```
[Out] int(((e + f*x)^m*(g + h*x)*(a + b*x)^3)/(c + d*x)^(m + 4), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(d*x+c)**(-4-m)*(f*x+e)**m*(h*x+g),x)
```

```
[Out] Timed out
```

3.133 $\int (a+bx)^2(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx$

Optimal. Leaf size=572

$$-\frac{(dg-ch)(c+dx)^{-m-2}(e+fx)^{m+1}\left(b^2(m+2)(de-cf)(cf(m+1)-de(m+3))-2df\left(a^2df+ab(cf(m+1)-\right.\right.}{d^3f(m+2)(m+3)(de-cf)^2}$$

```
[Out] (-a*d+b*c)*(-c*h+d*g)*(a*d*f+b*(c*f*(2+m)-d*e*(3+m)))*(d*x+c)^(-3-m)*(f*x+e
)^(1+m)/d^3/f/(-c*f+d*e)/(3+m)-b*(-c*h+d*g)*(b*x+a)*(d*x+c)^(-3-m)*(f*x+e)^
(1+m)/d^2/f-(-a*d+b*c)^2*h*(d*x+c)^(-2-m)*(f*x+e)^(1+m)/d^3/(-c*f+d*e)/(2+m
)-(-c*h+d*g)*(b^2*(-c*f+d*e)*(2+m)*(c*f*(1+m)-d*e*(3+m))-2*d*f*(b^2*c*e+a^2
*d*f+a*b*(c*f*(1+m)-d*e*(3+m))))*(d*x+c)^(-2-m)*(f*x+e)^(1+m)/d^3/f/(-c*f+d
*e)^2/(2+m)/(3+m)-(-a*d+b*c)*h*(a*d*f-b*(2*d*e*(2+m)-c*f*(3+2*m)))*(d*x+c)^
(-1-m)*(f*x+e)^(1+m)/d^3/(-c*f+d*e)^2/(1+m)/(2+m)+(-c*h+d*g)*(b^2*(-c*f+d*e
)*(2+m)*(c*f*(1+m)-d*e*(3+m))-2*d*f*(b^2*c*e+a^2*d*f+a*b*(c*f*(1+m)-d*e*(3+
m))))*(d*x+c)^(-1-m)*(f*x+e)^(1+m)/d^3/(-c*f+d*e)^3/(1+m)/(2+m)/(3+m)-b^2*h
*(f*x+e)^m*hypergeom([-m, -m],[1-m],-f*(d*x+c)/(-c*f+d*e))/d^4/m/((d*x+c)^m
)/((d*(f*x+e)/(-c*f+d*e))^m)
```

Rubi [A] time = 0.66, antiderivative size = 566, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {159, 89, 79, 70, 69, 90, 45, 37}

$$-\frac{(dg-ch)(c+dx)^{-m-2}(e+fx)^{m+1}\left(b^2(m+2)(de-cf)(cf(m+1)-de(m+3))-2df\left(a^2df+b(acf(m+1)-\right.\right.}{d^3f(m+2)(m+3)(de-cf)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(c + d*x)^(-4 - m)*(e + f*x)^m*(g + h*x),x]
```

```
[Out] ((b*c - a*d)*(d*g - c*h)*(a*d*f + b*c*f*(2 + m) - b*d*e*(3 + m))*(c + d*x)^
(-3 - m)*(e + f*x)^(1 + m))/(d^3*f*(d*e - c*f)*(3 + m)) - (b*(d*g - c*h)*(a
 + b*x)*(c + d*x)^(-3 - m)*(e + f*x)^(1 + m))/(d^2*f) - ((b*c - a*d)^2*h*(c
 + d*x)^(-2 - m)*(e + f*x)^(1 + m))/(d^3*(d*e - c*f)*(2 + m)) - ((d*g - c*h
)*(b^2*(d*e - c*f)*(2 + m)*(c*f*(1 + m) - d*e*(3 + m)) - 2*d*f*(a^2*d*f + b
*(b*c*e + a*c*f*(1 + m) - a*d*e*(3 + m))))*(c + d*x)^(-2 - m)*(e + f*x)^(1
+ m))/(d^3*f*(d*e - c*f)^2*(2 + m)*(3 + m)) - ((b*c - a*d)*h*(a*d*f - 2*b*d
*e*(2 + m) + b*c*f*(3 + 2*m))*(c + d*x)^(-1 - m)*(e + f*x)^(1 + m))/(d^3*(d
*e - c*f)^2*(1 + m)*(2 + m)) + ((d*g - c*h)*(b^2*(d*e - c*f)*(2 + m)*(c*f*(
1 + m) - d*e*(3 + m)) - 2*d*f*(a^2*d*f + b*(b*c*e + a*c*f*(1 + m) - a*d*e*(
3 + m))))*(c + d*x)^(-1 - m)*(e + f*x)^(1 + m))/(d^3*(d*e - c*f)^3*(1 + m)*
(2 + m)*(3 + m)) - (b^2*h*(e + f*x)^m*Hypergeometric2F1[-m, -m, 1 - m, -((f
*(c + d*x))/(d*e - c*f))])/(d^4*m*(c + d*x)^m*((d*(e + f*x))/(d*e - c*f))^m
)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
```

```
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m + 1)*(
c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[(a + b*x)^m*(c + d
*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] &&
 (SumSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))
```

Rubi steps

$$\int (a+bx)^2(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx = \frac{h\int (a+bx)^2(c+dx)^{-3-m}(e+fx)^m\,dx}{d} + \frac{(dg-ch)\int (a+bx)^2(\ldots}{\ldots}$$

$$= -\frac{b(dg-ch)(a+bx)(c+dx)^{-3-m}(e+fx)^{1+m}}{d^2f} - \frac{(bc-ad)^2h(c+\ldots}{d^3(de-\ldots}$$

$$= \frac{(bc-ad)(dg-ch)(adf+bcf(2+m)-bde(3+m))(c+dx)^{-3-\ldots}}{d^3f(de-cf)(3+m)}$$

$$= \frac{(bc-ad)(dg-ch)(adf+bcf(2+m)-bde(3+m))(c+dx)^{-3-\ldots}}{d^3f(de-cf)(3+m)}$$

$$= \frac{(bc-ad)(dg-ch)(adf+bcf(2+m)-bde(3+m))(c+dx)^{-3-\ldots}}{d^3f(de-cf)(3+m)}$$

Mathematica [A] time = 1.91, size = 422, normalized size = 0.74

$$\frac{(c+dx)^{-m-3}(e+fx)^m\left(-h(m+3)(c+dx)(de-cf)\left(df(m+1)(e+fx)(bc-ad)^2(de-cf)-(c+dx)\left(d(e+f\ldots\right.\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(c + d*x)^(-4 - m)*(e + f*x)^m*(g + h*x),x]
```

```
[Out] ((c + d*x)^(-3 - m)*(e + f*x)^m*(-(d*(d*g - c*h)*(e + f*x)*(-((b*c - a*d)*(
d*e - c*f)^2*(1 + m)*(2 + m)*(a*d*f + b*c*f*(2 + m) - b*d*e*(3 + m))) + b*d
*(d*e - c*f)^3*(1 + m)*(2 + m)*(3 + m)*(a + b*x) + (b^2*(d*e - c*f)*(2 + m)
*(c*f*(1 + m) - d*e*(3 + m)) + 2*d*f*(-(a^2*d*f) - b*(b*c*e + a*c*f*(1 + m)
 - a*d*e*(3 + m))))*(c + d*x)*(-(c*f*(2 + m)) + d*(e + e*m - f*x)))) - (d*e
 - c*f)*h*(3 + m)*(c + d*x)*(d*(b*c - a*d)^2*f*(d*e - c*f)*(1 + m)*(e + f*x
) - (c + d*x)*(d*(a^2*d^2*f^2 + 2*a*b*d*f*(c*f*(1 + m) - d*e*(2 + m)) + b^2
*(-(c^2*f^2*(1 + m)) + d^2*e^2*(2 + m)))*(e + f*x) - (b^2*(d*e - c*f)^3*(2
+ m)*Hypergeometric2F1[-1 - m, -1 - m, -m, (f*(c + d*x))/(-(d*e) + c*f)])/(
(d*(e + f*x))/(d*e - c*f))^m))))/(d^4*f*(d*e - c*f)^3*(1 + m)*(2 + m)*(3 +
m))
```

fricas [F] time = 1.05, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(b^2hx^3 + a^2g + \left(b^2g + 2\,abh\right)x^2 + \left(2\,abg + a^2h\right)x\right)(dx+c)^{-m-4}\left(fx+e\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="fricas")
```

```
[Out] integral((b^2*h*x^3 + a^2*g + (b^2*g + 2*a*b*h)*x^2 + (2*a*b*g + a^2*h)*x)*
(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^2\left(hx+g\right)(dx+c)^{-m-4}\left(fx+e\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^2*(h*x + g)*(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int (bx+a)^2 \left(hx+g\right) (dx+c)^{-m-4} \left(fx+e\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(d*x+c)^(-m-4)*(f*x+e)^m*(h*x+g),x)
[Out] int((b*x+a)^2*(d*x+c)^(-m-4)*(f*x+e)^m*(h*x+g),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^2 \left(hx+g\right)(dx+c)^{-m-4}\left(fx+e\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="maxima")
[Out] integrate((b*x + a)^2*(h*x + g)*(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e+f\,x\right)^m\,\left(g+h\,x\right)\,\left(a+b\,x\right)^2}{\left(c+d\,x\right)^{m+4}} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)^m*(g + h*x)*(a + b*x)^2)/(c + d*x)^(m + 4),x)
[Out] int(((e + f*x)^m*(g + h*x)*(a + b*x)^2)/(c + d*x)^(m + 4), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(d*x+c)**(-4-m)*(f*x+e)**m*(h*x+g),x)
[Out] Timed out
```

3.134 $\int (a+bx)(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx$

Optimal. Leaf size=363

$$\frac{(c+dx)^{-m-2}(e+fx)^{m+1}\left(adf(cfh(m+1)+d(2fg-eh(m+3)))+b\left(c^2f^2h\left(m^2+3m+2\right)+cdf(m+1)(fg-\right.\right.}{d^2f(m+2)(m+3)(de-cf)^2}$$

```
[Out] (b*(c^2*f^2*h*(m^2+3*m+2)-d^2*e*(3+m)*(f*g-e*h*(2+m))+c*d*f*(1+m)*(f*g-2*e*
h*(3+m)))+a*d*f*(c*f*h*(1+m)+d*(2*f*g-e*h*(3+m))))*(d*x+c)^(-2-m)*(f*x+e)^(
1+m)/d^2/f/(-c*f+d*e)^2/(2+m)/(3+m)-(b*(c^2*f^2*h*(m^2+3*m+2)-d^2*e*(3+m)*(
f*g-e*h*(2+m))+c*d*f*(1+m)*(f*g-2*e*h*(3+m)))+a*d*f*(c*f*h*(1+m)+d*(2*f*g-e
*h*(3+m))))*(d*x+c)^(-1-m)*(f*x+e)^(1+m)/d^2/(-c*f+d*e)^3/(1+m)/(2+m)/(3+m)
-(d*x+c)^(-3-m)*(f*x+e)^(1+m)*(a*d*f*(-c*h+d*g)-b*c*(c*f*h*(2+m)+d*(f*g-e*h
*(3+m)))+b*d*(-c*f+d*e)*h*(3+m)*x)/d^2/f/(-c*f+d*e)/(3+m)
```

Rubi [A] time = 0.40, antiderivative size = 360, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {146, 45, 37}

$$\frac{(c+dx)^{-m-2}(e+fx)^{m+1}\left(adf(cfh(m+1)-deh(m+3)+2dfg)+b\left(c^2f^2h\left(m^2+3m+2\right)+cdf(m+1)(fg-\right.\right.}{d^2f(m+2)(m+3)(de-cf)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(c + d*x)^(-4 - m)*(e + f*x)^m*(g + h*x),x]
```

```
[Out] ((a*d*f*(2*d*f*g + c*f*h*(1 + m) - d*e*h*(3 + m)) + b*(c^2*f^2*h*(2 + 3*m +
 m^2) - d^2*e*(3 + m)*(f*g - e*h*(2 + m)) + c*d*f*(1 + m)*(f*g - 2*e*h*(3 +
 m))))*(c + d*x)^(-2 - m)*(e + f*x)^(1 + m))/(d^2*f*(d*e - c*f)^2*(2 + m)*(
3 + m)) - ((a*d*f*(2*d*f*g + c*f*h*(1 + m) - d*e*h*(3 + m)) + b*(c^2*f^2*h*
(2 + 3*m + m^2) - d^2*e*(3 + m)*(f*g - e*h*(2 + m)) + c*d*f*(1 + m)*(f*g -
2*e*h*(3 + m))))*(c + d*x)^(-1 - m)*(e + f*x)^(1 + m))/(d^2*(d*e - c*f)^3*(
1 + m)*(2 + m)*(3 + m)) - ((c + d*x)^(-3 - m)*(e + f*x)^(1 + m)*(a*d*f*(d*g
 - c*h) - b*c*(d*f*g + c*f*h*(2 + m) - d*e*h*(3 + m)) + b*d*(d*e - c*f)*h*(
3 + m)*x))/(d^2*f*(d*e - c*f)*(3 + m))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 146

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m
 + 1)*(m + n + 3)), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
```

```
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rubi steps

$$\int (a+bx)(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx = -\frac{(c+dx)^{-3-m}(e+fx)^{1+m}(adf(dg-ch)-bc(dfg+cfh(2+m)-}{d^2f(de-cf)(3+m)}$$

$$= \frac{\left(adf(2dfg+cfh(1+m)-deh(3+m))+b\left(c^2f^2h\left(2+3m+m^2\right)\right.\right.}{d^2}$$

$$= \frac{\left(adf(2dfg+cfh(1+m)-deh(3+m))+b\left(c^2f^2h\left(2+3m+m^2\right)\right.\right.}{d^2}$$

Mathematica [A] time = 0.55, size = 227, normalized size = 0.63

$$\frac{(c+dx)^{-m-3}(e+fx)^{m+1}\left(\frac{(c+dx)(cf(m+2)-d(em+e-fx))\left(adf(cfh(m+1)-deh(m+3)+2dfg)+b\left(c^2f^2h\left(m^2+3m+2\right)+cdf(m+1)(fg-2eh(m+3))+\right.\right.}{(m+1)(m+2)(de-cf)^2}\right.}{d^2f(m+3)(c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(c + d*x)^(-4 - m)*(e + f*x)^m*(g + h*x),x]
```

```
[Out] ((c + d*x)^(-3 - m)*(e + f*x)^(1 + m)*(a*d*f*(d*g - c*h) + ((a*d*f*(2*d*f*g
 + c*f*h*(1 + m) - d*e*h*(3 + m)) + b*(c^2*f^2*h*(2 + 3*m + m^2) + d^2*e*(3
 + m)*(-(f*g) + e*h*(2 + m)) + c*d*f*(1 + m)*(f*g - 2*e*h*(3 + m))))*(c + d
*x)*(c*f*(2 + m) - d*(e + e*m - f*x)))/((d*e - c*f)^2*(1 + m)*(2 + m)) - b*
(c^2*f*h*(2 + m) - d^2*e*h*(3 + m)*x + c*d*(-(e*h*(3 + m)) + f*(g + h*(3 +
m)*x)))))/(d^2*f*(-(d*e) + c*f)*(3 + m))
```

fricas [B] time = 1.05, size = 1608, normalized size = 4.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="fricas")
```

```
[Out] -(((b*d^3*e^2*f - 2*b*c*d^2*e*f^2 + b*c^2*d*f^3)*h*m^2 - (3*b*d^3*e*f^2 - (
b*c*d^2 + 2*a*d^3)*f^3)*g + (6*b*d^3*e^2*f - 3*(2*b*c*d^2 + a*d^3)*e*f^2 +
(2*b*c^2*d + a*c*d^2)*f^3)*h - ((b*d^3*e*f^2 - b*c*d^2*f^3)*g - (5*b*d^3*e^
2*f - (8*b*c*d^2 + a*d^3)*e*f^2 + (3*b*c^2*d + a*c*d^2)*f^3)*h)*m)*x^4 + (a
*c*d^2*e^3 - 2*a*c^2*d*e^2*f + a*c^3*e*f^2)*g*m^2 + (((b*d^3*e^2*f - 2*b*c*
d^2*e*f^2 + b*c^2*d*f^3)*g + (b*d^3*e^3 - (b*c*d^2 - a*d^3)*e^2*f - (b*c^2*
d + 2*a*c*d^2)*e*f^2 + (b*c^3 + a*c^2*d)*f^3)*h)*m^2 - 4*(3*b*c*d^2*e*f^2 -
 (b*c^2*d + 2*a*c*d^2)*f^3)*g + 2*(3*b*d^3*e^3 + 3*b*c*d^2*e^2*f - 3*(b*c^2
*d + 2*a*c*d^2)*e*f^2 + (b*c^3 + 2*a*c^2*d)*f^3)*h + ((3*b*d^3*e^2*f - 2*(4
*b*c*d^2 + a*d^3)*e*f^2 + (5*b*c^2*d + 2*a*c*d^2)*f^3)*g + (5*b*d^3*e^3 - (
b*c*d^2 - 3*a*d^3)*e^2*f - (7*b*c^2*d + 8*a*c*d^2)*e*f^2 + (3*b*c^3 + 5*a*c
^2*d)*f^3)*h)*m)*x^3 + (((b*d^3*e^3 - (b*c*d^2 - a*d^3)*e^2*f - (b*c^2*d +
2*a*c*d^2)*e*f^2 + (b*c^3 + a*c^2*d)*f^3)*g + (a*c^3*f^3 + (b*c*d^2 + a*d^3
)*e^3 - (2*b*c^2*d + a*c*d^2)*e^2*f + (b*c^3 - a*c^2*d)*e*f^2)*h)*m^2 + 3*(
b*d^3*e^3 - 3*b*c*d^2*e^2*f - 3*b*c^2*d*e*f^2 + (b*c^3 + 4*a*c^2*d)*f^3)*g
- 3*(3*a*c*d^2*e^2*f + 3*a*c^2*d*e*f^2 - a*c^3*f^3 - (4*b*c*d^2 + a*d^3)*e^
```

```
3)*h + ((4*b*d^3*e^3 - (4*b*c*d^2 - a*d^3)*e^2*f - 4*(b*c^2*d + 2*a*c*d^2)*
e*f^2 + (4*b*c^3 + 7*a*c^2*d)*f^3)*g + (4*a*c^3*f^3 + (7*b*c*d^2 + 4*a*d^3)
*e^3 - 4*(2*b*c^2*d + a*c*d^2)*e^2*f + (b*c^3 - 4*a*c^2*d)*e*f^2)*h)*m)*x^2
 + (6*a*c^3*e*f^2 + (b*c^2*d + 2*a*c*d^2)*e^3 - 3*(b*c^3 + 2*a*c^2*d)*e^2*f
)*g - (3*a*c^3*e^2*f - (2*b*c^3 + a*c^2*d)*e^3)*h + ((5*a*c^3*e*f^2 + (b*c^
2*d + 3*a*c*d^2)*e^3 - (b*c^3 + 8*a*c^2*d)*e^2*f)*g + (a*c^2*d*e^3 - a*c^3*
e^2*f)*h)*m + (((a*c^3*f^3 + (b*c*d^2 + a*d^3)*e^3 - (2*b*c^2*d + a*c*d^2)*
e^2*f + (b*c^3 - a*c^2*d)*e*f^2)*g + (a*c*d^2*e^3 - 2*a*c^2*d*e^2*f + a*c^3
*e*f^2)*h)*m^2 + 2*(3*a*c^2*d*e*f^2 + 3*a*c^3*f^3 + (2*b*c*d^2 + a*d^3)*e^3
 - 3*(2*b*c^2*d + a*c*d^2)*e^2*f)*g - 4*(3*a*c^2*d*e^2*f - (2*b*c^2*d + a*c
*d^2)*e^3)*h + ((5*a*c^3*f^3 + (5*b*c*d^2 + 3*a*d^3)*e^3 - (8*b*c^2*d + 7*a
*c*d^2)*e^2*f + (3*b*c^3 - a*c^2*d)*e*f^2)*g + (3*a*c^3*e*f^2 + (2*b*c^2*d
+ 5*a*c*d^2)*e^3 - 2*(b*c^3 + 4*a*c^2*d)*e^2*f)*h)*m)*x)*(d*x + c)^(-m - 4)
*(f*x + e)^m/(6*d^3*e^3 - 18*c*d^2*e^2*f + 18*c^2*d*e*f^2 - 6*c^3*f^3 + (d^
3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*m^3 + 6*(d^3*e^3 - 3*c*d^2
*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*m^2 + 11*(d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2
*d*e*f^2 - c^3*f^3)*m)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)\left(hx+g\right)(dx+c)^{-m-4}\left(fx+e\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)*(h*x + g)*(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

maple [B] time = 0.01, size = 906, normalized size = 2.50

$$-\frac{\left(-b\,c^{2} f^{2} h\,m^{2} x^{2}+2 b c d e f h\,m^{2} x^{2}-b\,d^{2} e^{2} h\,m^{2} x^{2}-a\,c^{2} f^{2} h\,m^{2} x+2 a c d e f h\,m^{2} x-a c d\,f^{2} h m\,x^{2}-a\,d^{2} e^{2} h\,m^{2} x\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)^(-m-4)*(f*x+e)^m*(h*x+g),x)
```

```
[Out] -(d*x+c)^(-m-3)*(f*x+e)^(m+1)*(-b*c^2*f^2*h*m^2*x^2+2*b*c*d*e*f*h*m^2*x^2-b
*d^2*e^2*h*m^2*x^2-a*c^2*f^2*h*m^2*x+2*a*c*d*e*f*h*m^2*x-a*c*d*f^2*h*m*x^2-
a*d^2*e^2*h*m^2*x+a*d^2*e*f*h*m*x^2-b*c^2*f^2*g*m^2*x-3*b*c^2*f^2*h*m*x^2+2
*b*c*d*e*f*g*m^2*x+8*b*c*d*e*f*h*m*x^2-b*c*d*f^2*g*m*x^2-b*d^2*e^2*g*m^2*x-
5*b*d^2*e^2*h*m*x^2+b*d^2*e*f*g*m*x^2-a*c^2*f^2*g*m^2-4*a*c^2*f^2*h*m*x+2*a
*c*d*e*f*g*m^2+8*a*c*d*e*f*h*m*x-2*a*c*d*f^2*g*m*x-a*c*d*f^2*h*x^2-a*d^2*e^
2*g*m^2-4*a*d^2*e^2*h*m*x+2*a*d^2*e*f*g*m*x+3*a*d^2*e*f*h*x^2-2*a*d^2*f^2*g
*x^2+2*b*c^2*e*f*h*m*x-4*b*c^2*f^2*g*m*x-2*b*c^2*f^2*h*x^2-2*b*c*d*e^2*h*m*
x+8*b*c*d*e*f*g*m*x+6*b*c*d*e*f*h*x^2-b*c*d*f^2*g*x^2-4*b*d^2*e^2*g*m*x-6*b
*d^2*e^2*h*x^2+3*b*d^2*e*f*g*x^2+a*c^2*e*f*h*m-5*a*c^2*f^2*g*m-3*a*c^2*f^2*
h*x-a*c*d*e^2*h*m+8*a*c*d*e*f*g*m+10*a*c*d*e*f*h*x-6*a*c*d*f^2*g*x-3*a*d^2*
e^2*g*m-3*a*d^2*e^2*h*x+2*a*d^2*e*f*g*x+b*c^2*e*f*g*m+2*b*c^2*e*f*h*x-3*b*c
^2*f^2*g*x-b*c*d*e^2*g*m-6*b*c*d*e^2*h*x+10*b*c*d*e*f*g*x-3*b*d^2*e^2*g*x+3
*a*c^2*e*f*h-6*a*c^2*f^2*g-a*c*d*e^2*h+6*a*c*d*e*f*g-2*a*d^2*e^2*g-2*b*c^2*
e^2*h+3*b*c^2*e*f*g-b*c*d*e^2*g)/(c^3*f^3*m^3-3*c^2*d*e*f^2*m^3+3*c*d^2*e^2
*f*m^3-d^3*e^3*m^3+6*c^3*f^3*m^2-18*c^2*d*e*f^2*m^2+18*c*d^2*e^2*f*m^2-6*d^
3*e^3*m^2+11*c^3*f^3*m-33*c^2*d*e*f^2*m+33*c*d^2*e^2*f*m-11*d^3*e^3*m+6*c^3
*f^3-18*c^2*d*e*f^2+18*c*d^2*e^2*f-6*d^3*e^3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)\left(hx+g\right)(dx+c)^{-m-4}\left(fx+e\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)*(h*x + g)*(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

mupad [B] time = 4.28, size = 1890, normalized size = 5.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)^m*(g + h*x)*(a + b*x))/(c + d*x)^(m + 4),x)
```

```
[Out] ((e + f*x)^m*(2*b*c^3*e^3*h + 2*a*c*d^2*e^3*g + a*c^2*d*e^3*h + b*c^2*d*e^3
*g + 6*a*c^3*e*f^2*g - 3*a*c^3*e^2*f*h - 3*b*c^3*e^2*f*g - 6*a*c^2*d*e^2*f*
g + 3*a*c*d^2*e^3*g*m + a*c^2*d*e^3*h*m + b*c^2*d*e^3*g*m + 5*a*c^3*e*f^2*g
*m - a*c^3*e^2*f*h*m - b*c^3*e^2*f*g*m + a*c*d^2*e^3*g*m^2 + a*c^3*e*f^2*g*
m^2 - 2*a*c^2*d*e^2*f*g*m^2 - 8*a*c^2*d*e^2*f*g*m))/((c*f - d*e)^3*(c + d*x
)^(m + 4)*(11*m + 6*m^2 + m^3 + 6)) + (x*(e + f*x)^m*(6*a*c^3*f^3*g + 2*a*d
^3*e^3*g + 4*a*c*d^2*e^3*h + 4*b*c*d^2*e^3*g + 8*b*c^2*d*e^3*h + 5*a*c^3*f^
3*g*m + 3*a*d^3*e^3*g*m + a*c^3*f^3*g*m^2 + a*d^3*e^3*g*m^2 - 6*a*c*d^2*e^2
*f*g + 6*a*c^2*d*e*f^2*g - 12*a*c^2*d*e^2*f*h - 12*b*c^2*d*e^2*f*g + 5*a*c*
d^2*e^3*h*m + 5*b*c*d^2*e^3*g*m + 2*b*c^2*d*e^3*h*m + 3*a*c^3*e*f^2*h*m + 3
*b*c^3*e*f^2*g*m - 2*b*c^3*e^2*f*h*m + a*c*d^2*e^3*h*m^2 + b*c*d^2*e^3*g*m^
2 + a*c^3*e*f^2*h*m^2 + b*c^3*e*f^2*g*m^2 - a*c*d^2*e^2*f*g*m^2 - a*c^2*d*e
*f^2*g*m^2 - 2*a*c^2*d*e^2*f*h*m^2 - 2*b*c^2*d*e^2*f*g*m^2 - 7*a*c*d^2*e^2*
f*g*m - a*c^2*d*e*f^2*g*m - 8*a*c^2*d*e^2*f*h*m - 8*b*c^2*d*e^2*f*g*m))/((c
*f - d*e)^3*(c + d*x)^(m + 4)*(11*m + 6*m^2 + m^3 + 6)) + (x^4*(e + f*x)^m*
(2*a*d^3*f^3*g + a*c*d^2*f^3*h + b*c*d^2*f^3*g + 2*b*c^2*d*f^3*h - 3*a*d^3*
e*f^2*h - 3*b*d^3*e*f^2*g + 6*b*d^3*e^2*f*h - 6*b*c*d^2*e*f^2*h + a*c*d^2*f
^3*h*m + b*c*d^2*f^3*g*m + 3*b*c^2*d*f^3*h*m - a*d^3*e*f^2*h*m - b*d^3*e*f^
2*g*m + 5*b*d^3*e^2*f*h*m + b*c^2*d*f^3*h*m^2 + b*d^3*e^2*f*h*m^2 - 2*b*c*d
^2*e*f^2*h*m^2 - 8*b*c*d^2*e*f^2*h*m))/((c*f - d*e)^3*(c + d*x)^(m + 4)*(11
*m + 6*m^2 + m^3 + 6)) + (x^2*(e + f*x)^m*(3*a*c^3*f^3*h + 3*a*d^3*e^3*h +
3*b*c^3*f^3*g + 3*b*d^3*e^3*g + 12*a*c^2*d*f^3*g + 12*b*c*d^2*e^3*h + 4*a*c
^3*f^3*h*m + 4*a*d^3*e^3*h*m + 4*b*c^3*f^3*g*m + 4*b*d^3*e^3*g*m + a*c^3*f^
3*h*m^2 + a*d^3*e^3*h*m^2 + b*c^3*f^3*g*m^2 + b*d^3*e^3*g*m^2 - 9*a*c*d^2*e
^2*f*h - 9*a*c^2*d*e*f^2*h - 9*b*c*d^2*e^2*f*g - 9*b*c^2*d*e*f^2*g + 7*a*c^
2*d*f^3*g*m + 7*b*c*d^2*e^3*h*m + a*d^3*e^2*f*g*m + b*c^3*e*f^2*h*m + a*c^2
*d*f^3*g*m^2 + b*c*d^2*e^3*h*m^2 + a*d^3*e^2*f*g*m^2 + b*c^3*e*f^2*h*m^2 -
2*a*c*d^2*e*f^2*g*m^2 - a*c*d^2*e^2*f*h*m^2 - a*c^2*d*e*f^2*h*m^2 - b*c*d^2
*e^2*f*g*m^2 - b*c^2*d*e*f^2*g*m^2 - 2*b*c^2*d*e^2*f*h*m^2 - 8*a*c*d^2*e*f^
2*g*m - 4*a*c*d^2*e^2*f*h*m - 4*a*c^2*d*e*f^2*h*m - 4*b*c*d^2*e^2*f*g*m - 4
*b*c^2*d*e*f^2*g*m - 8*b*c^2*d*e^2*f*h*m))/((c*f - d*e)^3*(c + d*x)^(m + 4)
*(11*m + 6*m^2 + m^3 + 6)) + (x^3*(e + f*x)^m*(2*b*c^3*f^3*h + 6*b*d^3*e^3*
h + 8*a*c*d^2*f^3*g + 4*a*c^2*d*f^3*h + 4*b*c^2*d*f^3*g + 3*b*c^3*f^3*h*m +
 5*b*d^3*e^3*h*m + b*c^3*f^3*h*m^2 + b*d^3*e^3*h*m^2 - 12*a*c*d^2*e*f^2*h -
 12*b*c*d^2*e*f^2*g + 6*b*c*d^2*e^2*f*h - 6*b*c^2*d*e*f^2*h + 2*a*c*d^2*f^3
*g*m + 5*a*c^2*d*f^3*h*m + 5*b*c^2*d*f^3*g*m - 2*a*d^3*e*f^2*g*m + 3*a*d^3*
e^2*f*h*m + 3*b*d^3*e^2*f*g*m + a*c^2*d*f^3*h*m^2 + b*c^2*d*f^3*g*m^2 + a*d
^3*e^2*f*h*m^2 + b*d^3*e^2*f*g*m^2 - 2*a*c*d^2*e*f^2*h*m^2 - 2*b*c*d^2*e*f^
2*g*m^2 - b*c*d^2*e^2*f*h*m^2 - b*c^2*d*e*f^2*h*m^2 - 8*a*c*d^2*e*f^2*h*m -
 8*b*c*d^2*e*f^2*g*m - b*c*d^2*e^2*f*h*m - 7*b*c^2*d*e*f^2*h*m))/((c*f - d*
e)^3*(c + d*x)^(m + 4)*(11*m + 6*m^2 + m^3 + 6))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)**(-4-m)*(f*x+e)**m*(h*x+g),x)
```

```
[Out] Timed out
```

3.135 $\int (c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx$

Optimal. Leaf size=188

$$-\frac{(dg-ch)(c+dx)^{-m-3}(e+fx)^{m+1}}{d(m+3)(de-cf)}+\frac{(c+dx)^{-m-2}(e+fx)^{m+1}(cfh(m+1)+d(2fg-eh(m+3)))}{d(m+2)(m+3)(de-cf)^2}-\frac{f(c+dx)^{-m}}{}$$

```
[Out] -(-c*h+d*g)*(d*x+c)^(-3-m)*(f*x+e)^(1+m)/d/(-c*f+d*e)/(3+m)+(c*f*h*(1+m)+d*
(2*f*g-e*h*(3+m)))*(d*x+c)^(-2-m)*(f*x+e)^(1+m)/d/(-c*f+d*e)^2/(2+m)/(3+m)-
f*(c*f*h*(1+m)+d*(2*f*g-e*h*(3+m)))*(d*x+c)^(-1-m)*(f*x+e)^(1+m)/d/(-c*f+d*
e)^3/(1+m)/(2+m)/(3+m)
```

Rubi [A] time = 0.10, antiderivative size = 186, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {79, 45, 37}

$$-\frac{(dg-ch)(c+dx)^{-m-3}(e+fx)^{m+1}}{d(m+3)(de-cf)}+\frac{(c+dx)^{-m-2}(e+fx)^{m+1}(cfh(m+1)-deh(m+3)+2dfg)}{d(m+2)(m+3)(de-cf)^2}-\frac{f(c+dx)^{-m}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(-4 - m)*(e + f*x)^m*(g + h*x),x]
```

```
[Out] -(((d*g - c*h)*(c + d*x)^(-3 - m)*(e + f*x)^(1 + m))/(d*(d*e - c*f)*(3 + m)
)) + ((2*d*f*g + c*f*h*(1 + m) - d*e*h*(3 + m))*(c + d*x)^(-2 - m)*(e + f*x
)^(1 + m))/(d*(d*e - c*f)^2*(2 + m)*(3 + m)) - (f*(2*d*f*g + c*f*h*(1 + m)
- d*e*h*(3 + m))*(c + d*x)^(-1 - m)*(e + f*x)^(1 + m))/(d*(d*e - c*f)^3*(1
+ m)*(2 + m)*(3 + m))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rubi steps

$$\int (c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx = -\frac{(dg-ch)(c+dx)^{-3-m}(e+fx)^{1+m}}{d(de-cf)(3+m)} - \frac{(2dfg+cfh(1+m)-deh(3+m))\int}{d(de-cf)(3+}$$
$$= -\frac{(dg-ch)(c+dx)^{-3-m}(e+fx)^{1+m}}{d(de-cf)(3+m)} + \frac{(2dfg+cfh(1+m)-deh(3+m))(}{d(de-cf)^2(2+m)(}$$
$$= -\frac{(dg-ch)(c+dx)^{-3-m}(e+fx)^{1+m}}{d(de-cf)(3+m)} + \frac{(2dfg+cfh(1+m)-deh(3+m))(}{d(de-cf)^2(2+m)(}$$

Mathematica [A] time = 0.13, size = 181, normalized size = 0.96

$$\frac{(dg-ch)(c+dx)^{-m-3}(e+fx)^{m+1}}{d(-m-3)(de-cf)} - \frac{\left(\frac{(c+dx)^{-m-2}(e+fx)^{m+1}}{(-m-2)(de-cf)} + \frac{f(c+dx)^{-m-1}(e+fx)^{m+1}}{(-m-2)(-m-1)(de-cf)^2}\right)(-h(cf(m+1)+de(-m-3))-2df}{d(-m-3)(de-cf)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(-4 - m)*(e + f*x)^m*(g + h*x),x]
```

```
[Out] ((d*g - c*h)*(c + d*x)^(-3 - m)*(e + f*x)^(1 + m))/(d*(d*e - c*f)*(-3 - m))
 - ((-2*d*f*g - h*(d*e*(-3 - m) + c*f*(1 + m)))*(((c + d*x)^(-2 - m)*(e + f
*x)^(1 + m))/((d*e - c*f)*(-2 - m)) + (f*(c + d*x)^(-1 - m)*(e + f*x)^(1 +
m))/((d*e - c*f)^2*(-2 - m)*(-1 - m))))/(d*(d*e - c*f)*(-3 - m))
```

fricas [B] time = 0.87, size = 905, normalized size = 4.81

$$-\frac{\left(\left(2\,d^3f^3g-\left(d^3ef^2-cd^2f^3\right)hm-\left(3\,d^3ef^2-cd^2f^3\right)h\right)x^4+\left(cd^2e^3-2\,c^2de^2f+c^3ef^2\right)gm^2+\left(8\,cd^2f^3g+\left(d^3e^2f\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="fricas")
```

```
[Out] -((2*d^3*f^3*g - (d^3*e*f^2 - c*d^2*f^3)*h*m - (3*d^3*e*f^2 - c*d^2*f^3)*h)
*x^4 + (c*d^2*e^3 - 2*c^2*d*e^2*f + c^3*e*f^2)*g*m^2 + (8*c*d^2*f^3*g + (d^
3*e^2*f - 2*c*d^2*e*f^2 + c^2*d*f^3)*h*m^2 - 4*(3*c*d^2*e*f^2 - c^2*d*f^3)*
h - (2*(d^3*e*f^2 - c*d^2*f^3)*g - (3*d^3*e^2*f - 8*c*d^2*e*f^2 + 5*c^2*d*f
^3)*h)*m)*x^3 + (12*c^2*d*f^3*g + ((d^3*e^2*f - 2*c*d^2*e*f^2 + c^2*d*f^3)*
g + (d^3*e^3 - c*d^2*e^2*f - c^2*d*e*f^2 + c^3*f^3)*h)*m^2 + 3*(d^3*e^3 - 3
*c*d^2*e^2*f - 3*c^2*d*e*f^2 + c^3*f^3)*h + ((d^3*e^2*f - 8*c*d^2*e*f^2 + 7
*c^2*d*f^3)*g + 4*(d^3*e^3 - c*d^2*e^2*f - c^2*d*e*f^2 + c^3*f^3)*h)*m)*x^2
 + 2*(c*d^2*e^3 - 3*c^2*d*e^2*f + 3*c^3*e*f^2)*g + (c^2*d*e^3 - 3*c^3*e^2*f
)*h + ((3*c*d^2*e^3 - 8*c^2*d*e^2*f + 5*c^3*e*f^2)*g + (c^2*d*e^3 - c^3*e^2
*f)*h)*m + (((d^3*e^3 - c*d^2*e^2*f - c^2*d*e*f^2 + c^3*f^3)*g + (c*d^2*e^3
 - 2*c^2*d*e^2*f + c^3*e*f^2)*h)*m^2 + 2*(d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d
*e*f^2 + 3*c^3*f^3)*g + 4*(c*d^2*e^3 - 3*c^2*d*e^2*f)*h + ((3*d^3*e^3 - 7*c
*d^2*e^2*f - c^2*d*e*f^2 + 5*c^3*f^3)*g + (5*c*d^2*e^3 - 8*c^2*d*e^2*f + 3*
c^3*e*f^2)*h)*m)*x)*(d*x + c)^(-m - 4)*(f*x + e)^m/(6*d^3*e^3 - 18*c*d^2*e^
2*f + 18*c^2*d*e*f^2 - 6*c^3*f^3 + (d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2
 - c^3*f^3)*m^3 + 6*(d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*m^2
 + 11*(d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*m)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(hx+g\right)(dx+c)^{-m-4}\left(fx+e\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((h*x + g)*(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

maple [B] time = 0.01, size = 509, normalized size = 2.71

$$-\frac{\left(-c^2 f^2 h\, m^2 x+2 c d e f h\, m^2 x-c d\, f^2 h m\, x^2-d^2 e^2 h\, m^2 x+d^2 e f h m\, x^2-c^2 f^2 g\, m^2-4 c^2 f^2 h m x+2 c d e f g\, m^2+ 8}{c^3 f^3 m^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(-m-4)*(f*x+e)^m*(h*x+g),x)
```

```
[Out] -(d*x+c)^(-m-3)*(f*x+e)^(m+1)*(-c^2*f^2*h*m^2*x+2*c*d*e*f*h*m^2*x-c*d*f^2*h
*m*x^2-d^2*e^2*h*m^2*x+d^2*e*f*h*m*x^2-c^2*f^2*g*m^2-4*c^2*f^2*h*m*x+2*c*d*
e*f*g*m^2+8*c*d*e*f*h*m*x-2*c*d*f^2*g*m*x-c*d*f^2*h*x^2-d^2*e^2*g*m^2-4*d^2
*e^2*h*m*x+2*d^2*e*f*g*m*x+3*d^2*e*f*h*x^2-2*d^2*f^2*g*x^2+c^2*e*f*h*m-5*c^
2*f^2*g*m-3*c^2*f^2*h*x-c*d*e^2*h*m+8*c*d*e*f*g*m+10*c*d*e*f*h*x-6*c*d*f^2*
g*x-3*d^2*e^2*g*m-3*d^2*e^2*h*x+2*d^2*e*f*g*x+3*c^2*e*f*h-6*c^2*f^2*g-c*d*e
^2*h+6*c*d*e*f*g-2*d^2*e^2*g)/(c^3*f^3*m^3-3*c^2*d*e*f^2*m^3+3*c*d^2*e^2*f*
m^3-d^3*e^3*m^3+6*c^3*f^3*m^2-18*c^2*d*e*f^2*m^2+18*c*d^2*e^2*f*m^2-6*d^3*e
^3*m^2+11*c^3*f^3*m-33*c^2*d*e*f^2*m+33*c*d^2*e^2*f*m-11*d^3*e^3*m+6*c^3*f^
3-18*c^2*d*e*f^2+18*c*d^2*e^2*f-6*d^3*e^3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(hx+g\right)(dx+c)^{-m-4}\left(fx+e\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="maxima")
```

```
[Out] integrate((h*x + g)*(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

mupad [B] time = 3.41, size = 869, normalized size = 4.62

$$\frac{x^2\left(e+f\,x\right)^m\,\left(h\,c^3\,f^3\,m^2+4\,h\,c^3\,f^3\,m+3\,h\,c^3\,f^3-h\,c^2\,d\,e\,f^2\,m^2-4\,h\,c^2\,d\,e\,f^2\,m-9\,h\,c^2\,d\,e\,f^2+g\,c^2\,d\,f^3\,m^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)^m*(g + h*x))/(c + d*x)^(m + 4),x)
```

```
[Out] (x^2*(e + f*x)^m*(3*c^3*f^3*h + 3*d^3*e^3*h + c^3*f^3*h*m^2 + d^3*e^3*h*m^2
 + 12*c^2*d*f^3*g + 4*c^3*f^3*h*m + 4*d^3*e^3*h*m - 9*c*d^2*e^2*f*h - 9*c^2
*d*e*f^2*h + 7*c^2*d*f^3*g*m + d^3*e^2*f*g*m + c^2*d*f^3*g*m^2 + d^3*e^2*f*
g*m^2 - 8*c*d^2*e*f^2*g*m - 4*c*d^2*e^2*f*h*m - 4*c^2*d*e*f^2*h*m - 2*c*d^2
*e*f^2*g*m^2 - c*d^2*e^2*f*h*m^2 - c^2*d*e*f^2*h*m^2))/((c*f - d*e)^3*(c +
d*x)^(m + 4)*(11*m + 6*m^2 + m^3 + 6)) + (x*(e + f*x)^m*(6*c^3*f^3*g + 2*d^
3*e^3*g + c^3*f^3*g*m^2 + d^3*e^3*g*m^2 + 4*c*d^2*e^3*h + 5*c^3*f^3*g*m + 3
*d^3*e^3*g*m - 6*c*d^2*e^2*f*g + 6*c^2*d*e*f^2*g - 12*c^2*d*e^2*f*h + 5*c*d
^2*e^3*h*m + 3*c^3*e*f^2*h*m + c*d^2*e^3*h*m^2 + c^3*e*f^2*h*m^2 - 7*c*d^2*
e^2*f*g*m - c^2*d*e*f^2*g*m - 8*c^2*d*e^2*f*h*m - c*d^2*e^2*f*g*m^2 - c^2*d
*e*f^2*g*m^2 - 2*c^2*d*e^2*f*h*m^2))/((c*f - d*e)^3*(c + d*x)^(m + 4)*(11*m
 + 6*m^2 + m^3 + 6)) + (c*e*(e + f*x)^m*(6*c^2*f^2*g + 2*d^2*e^2*g + c^2*f^
2*g*m^2 + d^2*e^2*g*m^2 + c*d*e^2*h - 3*c^2*e*f*h + 5*c^2*f^2*g*m + 3*d^2*e
^2*g*m - 6*c*d*e*f*g + c*d*e^2*h*m - c^2*e*f*h*m - 2*c*d*e*f*g*m^2 - 8*c*d*
e*f*g*m))/((c*f - d*e)^3*(c + d*x)^(m + 4)*(11*m + 6*m^2 + m^3 + 6)) + (d^2
*f^2*x^4*(e + f*x)^m*(c*f*h - 3*d*e*h + 2*d*f*g + c*f*h*m - d*e*h*m))/((c*f
 - d*e)^3*(c + d*x)^(m + 4)*(11*m + 6*m^2 + m^3 + 6)) + (d*f*x^3*(e + f*x)^
```

```
m*(4*c*f + c*f*m - d*e*m)*(c*f*h - 3*d*e*h + 2*d*f*g + c*f*h*m - d*e*h*m))/
((c*f - d*e)^3*(c + d*x)^(m + 4)*(11*m + 6*m^2 + m^3 + 6))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(-4-m)*(f*x+e)**m*(h*x+g),x)
```

```
[Out] Timed out
```

3.136 $\int \frac{(A+Bx)(c+dx)^n(e+fx)^p}{a+bx} dx$

Optimal. Leaf size=177

$$-\frac{(Ab-aB)(c+dx)^{n+1}(e+fx)^p\left(\frac{d(e+fx)}{de-cf}\right)^{-p}F_1\left(n+1;1,-p;n+2;\frac{b(c+dx)}{bc-ad},-\frac{f(c+dx)}{de-cf}\right)}{b(n+1)(bc-ad)}-\frac{B(c+dx)^{n+1}(e+fx)^{p+1}\,{}_2}{b(p+}$$

```
[Out] -(A*b-B*a)*(d*x+c)^(1+n)*(f*x+e)^p*AppellF1(1+n,1,-p,2+n,b*(d*x+c)/(-a*d+b*
c),-f*(d*x+c)/(-c*f+d*e))/b/(-a*d+b*c)/(1+n)/((d*(f*x+e)/(-c*f+d*e))^p)-B*(
d*x+c)^(1+n)*(f*x+e)^(1+p)*hypergeom([1, 2+n+p],[2+p],d*(f*x+e)/(-c*f+d*e))
/b/(-c*f+d*e)/(1+p)
```

Rubi [A] time = 0.12, antiderivative size = 190, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {157, 70, 69, 137, 136}

$$\frac{B(c+dx)^{n+1}(e+fx)^p\left(\frac{d(e+fx)}{de-cf}\right)^{-p}{}_2F_1\left(n+1,-p;n+2;-\frac{f(c+dx)}{de-cf}\right)}{bd(n+1)}-\frac{(Ab-aB)(c+dx)^{n+1}(e+fx)^p\left(\frac{d(e+fx)}{de-cf}\right)^{-p}F}{b(n+1)(bc}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((A + B*x)*(c + d*x)^n*(e + f*x)^p)/(a + b*x),x]
```

```
[Out] -(((A*b - a*B)*(c + d*x)^(1 + n)*(e + f*x)^p*AppellF1[1 + n, -p, 1, 2 + n,
-((f*(c + d*x))/(d*e - c*f)), (b*(c + d*x))/(b*c - a*d)])/(b*(b*c - a*d)*(1
 + n)*((d*(e + f*x))/(d*e - c*f))^p)) + (B*(c + d*x)^(1 + n)*(e + f*x)^p*Hy
pergeometric2F1[1 + n, -p, 2 + n, -((f*(c + d*x))/(d*e - c*f))])/(b*d*(1 +
n)*((d*(e + f*x))/(d*e - c*f))^p)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
```

```
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !GtQ[b/(b*c
- a*d), 0] && !SimplerQ[c + d*x, a + b*x]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rubi steps

$$\int \frac{(A+Bx)(c+dx)^n(e+fx)^p}{a+bx}\,dx = \frac{B\int (c+dx)^n(e+fx)^p\,dx}{b} + \frac{(Ab-aB)\int \frac{(c+dx)^n(e+fx)^p}{a+bx}\,dx}{b}$$

$$= \frac{\left(B(e+fx)^p\left(\frac{d(e+fx)}{de-cf}\right)^{-p}\right)\int (c+dx)^n\left(\frac{de}{de-cf}+\frac{dfx}{de-cf}\right)^p dx}{b} + \frac{\left((Ab-aB)(e+fx\right.}{}$$

$$= -\frac{(Ab-aB)(c+dx)^{1+n}(e+fx)^p\left(\frac{d(e+fx)}{de-cf}\right)^{-p} F_1\left(1+n;-p,1;2+n;-\frac{f(c+dx)}{de-cf},\right.}{b(bc-ad)(1+n)}$$

Mathematica [A] time = 0.40, size = 199, normalized size = 1.12

$$\frac{(c+dx)^n(e+fx)^p\left(\frac{(Ab-aB)\left(\frac{b(c+dx)}{d(a+bx)}\right)^{-n}\left(\frac{b(e+fx)}{f(a+bx)}\right)^{-p}F_1\left(-n-p;-n,-p;-n-p+1;\frac{ad-bc}{d(a+bx)},\frac{af-be}{f(a+bx)}\right)}{n+p}+\frac{bB(e+fx)\left(\frac{f(c+dx)}{cf-de}\right)^{-n}{}_2F_1\left(-n,p+1;p+2;\frac{d(e+fx)}{de-cf}\right.}{f(p+1)}\right.}{b^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*x)*(c + d*x)^n*(e + f*x)^p)/(a + b*x),x]
```

```
[Out] ((c + d*x)^n*(e + f*x)^p*(((A*b - a*B)*AppellF1[-n - p, -n, -p, 1 - n - p,
(-(b*c) + a*d)/(d*(a + b*x)), (-(b*e) + a*f)/(f*(a + b*x))])/((n + p)*((b*(
c + d*x))/(d*(a + b*x)))^n*((b*(e + f*x))/(f*(a + b*x)))^p) + (b*B*(e + f*x
)*Hypergeometric2F1[-n, 1 + p, 2 + p, (d*(e + f*x))/(d*e - c*f)])/(f*(1 + p
)*((f*(c + d*x))/(-(d*e) + c*f))^n)))/b^2
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(Bx+A)(dx+c)^n\left(fx+e\right)^p}{bx+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(d*x+c)^n*(f*x+e)^p/(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((B*x + A)*(d*x + c)^n*(f*x + e)^p/(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx+A)(dx+c)^n\left(fx+e\right)^p}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(d*x+c)^n*(f*x+e)^p/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((B*x + A)*(d*x + c)^n*(f*x + e)^p/(b*x + a), x)
```

maple **[F]** time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{(Bx+A)\,(dx+c)^{n}\left(fx+e\right)^{p}}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(d*x+c)^n*(f*x+e)^p/(b*x+a),x)
```

```
[Out] int((B*x+A)*(d*x+c)^n*(f*x+e)^p/(b*x+a),x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx+A)(dx+c)^{n}\left(fx+e\right)^{p}}{bx+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(d*x+c)^n*(f*x+e)^p/(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((B*x + A)*(d*x + c)^n*(f*x + e)^p/(b*x + a), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(e+f\,x\right)^{p}\,\left(A+B\,x\right)\,\left(c+d\,x\right)^{n}}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)^p*(A + B*x)*(c + d*x)^n)/(a + b*x),x)
```

```
[Out] int(((e + f*x)^p*(A + B*x)*(c + d*x)^n)/(a + b*x), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(d*x+c)**n*(f*x+e)**p/(b*x+a),x)
```

```
[Out] Timed out
```

3.137 $\int \frac{(a+bx)^m(A+Bx)(c+dx)^{-m}}{e+fx}\,dx$

Optimal. Leaf size=233

$$-\frac{(a+bx)^m(Be-Af)(c+dx)^{-m}\,{}_2F_1\left(1,-m;1-m;\frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right)}{f^2m}-\frac{(a+bx)^{m+1}(c+dx)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^m\,{}_2F_1\left(m,m+1;\right.}{bf^2m(m}$$

```
[Out] -d*(-A*f+B*e)*(b*x+a)^(1+m)/(-a*d+b*c)/f^2/m/((d*x+c)^m)-(-A*f+B*e)*(b*x+a)
^m*hypergeom([1, -m],[1-m],(-a*f+b*e)*(d*x+c)/(-c*f+d*e)/(b*x+a))/f^2/m/((d
*x+c)^m)-(a*B*d*f*m-b*(B*c*f*m-A*d*f+B*d*e))*(b*x+a)^(1+m)*(b*(d*x+c)/(-a*d
+b*c))^m*hypergeom([m, 1+m],[2+m],-d*(b*x+a)/(-a*d+b*c))/b/(-a*d+b*c)/f^2/m
/(1+m)/((d*x+c)^m)
```

Rubi [A] time = 0.13, antiderivative size = 220, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {157, 70, 69, 105, 131}

$$\frac{(a+bx)^m(Be-Af)(c+dx)^{-m}\,{}_2F_1\left(1,m;m+1;\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}\right)}{f^2m}-\frac{(a+bx)^m(Be-Af)(c+dx)^{-m}\left(\frac{b(c+dx)}{bc-ad}\right)^m\,{}_2F_1\left(m,m\right.}{f^2m}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^m*(A + B*x))/((c + d*x)^m*(e + f*x)),x]
```

```
[Out] ((B*e - A*f)*(a + b*x)^m*Hypergeometric2F1[1, m, 1 + m, ((d*e - c*f)*(a + b
*x))/((b*e - a*f)*(c + d*x))])/(f^2*m*(c + d*x)^m) - ((B*e - A*f)*(a + b*x)
^m*((b*(c + d*x))/(b*c - a*d))^m*Hypergeometric2F1[m, m, 1 + m, -((d*(a + b
*x))/(b*c - a*d))])/(f^2*m*(c + d*x)^m) + (B*(a + b*x)^(1 + m)*((b*(c + d*x
))/(b*c - a*d))^m*Hypergeometric2F1[m, 1 + m, 2 + m, -((d*(a + b*x))/(b*c -
 a*d))])/(b*f*(1 + m)*(c + d*x)^m)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 131

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2
```

```
F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/(
(m + 1)*(b*e - a*f)^(n + 1)*(e + f*x)^(m + 1)), x] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && EqQ[m + n + p + 2, 0] && ILtQ[n, 0]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rubi steps

$$\int \frac{(a+bx)^m(A+Bx)(c+dx)^{-m}}{e+fx}\,dx = \frac{B\int (a+bx)^m(c+dx)^{-m}\,dx}{f} + \frac{(-Be+Af)\int \frac{(a+bx)^m(c+dx)^{-m}}{e+fx}\,dx}{f}$$

$$= -\frac{(b(Be-Af))\int (a+bx)^{-1+m}(c+dx)^{-m}\,dx}{f^2} + \frac{((be-af)(Be-Af))\int \frac{(a}{}}{f^2}$$

$$= \frac{(Be-Af)(a+bx)^m(c+dx)^{-m}\,{}_2F_1\left(1,m;1+m;\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}\right)}{f^2m} + \frac{B(a+b}{}$$

$$= \frac{(Be-Af)(a+bx)^m(c+dx)^{-m}\,{}_2F_1\left(1,m;1+m;\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}\right)}{f^2m} - \frac{(Be-A}{}$$

Mathematica [A] time = 0.14, size = 174, normalized size = 0.75

$$\frac{(a+bx)^m(c+dx)^{-m}\left(\left(\frac{b(c+dx)}{bc-ad}\right)^m\left(Bfm(a+bx)\,{}_2F_1\left(m,m+1;m+2;\frac{d(a+bx)}{ad-bc}\right)-b(m+1)(Be-Af)\,{}_2F_1\left(m,m;m\right.\right.\right.}{bf^2m(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^m*(A + B*x))/((c + d*x)^m*(e + f*x)),x]
```

```
[Out] ((a + b*x)^m*(b*(B*e - A*f)*(1 + m)*Hypergeometric2F1[1, m, 1 + m, ((d*e -
c*f)*(a + b*x))/((b*e - a*f)*(c + d*x))] + ((b*(c + d*x))/(b*c - a*d))^m*(-
(b*(B*e - A*f)*(1 + m)*Hypergeometric2F1[m, m, 1 + m, (d*(a + b*x))/(-(b*c)
 + a*d)]) + B*f*m*(a + b*x)*Hypergeometric2F1[m, 1 + m, 2 + m, (d*(a + b*x)
)/(-(b*c) + a*d)])))/(b*f^2*m*(1 + m)*(c + d*x)^m)
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(Bx+A)(bx+a)^m}{(fx+e)(dx+c)^m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)/((d*x+c)^m)/(f*x+e),x, algorithm="fricas")
```

```
[Out] integral((B*x + A)*(b*x + a)^m/((f*x + e)*(d*x + c)^m), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx+A)(bx+a)^m}{(fx+e)(dx+c)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)/((d*x+c)^m)/(f*x+e),x, algorithm="giac")
[Out] integrate((B*x + A)*(b*x + a)^m/((f*x + e)*(d*x + c)^m), x)
```

maple [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{(Bx+A)\,(bx+a)^{m}\,(dx+c)^{-m}}{fx+e}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(B*x+A)/((d*x+c)^m)/(f*x+e),x)
[Out] int((b*x+a)^m*(B*x+A)/((d*x+c)^m)/(f*x+e),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx+A)(bx+a)^{m}}{\left(fx+e\right)(dx+c)^{m}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)/((d*x+c)^m)/(f*x+e),x, algorithm="maxima")
[Out] integrate((B*x + A)*(b*x + a)^m/((f*x + e)*(d*x + c)^m), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\,x)\,(a+b\,x)^{m}}{\left(e+f\,x\right)\,(c+d\,x)^{m}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x)*(a + b*x)^m)/((e + f*x)*(c + d*x)^m),x)
[Out] int(((A + B*x)*(a + b*x)^m)/((e + f*x)*(c + d*x)^m), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(B*x+A)/((d*x+c)**m)/(f*x+e),x)
[Out] Exception raised: HeuristicGCDFailed
```

3.138 $\int \frac{(A+Bx)(c+dx)^n(e+fx)^p}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=250

$$\frac{2\sqrt{a+bx}\,(Ab-aB)(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(\frac{1}{2};-n,-p;\frac{3}{2};-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b^2}+\frac{2B(a+bx)^{3/}}{}$$

```
[Out] 2/3*B*(b*x+a)^(3/2)*(d*x+c)^n*(f*x+e)^p*AppellF1(3/2,-n,-p,5/2,-d*(b*x+a)/(
-a*d+b*c),-f*(b*x+a)/(-a*f+b*e))/b^2/((b*(d*x+c)/(-a*d+b*c))^n)/((b*(f*x+e)
/(-a*f+b*e))^p)+2*(A*b-B*a)*(d*x+c)^n*(f*x+e)^p*AppellF1(1/2,-n,-p,3/2,-d*(
b*x+a)/(-a*d+b*c),-f*(b*x+a)/(-a*f+b*e))*(b*x+a)^(1/2)/b^2/((b*(d*x+c)/(-a*
d+b*c))^n)/((b*(f*x+e)/(-a*f+b*e))^p)
```

Rubi [A] time = 0.22, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {159, 140, 139, 138}

$$\frac{2\sqrt{a+bx}\,(Ab-aB)(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(\frac{1}{2};-n,-p;\frac{3}{2};-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b^2}+\frac{2B(a+bx)^{3/}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*(c + d*x)^n*(e + f*x)^p)/Sqrt[a + b*x],x]
```

```
[Out] (2*(A*b - a*B)*Sqrt[a + b*x]*(c + d*x)^n*(e + f*x)^p*AppellF1[1/2, -n, -p,
3/2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b^2*((b*
(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p) + (2*B*(a + b*x)^(
3/2)*(c + d*x)^n*(e + f*x)^p*AppellF1[3/2, -n, -p, 5/2, -((d*(a + b*x))/(b*
c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(3*b^2*((b*(c + d*x))/(b*c - a*d)
)^n*((b*(e + f*x))/(b*e - a*f))^p)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
```

```
b*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m + 1)*(
c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[(a + b*x)^m*(c + d
*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] &&
 (SumSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))
```

Rubi steps

$$\int \frac{(A+Bx)(c+dx)^n(e+fx)^p}{\sqrt{a+bx}}\,dx = \frac{B\int \sqrt{a+bx}\,(c+dx)^n(e+fx)^p\,dx}{b} + \frac{(Ab-aB)\int \frac{(c+dx)^n(e+fx)^p}{\sqrt{a+bx}}\,dx}{b}$$

$$= \frac{\left(B(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\right)\int \sqrt{a+bx}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n(e+fx)^p\,dx}{b} + \frac{\Big((Ab-a}{}$$

$$= \frac{\left(B(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}\right)\int \sqrt{a+bx}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n\Big(\frac{b}{be-}}{b}$$

$$= \frac{2(Ab-aB)\sqrt{a+bx}\,(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\Big(\frac{1}{2};-n,-p;\frac{3}{2}}{b^2}$$

Mathematica [A] time = 0.23, size = 184, normalized size = 0.74

$$\frac{2\sqrt{a+bx}\,(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}\left(3(Ab-aB)F_1\left(\frac{1}{2};-n,-p;\frac{3}{2};\frac{d(a+bx)}{ad-bc},\frac{f(a+bx)}{af-be}\right)+B(a+bx)F_1\Big(\frac{3}{2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*(c + d*x)^n*(e + f*x)^p)/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(c + d*x)^n*(e + f*x)^p*(3*(A*b - a*B)*AppellF1[1/2, -n, -
p, 3/2, (d*(a + b*x))/(-(b*c) + a*d), (f*(a + b*x))/(-(b*e) + a*f)] + B*(a
+ b*x)*AppellF1[3/2, -n, -p, 5/2, (d*(a + b*x))/(-(b*c) + a*d), (f*(a + b*x
))/(-(b*e) + a*f)]))/(3*b^2*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b
*e - a*f))^p)
```

fricas [F] time = 1.03, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(Bx+A)(dx+c)^n\left(fx+e\right)^p}{\sqrt{bx+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(d*x+c)^n*(f*x+e)^p/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x + A)*(d*x + c)^n*(f*x + e)^p/sqrt(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx+A)(dx+c)^n\left(fx+e\right)^p}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(d*x+c)^n*(f*x+e)^p/(b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate((B*x + A)*(d*x + c)^n*(f*x + e)^p/sqrt(b*x + a), x)
```

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{(Bx + A)(dx + c)^{n} \left(fx + e\right)^{p}}{\sqrt{bx + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(d*x+c)^n*(f*x+e)^p/(b*x+a)^(1/2),x)
[Out] int((B*x+A)*(d*x+c)^n*(f*x+e)^p/(b*x+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx + A)(dx + c)^{n}\left(fx + e\right)^{p}}{\sqrt{bx + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(d*x+c)^n*(f*x+e)^p/(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x + A)*(d*x + c)^n*(f*x + e)^p/sqrt(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e + f\,x\right)^{p}\,\left(A + B\,x\right)\,\left(c + d\,x\right)^{n}}{\sqrt{a + b\,x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)^p*(A + B*x)*(c + d*x)^n)/(a + b*x)^(1/2),x)
[Out] int(((e + f*x)^p*(A + B*x)*(c + d*x)^n)/(a + b*x)^(1/2), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(d*x+c)**n*(f*x+e)**p/(b*x+a)**(1/2),x)
[Out] Exception raised: HeuristicGCDFailed
```

3.139 $\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)^3\,dx$

Optimal. Leaf size=530

$$\frac{3h^2(bg-ah)(a+bx)^{m+3}(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(m+3;-n,-p;m+4;-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b^4(m+3)}+\cdots$$

```
[Out] (-a*h+b*g)^3*(b*x+a)^(1+m)*(d*x+c)^n*(f*x+e)^p*AppellF1(1+m,-n,-p,2+m,-d*(b
*x+a)/(-a*d+b*c),-f*(b*x+a)/(-a*f+b*e))/b^4/(1+m)/((b*(d*x+c)/(-a*d+b*c))^n
)/((b*(f*x+e)/(-a*f+b*e))^p)+3*h*(-a*h+b*g)^2*(b*x+a)^(2+m)*(d*x+c)^n*(f*x+
e)^p*AppellF1(2+m,-n,-p,3+m,-d*(b*x+a)/(-a*d+b*c),-f*(b*x+a)/(-a*f+b*e))/b^
4/(2+m)/((b*(d*x+c)/(-a*d+b*c))^n)/((b*(f*x+e)/(-a*f+b*e))^p)+3*h^2*(-a*h+b
*g)*(b*x+a)^(3+m)*(d*x+c)^n*(f*x+e)^p*AppellF1(3+m,-n,-p,4+m,-d*(b*x+a)/(-a
*d+b*c),-f*(b*x+a)/(-a*f+b*e))/b^4/(3+m)/((b*(d*x+c)/(-a*d+b*c))^n)/((b*(f*
x+e)/(-a*f+b*e))^p)+h^3*(b*x+a)^(4+m)*(d*x+c)^n*(f*x+e)^p*AppellF1(4+m,-n,-
p,5+m,-d*(b*x+a)/(-a*d+b*c),-f*(b*x+a)/(-a*f+b*e))/b^4/(4+m)/((b*(d*x+c)/(-
a*d+b*c))^n)/((b*(f*x+e)/(-a*f+b*e))^p)
```

Rubi [A] time = 1.18, antiderivative size = 530, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {181, 159, 140, 139, 138}

$$\frac{3h^2(bg-ah)(a+bx)^{m+3}(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(m+3;-n,-p;m+4;-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b^4(m+3)}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^3,x]
```

```
[Out] ((b*g - a*h)^3*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[1 + m, -n
, -p, 2 + m, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(
b^4*(1 + m)*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p) +
(3*h*(b*g - a*h)^2*(a + b*x)^(2 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[2 + m
, -n, -p, 3 + m, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))
])/(b^4*(2 + m)*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p
) + (3*h^2*(b*g - a*h)*(a + b*x)^(3 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[3
 + m, -n, -p, 4 + m, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a
*f))])/(b^4*(3 + m)*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f
))^p) + (h^3*(a + b*x)^(4 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[4 + m, -n,
-p, 5 + m, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b^
4*(4 + m)*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m + 1)*(
c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[(a + b*x)^m*(c + d
*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] &&
 (SumSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))
```

Rule 181

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p*(g + h*x)^(q - 1), x], x] + Dist[(b*g - a*h)/b,
 Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^(q - 1), x], x] /; FreeQ
[{a, b, c, d, e, f, g, h, m, n, p}, x] && IGtQ[q, 0] && (SumSimplerQ[m, 1]
|| ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))
```

Rubi steps

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)^3\,dx = \frac{h\int (a+bx)^{1+m}(c+dx)^n(e+fx)^p(g+hx)^2\,dx}{b} + \frac{(bg-ah)\int (a+ \cdots}{\cdots}$$

$$= \frac{h^2\int (a+bx)^{2+m}(c+dx)^n(e+fx)^p(g+hx)\,dx}{b^2} + 2\frac{(h(bg-ah))\int \cdots}{\cdots}$$

$$= \frac{h^3\int (a+bx)^{3+m}(c+dx)^n(e+fx)^p\,dx}{b^3} + \frac{\left(h^2(bg-ah)\right)\int (a+bx)^{\cdots}}{b^{\cdots}}$$

$$= \frac{\left(h^3(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\right)\int (a+bx)^{3+m}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n(e+fx)^p \cdots}{b^3}$$

$$= \frac{\left(h^3(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}\right)\int (a+bx)^{3+m}\left(\frac{bc}{bc-a\cdots}\right.\cdots}{b^3}$$

$$= \frac{(bg-ah)^3(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}F\cdots}{b^4(1+m)}$$

Mathematica [F] time = 4.53, size = 0, normalized size = 0.00

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)^3\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^3,x]
```

```
[Out] Integrate[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^3, x]
```

fricas [F] time = 3.37, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(h^3x^3+3gh^2x^2+3g^2hx+g^3\right)(bx+a)^m(dx+c)^n\left(fx+e\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g)^3,x, algorithm="fricas")
```

```
[Out] integral((h^3*x^3 + 3*g*h^2*x^2 + 3*g^2*h*x + g^3)*(b*x + a)^m*(d*x + c)^n*
(f*x + e)^p, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \left(hx+g\right)^{3}(bx+a)^{m}(dx+c)^{n}\left(fx+e\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g)^3,x)
```

```
[Out] int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(hx+g\right)^{3}(bx+a)^{m}(dx+c)^{n}\left(fx+e\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g)^3,x, algorithm="maxima")
```

```
[Out] integrate((h*x + g)^3*(b*x + a)^m*(d*x + c)^n*(f*x + e)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(e+f\,x\right)}^p\,{\left(g+h\,x\right)}^3\,{\left(a+b\,x\right)}^m\,{\left(c+d\,x\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^p*(g + h*x)^3*(a + b*x)^m*(c + d*x)^n,x)
```

```
[Out] int((e + f*x)^p*(g + h*x)^3*(a + b*x)^m*(c + d*x)^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**n*(f*x+e)**p*(h*x+g)**3,x)
```

```
[Out] Timed out
```

3.140 $\int (a+bx)^m (c+dx)^n (e+fx)^p (g+hx)^2 \, dx$

Optimal. Leaf size=393

$$\frac{(bg-ah)^2(a+bx)^{m+1}(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(m+1;-n,-p;m+2;-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b^3(m+1)}+\cdots 2$$

```
[Out] (-a*h+b*g)^2*(b*x+a)^(1+m)*(d*x+c)^n*(f*x+e)^p*AppellF1(1+m,-n,-p,2+m,-d*(b
*x+a)/(-a*d+b*c),-f*(b*x+a)/(-a*f+b*e))/b^3/(1+m)/((b*(d*x+c)/(-a*d+b*c))^n
)/((b*(f*x+e)/(-a*f+b*e))^p)+2*h*(-a*h+b*g)*(b*x+a)^(2+m)*(d*x+c)^n*(f*x+e)
^p*AppellF1(2+m,-n,-p,3+m,-d*(b*x+a)/(-a*d+b*c),-f*(b*x+a)/(-a*f+b*e))/b^3/
(2+m)/((b*(d*x+c)/(-a*d+b*c))^n)/((b*(f*x+e)/(-a*f+b*e))^p)+h^2*(b*x+a)^(3+
m)*(d*x+c)^n*(f*x+e)^p*AppellF1(3+m,-n,-p,4+m,-d*(b*x+a)/(-a*d+b*c),-f*(b*x
+a)/(-a*f+b*e))/b^3/(3+m)/((b*(d*x+c)/(-a*d+b*c))^n)/((b*(f*x+e)/(-a*f+b*e)
)^p)
```

Rubi [A] time = 0.49, antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {181, 159, 140, 139, 138}

$$\frac{(bg-ah)^2(a+bx)^{m+1}(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(m+1;-n,-p;m+2;-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b^3(m+1)}+\cdots 2$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^2,x]
```

```
[Out] ((b*g - a*h)^2*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[1 + m, -n
, -p, 2 + m, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(
b^3*(1 + m)*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p) +
(2*h*(b*g - a*h)*(a + b*x)^(2 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[2 + m,
-n, -p, 3 + m, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])
/(b^3*(2 + m)*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p)
+ (h^2*(a + b*x)^(3 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[3 + m, -n, -p, 4
+ m, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b^3*(3 +
 m)*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m + 1)*(
c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[(a + b*x)^m*(c + d
*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] &&
 (SumSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] && !SumSimplerQ[p, 1]))
```

Rule 181

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p*(g + h*x)^(q - 1), x], x] + Dist[(b*g - a*h)/b,
 Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^(q - 1), x], x] /; FreeQ
[{a, b, c, d, e, f, g, h, m, n, p}, x] && IGtQ[q, 0] && (SumSimplerQ[m, 1]
|| ( !SumSimplerQ[n, 1] && !SumSimplerQ[p, 1]))
```

Rubi steps

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)^2\,dx = \frac{h\int (a+bx)^{1+m}(c+dx)^n(e+fx)^p(g+hx)\,dx}{b} + \frac{(bg-ah)\int (a+bx)^{\cdots}}{\cdots}$$

$$= \frac{h^2\int (a+bx)^{2+m}(c+dx)^n(e+fx)^p\,dx}{b^2} + 2\frac{(h(bg-ah))\int (a+bx)^{1+m\cdots}}{b^2}$$

$$= \frac{\left(h^2(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\right)\int (a+bx)^{2+m}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n(e+fx)^p\,dx\cdots}{b^2}$$

$$= \frac{\left(h^2(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}\right)\int (a+bx)^{2+m}\left(\frac{bc}{bc-ad}+\cdots\right.}{b^2}$$

$$= \frac{(bg-ah)^2(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(\cdots\right.}{b^3(1+m)}$$

Mathematica [F] time = 1.53, size = 0, normalized size = 0.00

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)^2\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^2,x]
[Out] Integrate[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^2, x]
```

fricas [F] time = 3.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(h^2x^2+2\,ghx+g^2\right)(bx+a)^m(dx+c)^n\left(fx+e\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g)^2,x, algorithm="fricas")
[Out] integral((h^2*x^2 + 2*g*h*x + g^2)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^p, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g)^2,x, algorithm="giac")
[Out] Timed out
```

maple [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \left(hx+g\right)^{2} \left(bx+a\right)^{m} \left(dx+c\right)^{n} \left(fx+e\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g)^2,x)
[Out] int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int {\left(hx+g\right)}^{2} {\left(bx+a\right)}^{m} {\left(dx+c\right)}^{n} {\left(fx+e\right)}^{p}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g)^2,x, algorithm="maxima")
[Out] integrate((h*x + g)^2*(b*x + a)^m*(d*x + c)^n*(f*x + e)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(e+f\,x\right)}^p\,{\left(g+h\,x\right)}^2\,{\left(a+b\,x\right)}^m\,{\left(c+d\,x\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^p*(g + h*x)^2*(a + b*x)^m*(c + d*x)^n,x)
[Out] int((e + f*x)^p*(g + h*x)^2*(a + b*x)^m*(c + d*x)^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**n*(f*x+e)**p*(h*x+g)**2,x)
[Out] Timed out
```

3.141 $\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)\,dx$

Optimal. Leaf size=256

$$\frac{(bg-ah)(a+bx)^{m+1}(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(m+1;-n,-p;m+2;-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b^2(m+1)}+\frac{h(a+$$

```
[Out] (-a*h+b*g)*(b*x+a)^(1+m)*(d*x+c)^n*(f*x+e)^p*AppellF1(1+m,-n,-p,2+m,-d*(b*x
+a)/(-a*d+b*c),-f*(b*x+a)/(-a*f+b*e))/b^2/(1+m)/((b*(d*x+c)/(-a*d+b*c))^n)/
((b*(f*x+e)/(-a*f+b*e))^p)+h*(b*x+a)^(2+m)*(d*x+c)^n*(f*x+e)^p*AppellF1(2+m
,-n,-p,3+m,-d*(b*x+a)/(-a*d+b*c),-f*(b*x+a)/(-a*f+b*e))/b^2/(2+m)/((b*(d*x+
c)/(-a*d+b*c))^n)/((b*(f*x+e)/(-a*f+b*e))^p)
```

Rubi [A] time = 0.21, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {159, 140, 139, 138}

$$\frac{(bg-ah)(a+bx)^{m+1}(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(m+1;-n,-p;m+2;-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b^2(m+1)}+\frac{h(a+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x),x]
```

```
[Out] ((b*g - a*h)*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[1 + m, -n,
-p, 2 + m, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b^
2*(1 + m)*((b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p) + (h
*(a + b*x)^(2 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[2 + m, -n, -p, 3 + m, -
((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b^2*(2 + m)*((
b*(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
```

```
b*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m + 1)*(
c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[(a + b*x)^m*(c + d
*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] &&
 (SumSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))
```

Rubi steps

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)\,dx = \frac{h\int (a+bx)^{1+m}(c+dx)^n(e+fx)^p\,dx}{b} + \frac{(bg-ah)\int (a+bx)^m(c+
$$

$$= \frac{\left(h(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\right)\int (a+bx)^{1+m}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n(e+fx)^p\,d
}{b}$$

$$= \frac{\left(h(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}\right)\int (a+bx)^{1+m}\left(\frac{bc}{bc-ad}\right.}{b}$$

$$= \frac{(bg-ah)(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1(}{b^2(1+m)}$$

Mathematica [F] time = 0.83, size = 0, normalized size = 0.00

$$\int (a+bx)^m(c+dx)^n(e+fx)^p(g+hx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x),x]
[Out] Integrate[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x), x]
```

fricas [F] time = 2.27, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(hx+g\right)(bx+a)^m(dx+c)^n\left(fx+e\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g),x, algorithm="fricas")
[Out] integral((h*x + g)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(hx+g\right)(bx+a)^m(dx+c)^n\left(fx+e\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g),x, algorithm="giac")
[Out] integrate((h*x + g)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^p, x)
```

maple [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \left(hx+g\right)(bx+a)^m(dx+c)^n\left(fx+e\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g),x)
[Out] int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (hx+g)(bx+a)^m(dx+c)^n(fx+e)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p*(h*x+g),x, algorithm="maxima")
[Out] integrate((h*x + g)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (e+f\,x)^p\,(g+h\,x)\,(a+b\,x)^m\,(c+d\,x)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^p*(g + h*x)*(a + b*x)^m*(c + d*x)^n,x)
[Out] int((e + f*x)^p*(g + h*x)*(a + b*x)^m*(c + d*x)^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**n*(f*x+e)**p*(h*x+g),x)
[Out] Timed out
```

3.142 $\int (a+bx)^m (c+dx)^n (e+fx)^p \, dx$

Optimal. Leaf size=123

$$\frac{(a+bx)^{m+1}(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(m+1;-n,-p;m+2;-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b(m+1)}$$

```
[Out] (b*x+a)^(1+m)*(d*x+c)^n*(f*x+e)^p*AppellF1(1+m,-n,-p,2+m,-d*(b*x+a)/(-a*d+b
*c),-f*(b*x+a)/(-a*f+b*e))/b/(1+m)/((b*(d*x+c)/(-a*d+b*c))^n)/((b*(f*x+e)/(
-a*f+b*e))^p)
```

Rubi [A] time = 0.08, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {140, 139, 138}

$$\frac{(a+bx)^{m+1}(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(m+1;-n,-p;m+2;-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p,x]
```

```
[Out] ((a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[1 + m, -n, -p, 2 + m, -
((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(1 + m)*((b*
(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rubi steps

$$\int (a+bx)^m(c+dx)^n(e+fx)^p\,dx = \left((c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\right)\int (a+bx)^m\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n(e+fx)^p\,dx$$
$$= \left((c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}\right)\int (a+bx)^m\left(\frac{bc}{bc-ad}+\frac{b}{bc}\right.$$
$$= \frac{(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^p\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(1+m;-n,-p;2+m\right.}{b(1+m)}$$

Mathematica [A] time = 0.13, size = 121, normalized size = 0.98

$$\frac{(a+bx)^{m+1}(c+dx)^n(e+fx)^p\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{-p}F_1\left(m+1;-n,-p;m+2;\frac{d(a+bx)}{ad-bc},\frac{f(a+bx)}{af-be}\right)}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p,x]
```

```
[Out] ((a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^p*AppellF1[1 + m, -n, -p, 2 + m, (
d*(a + b*x))/(-(b*c) + a*d), (f*(a + b*x))/(-(b*e) + a*f)])/(b*(1 + m)*((b*
(c + d*x))/(b*c - a*d))^n*((b*(e + f*x))/(b*e - a*f))^p)
```

fricas [F] time = 1.06, size = 0, normalized size = 0.00

$$\text{integral}\left((bx+a)^m(dx+c)^n\left(fx+e\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x + a)^m*(d*x + c)^n*(f*x + e)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^m(dx+c)^n\left(fx+e\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^m*(d*x + c)^n*(f*x + e)^p, x)
```

maple [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^m\,(dx+c)^n\,\left(fx+e\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p,x)
```

```
[Out] int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^m(dx+c)^n\left(fx+e\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^m*(d*x + c)^n*(f*x + e)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(e + f\,x\right)^p\,{\left(a + b\,x\right)}^m\,{\left(c + d\,x\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^p*(a + b*x)^m*(c + d*x)^n,x)
```

```
[Out] int((e + f*x)^p*(a + b*x)^m*(c + d*x)^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**n*(f*x+e)**p,x)
```

```
[Out] Timed out
```

3.143 $\int \frac{(a+bx)^m(c+dx)^n(e+fx)^p}{g+hx}\,dx$

Optimal. Leaf size=32

$$\text{Int}\left(\frac{(a+bx)^m(c+dx)^n(e+fx)^p}{g+hx},x\right)$$

```
[Out] CannotIntegrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p/(h*x+g),x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^p}{g+hx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((a + b*x)^m*(c + d*x)^n*(e + f*x)^p)/(g + h*x),x]
[Out] Defer[Int][((a + b*x)^m*(c + d*x)^n*(e + f*x)^p)/(g + h*x), x]
```

Rubi steps

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^p}{g+hx}\,dx = \int \frac{(a+bx)^m(c+dx)^n(e+fx)^p}{g+hx}\,dx$$

Mathematica [A] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^m(c+dx)^n(e+fx)^p}{g+hx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((a + b*x)^m*(c + d*x)^n*(e + f*x)^p)/(g + h*x),x]
[Out] Integrate[((a + b*x)^m*(c + d*x)^n*(e + f*x)^p)/(g + h*x), x]
```

fricas [A] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(bx+a)^m(dx+c)^n\left(fx+e\right)^p}{hx+g},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p/(h*x+g),x, algorithm="fricas")
[Out] integral((b*x + a)^m*(d*x + c)^n*(f*x + e)^p/(h*x + g), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^m(dx+c)^n\left(fx+e\right)^p}{hx+g}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p/(h*x+g),x, algorithm="giac")
[Out] integrate((b*x + a)^m*(d*x + c)^n*(f*x + e)^p/(h*x + g), x)
```

maple [A] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^m (dx+c)^n \left(fx+e\right)^p}{hx+g}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p/(h*x+g),x)
[Out] int((b*x+a)^m*(d*x+c)^n*(f*x+e)^p/(h*x+g),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^m (dx+c)^n \left(fx+e\right)^p}{hx+g}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^n*(f*x+e)^p/(h*x+g),x, algorithm="maxima")
[Out] integrate((b*x + a)^m*(d*x + c)^n*(f*x + e)^p/(h*x + g), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(e+f\,x\right)^p\,(a+b\,x)^m\,(c+d\,x)^n}{g+h\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)^p*(a + b*x)^m*(c + d*x)^n)/(g + h*x),x)
[Out] int(((e + f*x)^p*(a + b*x)^m*(c + d*x)^n)/(g + h*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**n*(f*x+e)**p/(h*x+g),x)
[Out] Timed out
```

3.144 $\int (a+bx)^m (A+Bx)(c+dx)^n (e+fx)^{-m-n}\,dx$

Optimal. Leaf size=268

$$\frac{(Ab-aB)(a+bx)^{m+1}(c+dx)^n(e+fx)^{-m-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}F_1\left(m+1;-n,m+n;m+2;-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b^2(m+1)}$$

```
[Out] (A*b-B*a)*(b*x+a)^(1+m)*(d*x+c)^n*(f*x+e)^(-m-n)*(b*(f*x+e)/(-a*f+b*e))^(m+
n)*AppellF1(1+m,-n,m+n,2+m,-d*(b*x+a)/(-a*d+b*c),-f*(b*x+a)/(-a*f+b*e))/b^2
/(1+m)/((b*(d*x+c)/(-a*d+b*c))^n)+B*(b*x+a)^(2+m)*(d*x+c)^n*(f*x+e)^(-m-n)*
(b*(f*x+e)/(-a*f+b*e))^(m+n)*AppellF1(2+m,-n,m+n,3+m,-d*(b*x+a)/(-a*d+b*c),
-f*(b*x+a)/(-a*f+b*e))/b^2/(2+m)/((b*(d*x+c)/(-a*d+b*c))^n)
```

Rubi [A] time = 0.21, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {159, 140, 139, 138}

$$\frac{(Ab-aB)(a+bx)^{m+1}(c+dx)^n(e+fx)^{-m-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}F_1\left(m+1;-n,m+n;m+2;-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{b^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-m - n),x]
```

```
[Out] ((A*b - a*B)*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-m - n)*((b*(e + f*x)
)/(b*e - a*f))^(m + n)*AppellF1[1 + m, -n, m + n, 2 + m, -((d*(a + b*x))/(b
*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b^2*(1 + m)*((b*(c + d*x))/(b*c
 - a*d))^n) + (B*(a + b*x)^(2 + m)*(c + d*x)^n*(e + f*x)^(-m - n)*((b*(e +
f*x))/(b*e - a*f))^(m + n)*AppellF1[2 + m, -n, m + n, 3 + m, -((d*(a + b*x)
)/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b^2*(2 + m)*((b*(c + d*x))/
(b*c - a*d))^n)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
```

```
(b*c - a*d), 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !SimplerQ[e + f*x, a +
b*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m + 1)*(
c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[(a + b*x)^m*(c + d
*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] &&
 (SumSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))
```

Rubi steps

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-m-n}\,dx = \frac{B\int (a+bx)^{1+m}(c+dx)^n(e+fx)^{-m-n}\,dx}{b} + \frac{(Ab-aB)\int (a+}{}$$

$$= \frac{\left(B(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\right)\int (a+bx)^{1+m}\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n(e+f}{b}$$

$$= \frac{\left(B(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^{-m-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}\right)\int (a+bx)^{1}}{b}$$

$$= \frac{(Ab-aB)(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^{-m-n}\left(\frac{b(e+}{be-}\right.}{b^2(1+}$$

Mathematica [F] time = 0.65, size = 0, normalized size = 0.00

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-m-n}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-m - n),x]
[Out] Integrate[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-m - n), x]
```

fricas [F] time = 1.19, size = 0, normalized size = 0.00

$$\text{integral}\left((Bx+A)(bx+a)^m(dx+c)^n\left(fx+e\right)^{-m-n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-m-n),x, algorithm="fricas")
[Out] integral((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (Bx+A)(bx+a)^m(dx+c)^n\left(fx+e\right)^{-m-n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-m-n),x, algorithm="giac")
[Out] integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n), x)
```

maple [F] time = 0.25, size = 0, normalized size = 0.00

$$\int (Bx + A)(bx + a)^m (dx + c)^n \left(fx + e\right)^{-m-n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-m-n),x)
[Out] int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-m-n),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (Bx + A)(bx + a)^m (dx + c)^n \left(fx + e\right)^{-m-n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-m-n),x, algorithm="maxima")
[Out] integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B\,x)\,(a + b\,x)^m\,(c + d\,x)^n}{\left(e + f\,x\right)^{m+n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x)*(a + b*x)^m*(c + d*x)^n)/(e + f*x)^(m + n),x)
[Out] int(((A + B*x)*(a + b*x)^m*(c + d*x)^n)/(e + f*x)^(m + n), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(B*x+A)*(d*x+c)**n*(f*x+e)**(-m-n),x)
[Out] Timed out
```

3.145 $\int (a+bx)^m (A+Bx)(c+dx)^n (e+fx)^{-1-m-n}\,dx$

Optimal. Leaf size=283

$$\frac{B(a+bx)^{m+1}(c+dx)^n(e+fx)^{-m-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}F_1\left(m+1;-n,m+n;m+2;-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{bf(m+1)}-\cdots$$

```
[Out] B*(b*x+a)^(1+m)*(d*x+c)^n*(f*x+e)^(-m-n)*(b*(f*x+e)/(-a*f+b*e))^(m+n)*Appel
lF1(1+m,-n,m+n,2+m,-d*(b*x+a)/(-a*d+b*c),-f*(b*x+a)/(-a*f+b*e))/b/f/(1+m)/(
(b*(d*x+c)/(-a*d+b*c))^n)-(-A*f+B*e)*(b*x+a)^(1+m)*(d*x+c)^n*(f*x+e)^(-m-n)
*(b*(f*x+e)/(-a*f+b*e))^(m+n)*AppellF1(1+m,-n,1+m+n,2+m,-d*(b*x+a)/(-a*d+b*
c),-f*(b*x+a)/(-a*f+b*e))/f/(-a*f+b*e)/(1+m)/((b*(d*x+c)/(-a*d+b*c))^n)
```

Rubi [A] time = 0.21, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {159, 140, 139, 138}

$$\frac{B(a+bx)^{m+1}(c+dx)^n(e+fx)^{-m-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}F_1\left(m+1;-n,m+n;m+2;-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{bf(m+1)}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-1 - m - n),x]
```

```
[Out] (B*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-m - n)*((b*(e + f*x))/(b*e - a
*f))^(m + n)*AppellF1[1 + m, -n, m + n, 2 + m, -((d*(a + b*x))/(b*c - a*d))
, -((f*(a + b*x))/(b*e - a*f))])/(b*f*(1 + m)*((b*(c + d*x))/(b*c - a*d))^n
) - ((B*e - A*f)*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-m - n)*((b*(e +
f*x))/(b*e - a*f))^(m + n)*AppellF1[1 + m, -n, 1 + m + n, 2 + m, -((d*(a +
b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(f*(b*e - a*f)*(1 + m)*(
(b*(c + d*x))/(b*c - a*d))^n)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &&  !GtQ[b/
```

```
(b*c - a*d), 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !SimplerQ[e + f*x, a +
b*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m + 1)*(
c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[(a + b*x)^m*(c + d
*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] &&
 (SumSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))
```

Rubi steps

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-1-m-n}\,dx = \frac{B\int (a+bx)^m(c+dx)^n(e+fx)^{-m-n}\,dx}{f} + \frac{(-Be+Af)\int (a+b\!\!\!\!}{}$$

$$= \frac{\left(B(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\right)\int (a+bx)^m\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n(e+fx)^{-}}{f}$$

$$= \frac{\left(B(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^{-m-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}\right)\int (a+bx)^m}{f}$$

$$= \frac{B(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^{-m-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}F_1}{bf(1+m)}$$

Mathematica [A] time = 0.33, size = 208, normalized size = 0.73

$$\frac{(a+bx)^{m+1}(c+dx)^n(e+fx)^{-m-n+1}\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n-1}\left(b(Af-Be)F_1\left(m+1;-n,m+n+1;m+2;\frac{d(a+bx}{ad-bc}\right.\right.}{f(m+1)(be-af)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-1 - m - n),x]
```

```
[Out] ((a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(1 - m - n)*((b*(e + f*x))/(b*e -
a*f))^(-1 + m + n)*(B*(b*e - a*f)*AppellF1[1 + m, -n, m + n, 2 + m, (d*(a +
 b*x))/(-(b*c) + a*d), (f*(a + b*x))/(-(b*e) + a*f)] + b*(-(B*e) + A*f)*App
ellF1[1 + m, -n, 1 + m + n, 2 + m, (d*(a + b*x))/(-(b*c) + a*d), (f*(a + b*
x))/(-(b*e) + a*f)]))/(f*(b*e - a*f)^2*(1 + m)*((b*(c + d*x))/(b*c - a*d))^
n)
```

fricas [F] time = 1.24, size = 0, normalized size = 0.00

$$\text{integral}\left((Bx+A)(bx+a)^m(dx+c)^n\left(fx+e\right)^{-m-n-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-1-m-n),x, algorithm="fricas
")
```

```
[Out] integral((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (Bx+A)(bx+a)^m(dx+c)^n\left(fx+e\right)^{-m-n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-1-m-n),x, algorithm="giac")
```

```
[Out] integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 1), x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int (Bx + A)(bx + a)^m (dx + c)^n (fx + e)^{-m-n-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-m-n-1),x)
```

```
[Out] int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-m-n-1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (Bx + A)(bx + a)^m (dx + c)^n (fx + e)^{-m-n-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-1-m-n),x, algorithm="maxima
")
```

```
[Out] integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B\,x)\,(a + b\,x)^m\,(c + d\,x)^n}{(e + f\,x)^{m+n+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x)*(a + b*x)^m*(c + d*x)^n)/(e + f*x)^(m + n + 1),x)
```

```
[Out] int(((A + B*x)*(a + b*x)^m*(c + d*x)^n)/(e + f*x)^(m + n + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(B*x+A)*(d*x+c)**n*(f*x+e)**(-1-m-n),x)
```

```
[Out] Timed out
```

3.146 $\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-2-m-n}\,dx$

Optimal. Leaf size=277

$$\frac{B(a+bx)^{m+1}(c+dx)^n(e+fx)^{-m-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}F_1\left(m+1;-n,m+n+1;m+2;-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{f(m+1)(be-af)}$$

```
[Out] B*(b*x+a)^(1+m)*(d*x+c)^n*(f*x+e)^(-m-n)*(b*(f*x+e)/(-a*f+b*e))^(m+n)*Appel
lF1(1+m,-n,1+m+n,2+m,-d*(b*x+a)/(-a*d+b*c),-f*(b*x+a)/(-a*f+b*e))/f/(-a*f+b
*e)/(1+m)/((b*(d*x+c)/(-a*d+b*c))^n)-(-A*f+B*e)*(b*x+a)^(1+m)*(d*x+c)^n*(f*
x+e)^(-1-m-n)*hypergeom([-n, 1+m],[2+m],-(-c*f+d*e)*(b*x+a)/(-a*d+b*c)/(f*x
+e))/f/(-a*f+b*e)/(1+m)/(((-a*f+b*e)*(d*x+c)/(-a*d+b*c)/(f*x+e))^n)
```

Rubi [A] time = 0.15, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {159, 140, 139, 138, 132}

$$\frac{B(a+bx)^{m+1}(c+dx)^n(e+fx)^{-m-n}\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+n}F_1\left(m+1;-n,m+n+1;m+2;-\frac{d(a+bx)}{bc-ad},-\frac{f(a+bx)}{be-af}\right)}{f(m+1)(be-af)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-2 - m - n),x]
```

```
[Out] (B*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-m - n)*((b*(e + f*x))/(b*e - a
*f))^(m + n)*AppellF1[1 + m, -n, 1 + m + n, 2 + m, -((d*(a + b*x))/(b*c - a
*d)), -((f*(a + b*x))/(b*e - a*f))])/(f*(b*e - a*f)*(1 + m)*((b*(c + d*x))/
(b*c - a*d))^n) - ((B*e - A*f)*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-1
- m - n)*Hypergeometric2F1[1 + m, -n, 2 + m, -(((d*e - c*f)*(a + b*x))/((b*
c - a*d)*(e + f*x)))])/(f*(b*e - a*f)*(1 + m)*(((b*e - a*f)*(c + d*x))/((b*
c - a*d)*(e + f*x)))^n)
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] && !IntegerQ[n]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &&  !GtQ[b/
(b*c - a*d), 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !SimplerQ[e + f*x, a +
b*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(a + b*x)^(m + 1)*(
c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[(a + b*x)^m*(c + d
*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p}, x] &&
 (SumSimplerQ[m, 1] || ( !SumSimplerQ[n, 1] &&  !SumSimplerQ[p, 1]))
```

Rubi steps

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-2-m-n}\,dx = \frac{B\int (a+bx)^m(c+dx)^n(e+fx)^{-1-m-n}\,dx}{f} + \frac{(-Be+Af)\int (a$$

$$= -\frac{(Be-Af)(a+bx)^{1+m}(c+dx)^n\left(\frac{(be-af)(c+dx)}{(bc-ad)(e+fx)}\right)^{-n}(e+fx)^{-1}}{f(be-af)(1+m}$$

$$= -\frac{(Be-Af)(a+bx)^{1+m}(c+dx)^n\left(\frac{(be-af)(c+dx)}{(bc-ad)(e+fx)}\right)^{-n}(e+fx)^{-1}}{f(be-af)(1+m}$$

$$= \frac{B(a+bx)^{1+m}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}(e+fx)^{-m-n}\left(\frac{b(e+fx)}{be-af}\right)^{m+}}{f(be-af)(1}$$

Mathematica [A] time = 0.44, size = 215, normalized size = 0.78

$$-\frac{(a+bx)^{m+1}(c+dx)^n(e+fx)^{-m-n-1}\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{b(e+fx)}{be-af}\right)^n\left((Af-Be)\,{}_2F_1\left(m+1,-n;m+2;\frac{(cf-de)(a+bx)}{(bc-ad)(e+fx)}\right)+B}{f(m+1)(af-be)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-2 - m - n),x]
```

```
[Out] -(((a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-1 - m - n)*((b*(e + f*x))/(b*e
 - a*f))^n*(B*(e + f*x)*((b*(e + f*x))/(b*e - a*f))^m*AppellF1[1 + m, -n, 1
 + m + n, 2 + m, (d*(a + b*x))/(-(b*c) + a*d), (f*(a + b*x))/(-(b*e) + a*f)
] + (-(B*e) + A*f)*Hypergeometric2F1[1 + m, -n, 2 + m, ((-(d*e) + c*f)*(a +
 b*x))/((b*c - a*d)*(e + f*x))]))/(f*(-(b*e) + a*f)*(1 + m)*((b*(c + d*x))/
(b*c - a*d))^n))
```

fricas [F] time = 1.49, size = 0, normalized size = 0.00

$$\text{integral}\left((Bx+A)(bx+a)^m(dx+c)^n\left(fx+e\right)^{-m-n-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-2-m-n),x, algorithm="fricas
")
```

```
[Out] integral((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (Bx + A)(bx + a)^{m}(dx + c)^{n}\left(fx + e\right)^{-m-n-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-2-m-n),x, algorithm="giac")
```

```
[Out] integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 2), x)
```

maple [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \left(Bx + A\right)\left(bx + a\right)^{m}\left(dx + c\right)^{n}\left(fx + e\right)^{-m-n-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-m-n-2),x)
```

```
[Out] int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-m-n-2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (Bx + A)(bx + a)^{m}(dx + c)^{n}\left(fx + e\right)^{-m-n-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-2-m-n),x, algorithm="maxima
")
```

```
[Out] integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + B\,x\right)\,\left(a + b\,x\right)^{m}\,\left(c + d\,x\right)^{n}}{\left(e + f\,x\right)^{m+n+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x)*(a + b*x)^m*(c + d*x)^n)/(e + f*x)^(m + n + 2),x)
```

```
[Out] int(((A + B*x)*(a + b*x)^m*(c + d*x)^n)/(e + f*x)^(m + n + 2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(B*x+A)*(d*x+c)**n*(f*x+e)**(-2-m-n),x)
```

```
[Out] Timed out
```

3.147 $\int (a+bx)^m (A+Bx)(c+dx)^n (e+fx)^{-3-m-n} \, dx$

Optimal. Leaf size=263

$$\frac{(a+bx)^{m+1}(Be-Af)(c+dx)^{n+1}(e+fx)^{-m-n-2}}{(m+n+2)(be-af)(de-cf)} - \frac{(a+bx)^{m+1}(c+dx)^n(e+fx)^{-m-n-1}\left(\frac{(c+dx)(be-af)}{(e+fx)(bc-ad)}\right)^{-n}(a(Adf(}{}$$

```
[Out] (-A*f+B*e)*(b*x+a)^(1+m)*(d*x+c)^(1+n)*(f*x+e)^(-2-m-n)/(-a*f+b*e)/(-c*f+d*
e)/(2+m+n)-(b*(B*c*e*(1+m)+A*(c*f*(1+n)-d*e*(2+m+n)))+a*(A*d*f*(1+m)+B*(d*e
*(1+n)-c*f*(2+m+n))))*(b*x+a)^(1+m)*(d*x+c)^n*(f*x+e)^(-1-m-n)*hypergeom([-
n, 1+m],[2+m],-(-c*f+d*e)*(b*x+a)/(-a*d+b*c)/(f*x+e))/(-a*f+b*e)^2/(-c*f+d*
e)/(1+m)/(2+m+n)/(((-a*f+b*e)*(d*x+c)/(-a*d+b*c)/(f*x+e))^n)
```

Rubi [A] time = 0.23, antiderivative size = 261, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {155, 12, 132}

$$\frac{(a+bx)^{m+1}(Be-Af)(c+dx)^{n+1}(e+fx)^{-m-n-2}}{(m+n+2)(be-af)(de-cf)} - \frac{(a+bx)^{m+1}(c+dx)^n(e+fx)^{-m-n-1}\left(\frac{(c+dx)(be-af)}{(e+fx)(bc-ad)}\right)^{-n}(a(Adf(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-3 - m - n),x]
```

```
[Out] ((B*e - A*f)*(a + b*x)^(1 + m)*(c + d*x)^(1 + n)*(e + f*x)^(-2 - m - n))/((
b*e - a*f)*(d*e - c*f)*(2 + m + n)) - ((b*(B*c*e*(1 + m) + A*c*f*(1 + n) -
A*d*e*(2 + m + n)) + a*(A*d*f*(1 + m) + B*d*e*(1 + n) - B*c*f*(2 + m + n)))
*(a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-1 - m - n)*Hypergeometric2F1[1 +
 m, -n, 2 + m, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/((b*e -
 a*f)^2*(d*e - c*f)*(1 + m)*(2 + m + n)*(((b*e - a*f)*(c + d*x))/((b*c - a*
d)*(e + f*x)))^n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rubi steps

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-3-m-n}\,dx = \frac{(Be-Af)(a+bx)^{1+m}(c+dx)^{1+n}(e+fx)^{-2-m-n}}{(be-af)(de-cf)(2+m+n)} - \int (b(Bce(1 \ldots$$

$$= \frac{(Be-Af)(a+bx)^{1+m}(c+dx)^{1+n}(e+fx)^{-2-m-n}}{(be-af)(de-cf)(2+m+n)} - (b(Bce(1+ \ldots$$

$$= \frac{(Be-Af)(a+bx)^{1+m}(c+dx)^{1+n}(e+fx)^{-2-m-n}}{(be-af)(de-cf)(2+m+n)} - (b(Bce(1+ \ldots$$

Mathematica [**A**] time = 0.27, size = 223, normalized size = 0.85

$$-\frac{(a+bx)^{m+1}(c+dx)^n(e+fx)^{-m-n-2}\left(\frac{(e+fx)\left(\frac{(c+dx)(be-af)}{(e+fx)(bc-ad)}\right)^{-n}(a(Adf(m+1)-Bcf(m+n+2)+Bde(n+1))+b(Acf(n+1)-Ade(m+n+2)+Bce(\ldots}{(m+1)(be-af)} \ldots\right.}{(m+n+2)(be-af)(de-cf)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-3 - m - n),x]
```

```
[Out] -(((a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-2 - m - n)*((-(B*e) + A*f)*(c
+ d*x) + ((b*(B*c*e*(1 + m) + A*c*f*(1 + n) - A*d*e*(2 + m + n)) + a*(A*d*f
*(1 + m) + B*d*e*(1 + n) - B*c*f*(2 + m + n)))*(e + f*x)*Hypergeometric2F1[
1 + m, -n, 2 + m, ((-(d*e) + c*f)*(a + b*x))/((b*c - a*d)*(e + f*x))])/((b*
e - a*f)*(1 + m)*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*(e + f*x)))^n)))/((b
*e - a*f)*(d*e - c*f)*(2 + m + n)))
```

fricas [**F**] time = 2.17, size = 0, normalized size = 0.00

$$\text{integral}\left((Bx+A)(bx+a)^m(dx+c)^n\left(fx+e\right)^{-m-n-3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-3-m-n),x, algorithm="fricas
")
```

```
[Out] integral((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 3), x)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int (Bx+A)(bx+a)^m(dx+c)^n\left(fx+e\right)^{-m-n-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-3-m-n),x, algorithm="giac")
```

```
[Out] integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 3), x)
```

maple [**F**] time = 0.25, size = 0, normalized size = 0.00

$$\int (Bx+A)\,(bx+a)^m\,(dx+c)^n\,\left(fx+e\right)^{-m-n-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-m-n-3),x)
```

```
[Out] int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-m-n-3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (Bx + A)(bx + a)^m (dx + c)^n (fx + e)^{-m-n-3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-3-m-n),x, algorithm="maxima
")
```

```
[Out] integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B\,x)\,(a + b\,x)^m\,(c + d\,x)^n}{\left(e + f\,x\right)^{m+n+3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x)*(a + b*x)^m*(c + d*x)^n)/(e + f*x)^(m + n + 3),x)
```

```
[Out] int(((A + B*x)*(a + b*x)^m*(c + d*x)^n)/(e + f*x)^(m + n + 3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(B*x+A)*(d*x+c)**n*(f*x+e)**(-3-m-n),x)
```

```
[Out] Timed out
```

3.148 $\int (a+bx)^m (A+Bx)(c+dx)^n (e+fx)^{-4-m-n}\,dx$

Optimal. Leaf size=558

$$(a+bx)^{m+1}(c+dx)^n(e+fx)^{-m-n-1}\left(\frac{(c+dx)(be-af)}{(e+fx)(bc-ad)}\right)^{-n}((m+n+2)(bde((m+n+3)(A(-adf-bcf+bde)+aBcf)$$

```
[Out] (-A*f+B*e)*(b*x+a)^(1+m)*(d*x+c)^(1+n)*(f*x+e)^(-3-m-n)/(-a*f+b*e)/(-c*f+d*
e)/(3+m+n)+(a*f*(A*d*f*(2+m)+B*(d*e*(1+n)-c*f*(3+m+n)))+b*(B*e*(d*e+c*f*(1+
m))+A*f*(c*f*(2+n)-d*e*(4+m+n))))*(b*x+a)^(1+m)*(d*x+c)^(1+n)*(f*x+e)^(-2-m
-n)/(-a*f+b*e)^2/(-c*f+d*e)^2/(2+m+n)/(3+m+n)+((2+m+n)*(a*b*c*d*f*(-A*f+B*e
)+b*d*e*((a*B*c*f+A*(-a*d*f-b*c*f+b*d*e))*(3+m+n)-(-A*f+B*e)*(b*c*(1+m)+a*d
*(1+n)))-(a*d+b*c)*f*((a*B*c*f+A*(-a*d*f-b*c*f+b*d*e))*(3+m+n)-(-A*f+B*e)*(
b*c*(1+m)+a*d*(1+n))))-(b*c*(1+m)+a*d*(1+n))*(a*f*(A*d*f*(2+m)+B*(d*e*(1+n)
-c*f*(3+m+n)))+b*(B*e*(d*e+c*f*(1+m))+A*f*(c*f*(2+n)-d*e*(4+m+n)))))*(b*x+a
)^(1+m)*(d*x+c)^n*(f*x+e)^(-1-m-n)*hypergeom([-n, 1+m],[2+m],-(-c*f+d*e)*(b
*x+a)/(-a*d+b*c)/(f*x+e))/(-a*f+b*e)^3/(-c*f+d*e)^2/(1+m)/(2+m+n)/(3+m+n)/(
((-a*f+b*e)*(d*x+c)/(-a*d+b*c)/(f*x+e))^n)
```

Rubi [A] time = 0.98, antiderivative size = 558, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {155, 12, 132}

$$(a+bx)^{m+1}(c+dx)^n(e+fx)^{-m-n-1}\left(\frac{(c+dx)(be-af)}{(e+fx)(bc-ad)}\right)^{-n}((m+n+2)(-bde(a(Adf(m+2)-Bcf(m+n+3)+Bde(n$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-4 - m - n),x]
```

```
[Out] ((B*e - A*f)*(a + b*x)^(1 + m)*(c + d*x)^(1 + n)*(e + f*x)^(-3 - m - n))/((
b*e - a*f)*(d*e - c*f)*(3 + m + n)) + ((a*f*(A*d*f*(2 + m) + B*d*e*(1 + n)
- B*c*f*(3 + m + n)) + b*(B*e*(d*e + c*f*(1 + m)) + A*f*(c*f*(2 + n) - d*e*
(4 + m + n))))*(a + b*x)^(1 + m)*(c + d*x)^(1 + n)*(e + f*x)^(-2 - m - n))/
((b*e - a*f)^2*(d*e - c*f)^2*(2 + m + n)*(3 + m + n)) + (((2 + m + n)*(a*b*
c*d*f*(B*e - A*f) - b*d*e*(b*(B*c*e*(1 + m) + A*c*f*(2 + n) - A*d*e*(3 + m
+ n)) + a*(A*d*f*(2 + m) + B*d*e*(1 + n) - B*c*f*(3 + m + n))) + (b*c + a*d
)*f*(b*(B*c*e*(1 + m) + A*c*f*(2 + n) - A*d*e*(3 + m + n)) + a*(A*d*f*(2 +
m) + B*d*e*(1 + n) - B*c*f*(3 + m + n)))) - (b*c*(1 + m) + a*d*(1 + n))*(a*
f*(A*d*f*(2 + m) + B*d*e*(1 + n) - B*c*f*(3 + m + n)) + b*(B*e*(d*e + c*f*(
1 + m)) + A*f*(c*f*(2 + n) - d*e*(4 + m + n)))))*(a + b*x)^(1 + m)*(c + d*x
)^n*(e + f*x)^(-1 - m - n)*Hypergeometric2F1[1 + m, -n, 2 + m, -(((d*e - c*
f)*(a + b*x))/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)^3*(d*e - c*f)^2*(1 +
m)*(2 + m + n)*(3 + m + n)*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*(e + f*x))
)^n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
```

```
p + 2, 0] &&  !IntegerQ[n]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rubi steps

$$\int (a+bx)^m(A+Bx)(c+dx)^n(e+fx)^{-4-m-n}\,dx = \frac{(Be-Af)(a+bx)^{1+m}(c+dx)^{1+n}(e+fx)^{-3-m-n}}{(be-af)(de-cf)(3+m+n)} - \int (a+b \ldots$$

$$= \frac{(Be-Af)(a+bx)^{1+m}(c+dx)^{1+n}(e+fx)^{-3-m-n}}{(be-af)(de-cf)(3+m+n)} + \frac{(af(Ad \ldots}{\ldots}$$

$$= \frac{(Be-Af)(a+bx)^{1+m}(c+dx)^{1+n}(e+fx)^{-3-m-n}}{(be-af)(de-cf)(3+m+n)} + \frac{(af(Ad \ldots}{\ldots}$$

$$= \frac{(Be-Af)(a+bx)^{1+m}(c+dx)^{1+n}(e+fx)^{-3-m-n}}{(be-af)(de-cf)(3+m+n)} + \frac{(af(Ad \ldots}{\ldots}$$

Mathematica [A] time = 1.94, size = 508, normalized size = 0.91

$$-\frac{(a+bx)^{m+1}(c+dx)^n(e+fx)^{-m-n-3}\left(-\frac{(e+fx)^2\left(\frac{(c+dx)(be-af)}{(e+fx)(bc-ad)}\right)^{-n}((m+n+2)(-bde(a(Adf(m+2)-Bcf(m+n+3)+Bde(n+1))+b(Acf(n+ \ldots}{\ldots}\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(A + B*x)*(c + d*x)^n*(e + f*x)^(-4 - m - n),x]
```

```
[Out] -(((a + b*x)^(1 + m)*(c + d*x)^n*(e + f*x)^(-3 - m - n)*(-((B*e - A*f)*(c +
 d*x)) - ((a*f*(A*d*f*(2 + m) + B*d*e*(1 + n) - B*c*f*(3 + m + n)) + b*(B*e
*(d*e + c*f*(1 + m)) + A*f*(c*f*(2 + n) - d*e*(4 + m + n))))*(c + d*x)*(e +
 f*x))/((b*e - a*f)*(d*e - c*f)*(2 + m + n)) - (((2 + m + n)*(a*b*c*d*f*(B*
e - A*f) - b*d*e*(b*(B*c*e*(1 + m) + A*c*f*(2 + n) - A*d*e*(3 + m + n)) + a
*(A*d*f*(2 + m) + B*d*e*(1 + n) - B*c*f*(3 + m + n))) + (b*c + a*d)*f*(b*(B
*c*e*(1 + m) + A*c*f*(2 + n) - A*d*e*(3 + m + n)) + a*(A*d*f*(2 + m) + B*d*
e*(1 + n) - B*c*f*(3 + m + n)))) - (b*c*(1 + m) + a*d*(1 + n))*(a*f*(A*d*f*
(2 + m) + B*d*e*(1 + n) - B*c*f*(3 + m + n)) + b*(B*e*(d*e + c*f*(1 + m)) +
 A*f*(c*f*(2 + n) - d*e*(4 + m + n)))))*(e + f*x)^2*Hypergeometric2F1[1 + m
, -n, 2 + m, ((-(d*e) + c*f)*(a + b*x))/((b*c - a*d)*(e + f*x))])/((b*e - a
*f)^2*(d*e - c*f)*(1 + m)*(2 + m + n)*(((b*e - a*f)*(c + d*x))/((b*c - a*d)
*(e + f*x)))^n)))/((b*e - a*f)*(d*e - c*f)*(3 + m + n)))
```

fricas [F] time = 2.55, size = 0, normalized size = 0.00

$$\text{integral}\left((Bx+A)(bx+a)^m(dx+c)^n\left(fx+e\right)^{-m-n-4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-4-m-n),x, algorithm="fricas
")
```

```
[Out] integral((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (Bx + A)(bx + a)^m (dx + c)^n \left(fx + e\right)^{-m-n-4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-4-m-n),x, algorithm="giac")
```

```
[Out] integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 4), x)
```

maple [F] time = 0.25, size = 0, normalized size = 0.00

$$\int (Bx + A)\,(bx + a)^m\,(dx + c)^n\,\left(fx + e\right)^{-m-n-4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-m-n-4),x)
```

```
[Out] int((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-m-n-4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (Bx + A)(bx + a)^m (dx + c)^n \left(fx + e\right)^{-m-n-4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n*(f*x+e)^(-4-m-n),x, algorithm="maxima
")
```

```
[Out] integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n*(f*x + e)^(-m - n - 4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B\,x)\,(a + b\,x)^m\,(c + d\,x)^n}{\left(e + f\,x\right)^{m+n+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x)*(a + b*x)^m*(c + d*x)^n)/(e + f*x)^(m + n + 4),x)
```

```
[Out] int(((A + B*x)*(a + b*x)^m*(c + d*x)^n)/(e + f*x)^(m + n + 4), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(B*x+A)*(d*x+c)**n*(f*x+e)**(-4-m-n),x)
```

```
[Out] Timed out
```

3.149 $\int \frac{x(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=79

$$-\frac{\sqrt{1-d^2x^2}\left(2\left(3ad^2+2c\right)+3bd^2x\right)}{6d^4}+\frac{b\sin^{-1}(dx)}{2d^3}-\frac{cx^2\sqrt{1-d^2x^2}}{3d^2}$$

```
[Out] 1/2*b*arcsin(d*x)/d^3-1/3*c*x^2*(-d^2*x^2+1)^(1/2)/d^2-1/6*(3*b*d^2*x+6*a*d
^2+4*c)*(-d^2*x^2+1)^(1/2)/d^4
```

Rubi [A] time = 0.14, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1609, 1809, 780, 216}

$$-\frac{\sqrt{1-d^2x^2}\left(2\left(3ad^2+2c\right)+3bd^2x\right)}{6d^4}+\frac{b\sin^{-1}(dx)}{2d^3}-\frac{cx^2\sqrt{1-d^2x^2}}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x + c*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -(c*x^2*Sqrt[1 - d^2*x^2])/(3*d^2) - ((2*(2*c + 3*a*d^2) + 3*b*d^2*x)*Sqrt[
1 - d^2*x^2])/(6*d^4) + (b*ArcSin[d*x])/(2*d^3)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1609

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{x\left(a+bx+cx^2\right)}{\sqrt{1-dx}\,\sqrt{1+dx}}\,dx = \int \frac{x\left(a+bx+cx^2\right)}{\sqrt{1-d^2x^2}}\,dx$$
$$= -\frac{cx^2\sqrt{1-d^2x^2}}{3d^2} - \frac{\int \frac{x\left(-2c-3ad^2-3bd^2x\right)}{\sqrt{1-d^2x^2}}\,dx}{3d^2}$$
$$= -\frac{cx^2\sqrt{1-d^2x^2}}{3d^2} - \frac{\left(2\left(2c+3ad^2\right)+3bd^2x\right)\sqrt{1-d^2x^2}}{6d^4} + \frac{b\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{2d^2}$$
$$= -\frac{cx^2\sqrt{1-d^2x^2}}{3d^2} - \frac{\left(2\left(2c+3ad^2\right)+3bd^2x\right)\sqrt{1-d^2x^2}}{6d^4} + \frac{b\sin^{-1}(dx)}{2d^3}$$

Mathematica [A] time = 0.07, size = 57, normalized size = 0.72

$$\frac{3bd\sin^{-1}(dx) - \sqrt{1-d^2x^2}\left(3d^2(2a+bx)+2c\left(d^2x^2+2\right)\right)}{6d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x + c*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (-(Sqrt[1 - d^2*x^2]*(3*d^2*(2*a + b*x) + 2*c*(2 + d^2*x^2))) + 3*b*d*ArcSi
n[d*x])/(6*d^4)
```

fricas [A] time = 0.79, size = 78, normalized size = 0.99

$$-\frac{6\,bd\arctan\left(\frac{\sqrt{dx+1}\,\sqrt{-dx+1}-1}{dx}\right)+\left(2\,cd^2x^2+3\,bd^2x+6\,ad^2+4\,c\right)\sqrt{dx+1}\,\sqrt{-dx+1}}{6\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/6*(6*b*d*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)) + (2*c*d^2*x^2
 + 3*b*d^2*x + 6*a*d^2 + 4*c)*sqrt(d*x + 1)*sqrt(-d*x + 1))/d^4
```

giac [A] time = 1.34, size = 101, normalized size = 1.28

$$-\frac{\sqrt{dx+1}\,\sqrt{-dx+1}\left((dx+1)\left(\frac{2\,(dx+1)c}{d^3}+\frac{3\,bd^{10}-4\,cd^9}{d^{12}}\right)+\frac{3\left(2\,ad^{11}-bd^{10}+2\,cd^9\right)}{d^{12}}\right)-\frac{6\,b\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{dx+1}\right)}{d^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*(sqrt(d*x + 1)*sqrt(-d*x + 1)*((d*x + 1)*(2*(d*x + 1)*c/d^3 + (3*b*d^1
0 - 4*c*d^9)/d^12) + 3*(2*a*d^11 - b*d^10 + 2*c*d^9)/d^12) - 6*b*arcsin(1/2
*sqrt(2)*sqrt(d*x + 1))/d^2)/d
```

maple [C] time = 0.04, size = 139, normalized size = 1.76

$$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(2\sqrt{-d^2x^2+1}\,c\,d^2x^2\mathrm{csgn}(d)+3\sqrt{-d^2x^2+1}\,b\,d^2x\,\mathrm{csgn}(d)+6\sqrt{-d^2x^2+1}\,a\,d^2\mathrm{csgn}(d)-3\right.}{6\sqrt{-d^2x^2+1}\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] -1/6*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*(2*csgn(d)*x^2*c*d^2*(-d^2*x^2+1)^(1/2)+3
*(-d^2*x^2+1)^(1/2)*csgn(d)*x*b*d^2+6*(-d^2*x^2+1)^(1/2)*csgn(d)*a*d^2+4*(-
d^2*x^2+1)^(1/2)*csgn(d)*c-3*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*b*d)*cs
gn(d)/d^4/(-d^2*x^2+1)^(1/2)
```

maxima [A] time = 0.97, size = 87, normalized size = 1.10

$$-\frac{\sqrt{-d^2x^2+1}\,cx^2}{3\,d^2}-\frac{\sqrt{-d^2x^2+1}\,bx}{2\,d^2}-\frac{\sqrt{-d^2x^2+1}\,a}{d^2}+\frac{b\arcsin(dx)}{2\,d^3}-\frac{2\,\sqrt{-d^2x^2+1}\,c}{3\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/3*sqrt(-d^2*x^2 + 1)*c*x^2/d^2 - 1/2*sqrt(-d^2*x^2 + 1)*b*x/d^2 - sqrt(-
d^2*x^2 + 1)*a/d^2 + 1/2*b*arcsin(d*x)/d^3 - 2/3*sqrt(-d^2*x^2 + 1)*c/d^4
```

mupad [B] time = 7.44, size = 244, normalized size = 3.09

$$-\frac{\sqrt{1-d\,x}\,\left(\frac{a}{d^2}+\frac{a\,x}{d}\right)}{\sqrt{d\,x+1}}-\frac{2\,b\,\mathrm{atan}\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{d^3}-\frac{\frac{14\,b\,{\left(\sqrt{1-d\,x}-1\right)}^3}{{\left(\sqrt{d\,x+1}-1\right)}^3}-\frac{14\,b\,{\left(\sqrt{1-d\,x}-1\right)}^5}{{\left(\sqrt{d\,x+1}-1\right)}^5}+\frac{2\,b\,{\left(\sqrt{1-d\,x}-1\right)}^7}{{\left(\sqrt{d\,x+1}-1\right)}^7}-\frac{2\,b\,\left(\sqrt{1-d\,x}-1\right)}{\sqrt{d\,x+1}-1}}{d^3\,{\left(\frac{{\left(\sqrt{1-d\,x}-1\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+1\right)}^4}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x + c*x^2))/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] - ((1 - d*x)^(1/2)*(a/d^2 + (a*x)/d))/(d*x + 1)^(1/2) - (2*b*atan(((1 - d*x
)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/d^3 - ((14*b*((1 - d*x)^(1/2) - 1)^3)/
((d*x + 1)^(1/2) - 1)^3 - (14*b*((1 - d*x)^(1/2) - 1)^5)/((d*x + 1)^(1/2) -
 1)^5 + (2*b*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7 - (2*b*((1 -
d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1))/(d^3*(((1 - d*x)^(1/2) - 1)^2/((d*x
 + 1)^(1/2) - 1)^2 + 1)^4) - ((1 - d*x)^(1/2)*((2*c)/(3*d^4) + (c*x^3)/(3*d
) + (c*x^2)/(3*d^2) + (2*c*x)/(3*d^3)))/(d*x + 1)^(1/2)
```

sympy [C] time = 82.40, size = 313, normalized size = 3.96

$$-\frac{iaG_{6,6}^{6,2}\left(\begin{matrix}-\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1\\-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}-\frac{aG_{6,6}^{2,6}\left(\begin{matrix}-1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\-\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0\end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}-\frac{ibG_{6,6}^{6,2}\left(\begin{matrix}-\frac{3}{4},\cdots\\-1,-\frac{3}{4},\cdots\end{matrix}\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2+b*x+a)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] -I*a*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0),
()), 1/(d**2*x**2))/(4*pi**(3/2)*d**2) - a*meijerg(((-1, -3/4, -1/2, -1/4,
0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(-2*I*pi)/(d**2*x
**2))/(4*pi**(3/2)*d**2) - I*b*meijerg(((-3/4, -1/4), (-1/2, -1/2, 0, 1)),
((-1, -3/4, -1/2, -1/4, 0, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**3) + b*m
eijerg(((-3/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-3/2, -1, -1,
 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**3) - I*c*meijerg(((-5
/4, -3/4), (-1, -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/4, -1/2, 0), ()), 1/(d*
*2*x**2))/(4*pi**(3/2)*d**4) - c*meijerg(((-2, -7/4, -3/2, -5/4, -1, 1), ()
), ((-7/4, -5/4), (-2, -3/2, -3/2, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*
pi**(3/2)*d**4)
```

3.150 $\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=63

$$\frac{(2ad^2+c)\sin^{-1}(dx)}{2d^3}-\frac{b\sqrt{1-d^2x^2}}{d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}$$

```
[Out] 1/2*(2*a*d^2+c)*arcsin(d*x)/d^3-b*(-d^2*x^2+1)^(1/2)/d^2-1/2*c*x*(-d^2*x^2+
1)^(1/2)/d^2
```

Rubi [A] time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {899, 1815, 641, 216}

$$\frac{(2ad^2+c)\sin^{-1}(dx)}{2d^3}-\frac{b\sqrt{1-d^2x^2}}{d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((b*Sqrt[1 - d^2*x^2])/d^2) - (c*x*Sqrt[1 - d^2*x^2])/(2*d^2) + ((c + 2*a*
d^2)*ArcSin[d*x])/(2*d^3)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 899

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\,\sqrt{1+dx}}\,dx = \int \frac{a+bx+cx^2}{\sqrt{1-d^2x^2}}\,dx$$
$$= -\frac{cx\sqrt{1-d^2x^2}}{2d^2} - \frac{\int \frac{-c-2ad^2-2bd^2x}{\sqrt{1-d^2x^2}}\,dx}{2d^2}$$
$$= -\frac{b\sqrt{1-d^2x^2}}{d^2} - \frac{cx\sqrt{1-d^2x^2}}{2d^2} - \frac{\left(-c-2ad^2\right)\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{2d^2}$$
$$= -\frac{b\sqrt{1-d^2x^2}}{d^2} - \frac{cx\sqrt{1-d^2x^2}}{2d^2} + \frac{\left(c+2ad^2\right)\sin^{-1}(dx)}{2d^3}$$

Mathematica [A] time = 0.04, size = 45, normalized size = 0.71

$$\frac{\left(2ad^2+c\right)\sin^{-1}(dx) - d\sqrt{1-d^2x^2}\,(2b+cx)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (-(d*(2*b + c*x)*Sqrt[1 - d^2*x^2]) + (c + 2*a*d^2)*ArcSin[d*x])/(2*d^3)
```

fricas [A] time = 0.54, size = 67, normalized size = 1.06

$$-\frac{(cdx+2\,bd)\sqrt{dx+1}\,\sqrt{-dx+1} + 2\left(2\,ad^2+c\right)\arctan\left(\frac{\sqrt{dx+1}\,\sqrt{-dx+1}-1}{dx}\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*((c*d*x + 2*b*d)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(2*a*d^2 + c)*arctan
((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/d^3
```

giac [A] time = 1.31, size = 76, normalized size = 1.21

$$-\frac{\sqrt{dx+1}\,\sqrt{-dx+1}\left(\frac{(dx+1)c}{d^2} + \frac{2\,bd^5-cd^4}{d^6}\right) - \frac{2\left(2\,ad^2+c\right)\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{dx+1}\right)}{d^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(d*x + 1)*sqrt(-d*x + 1)*((d*x + 1)*c/d^2 + (2*b*d^5 - c*d^4)/d^6
) - 2*(2*a*d^2 + c)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^2)/d
```

maple [C] time = 0.02, size = 117, normalized size = 1.86

$$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(-2a\,d^2\arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^2x^2+1}}\right) + \sqrt{-d^2x^2+1}\,cdx\,\mathrm{csgn}(d) + 2\sqrt{-d^2x^2+1}\,bd\,\mathrm{csgn}(d) - c\,\mathrm{ar}\right.}{2\sqrt{-d^2x^2+1}\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] -1/2*(-d*x+1)^(1/2)*(d*x+1)^(1/2)/d^3*(csgn(d)*d*(-d^2*x^2+1)^(1/2)*x*c-2*a
rctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))*a*d^2+2*csgn(d)*d*(-d^2*x^2+1)^(1/2
)*b-arctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))*c)/(-d^2*x^2+1)^(1/2)*csgn(d)
```

maxima [A] time = 0.97, size = 57, normalized size = 0.90

$$\frac{a\arcsin(dx)}{d}-\frac{\sqrt{-d^2x^2+1}\,cx}{2\,d^2}-\frac{\sqrt{-d^2x^2+1}\,b}{d^2}+\frac{c\arcsin(dx)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] a*arcsin(d*x)/d - 1/2*sqrt(-d^2*x^2 + 1)*c*x/d^2 - sqrt(-d^2*x^2 + 1)*b/d^2
 + 1/2*c*arcsin(d*x)/d^3
```

mupad [B] time = 6.99, size = 232, normalized size = 3.68

$$-\frac{\sqrt{1-d\,x}\,\left(\frac{b}{d^2}+\frac{b\,x}{d}\right)}{\sqrt{d\,x+1}}-\frac{4\,a\,\mathrm{atan}\left(\frac{d\left(\sqrt{1-d\,x}-1\right)}{\left(\sqrt{d\,x+1}-1\right)\sqrt{d^2}}\right)}{\sqrt{d^2}}-\frac{2\,c\,\mathrm{atan}\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{d^3}-\frac{\frac{14\,c\left(\sqrt{1-d\,x}-1\right)^3}{\left(\sqrt{d\,x+1}-1\right)^3}-\frac{14\,c\left(\sqrt{1-d\,x}-1\right)^5}{\left(\sqrt{d\,x+1}-1\right)^5}+\frac{2\,c\left(\sqrt{1-d\,x}\right.}{\left(\sqrt{d\,x+1}-\right.}}{d^3\left(\frac{\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] - ((1 - d*x)^(1/2)*(b/d^2 + (b*x)/d))/(d*x + 1)^(1/2) - (4*a*atan((d*((1 -
d*x)^(1/2) - 1))/(((d*x + 1)^(1/2) - 1)*(d^2)^(1/2))))/(d^2)^(1/2) - (2*c*a
tan(((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/d^3 - ((14*c*((1 - d*x)^(
1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 - (14*c*((1 - d*x)^(1/2) - 1)^5)/((d*x
 + 1)^(1/2) - 1)^5 + (2*c*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7
- (2*c*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1))/(d^3*(((1 - d*x)^(1/2)
 - 1)^2/((d*x + 1)^(1/2) - 1)^2 + 1)^4)
```

sympy [C] time = 49.79, size = 282, normalized size = 4.48

$$-\frac{iaG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}+\frac{aG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}-\frac{ibG_{6,6}^{6,2}\left(\begin{matrix}-\frac{1}{4},\frac{1}{4} & 0,0\ldots\\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\right.}{4\pi^{\frac{3}{2}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] -I*a*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ())
, 1/(d**2*x**2))/(4*pi**(3/2)*d) + a*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1),
 ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi
**(3/2)*d) - I*b*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/
4, 1/2, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**2) - b*meijerg(((-1, -3/4,
-1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(-2*I
*pi)/(d**2*x**2))/(4*pi**(3/2)*d**2) - I*c*meijerg(((-3/4, -1/4), (-1/2, -1
/2, 0, 1)), ((-1, -3/4, -1/2, -1/4, 0, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)
*d**3) + c*meijerg(((-3/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-
3/2, -1, -1, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**3)
```

3.151 $\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=48

$$-a\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)+\frac{b\sin^{-1}(dx)}{d}-\frac{c\sqrt{1-d^2x^2}}{d^2}$$

```
[Out] b*arcsin(d*x)/d-a*arctanh((-d^2*x^2+1)^(1/2))-c*(-d^2*x^2+1)^(1/2)/d^2
```

Rubi [A] time = 0.18, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {1609, 1809, 844, 216, 266, 63, 208}

$$-a\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)+\frac{b\sin^{-1}(dx)}{d}-\frac{c\sqrt{1-d^2x^2}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(x*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((c*Sqrt[1 - d^2*x^2])/d^2) + (b*ArcSin[d*x])/d - a*ArcTanh[Sqrt[1 - d^2*x
^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1609

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
```

```
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\sqrt{1+dx}}\,dx &= \int \frac{a+bx+cx^2}{x\sqrt{1-d^2x^2}}\,dx \\
&= -\frac{c\sqrt{1-d^2x^2}}{d^2} - \frac{\int \frac{-ad^2-bd^2x}{x\sqrt{1-d^2x^2}}\,dx}{d^2} \\
&= -\frac{c\sqrt{1-d^2x^2}}{d^2} + a\int \frac{1}{x\sqrt{1-d^2x^2}}\,dx + b\int \frac{1}{\sqrt{1-d^2x^2}}\,dx \\
&= -\frac{c\sqrt{1-d^2x^2}}{d^2} + \frac{b\sin^{-1}(dx)}{d} + \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-d^2x}}\,dx, x, x^2\right) \\
&= -\frac{c\sqrt{1-d^2x^2}}{d^2} + \frac{b\sin^{-1}(dx)}{d} - \frac{a\,\text{Subst}\left(\int \frac{1}{\frac{1}{d^2}-\frac{x^2}{d^2}}\,dx, x, \sqrt{1-d^2x^2}\right)}{d^2} \\
&= -\frac{c\sqrt{1-d^2x^2}}{d^2} + \frac{b\sin^{-1}(dx)}{d} - a\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 48, normalized size = 1.00

$$-a\tanh^{-1}\left(\sqrt{1-d^2x^2}\right) + \frac{b\sin^{-1}(dx)}{d} - \frac{c\sqrt{1-d^2x^2}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(x*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((c*Sqrt[1 - d^2*x^2])/d^2) + (b*ArcSin[d*x])/d - a*ArcTanh[Sqrt[1 - d^2*x
^2]]
```

fricas [A] time = 0.61, size = 81, normalized size = 1.69

$$\frac{ad^2\log\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{x}\right) - 2\,bd\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{dx}\right) - \sqrt{dx+1}\sqrt{-dx+1}\,c}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas
")
```

```
[Out] (a*d^2*log((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/x) - 2*b*d*arctan((sqrt(d*x +
 1)*sqrt(-d*x + 1) - 1)/(d*x)) - sqrt(d*x + 1)*sqrt(-d*x + 1)*c)/d^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,-4,0,%%%{4,[2,2]%%%}] at parameters values [70,22
]Warning, choosing root of [1,0,-4,0,%%%{4,[2,2]%%%}] at parameters values
[42,56]-a*ln(abs(2*sqrt(d*x+1)/(-2*sqrt(-d*x+1)+2*sqrt(2))+2-1/2*(-2*sqrt(-
d*x+1)+2*sqrt(2))/sqrt(d*x+1)))+a*ln(abs(2*sqrt(d*x+1)/(-2*sqrt(-d*x+1)+2*s
qrt(2))-2-1/2*(-2*sqrt(-d*x+1)+2*sqrt(2))/sqrt(d*x+1)))-2*b*(-1/2*pi-atan(s
qrt(d*x+1)*((-1/2*(-2*sqrt(-d*x+1)+2*sqrt(2))/sqrt(d*x+1))^2-1)/(-2*sqrt(-d
*x+1)+2*sqrt(2))))/d-2*c*d^2/2/d^4*sqrt(d*x+1)*sqrt(-d*x+1)
```

maple [C] time = 0.03, size = 96, normalized size = 2.00

$$\frac{\left(-a\,d^{2}\operatorname{arctanh}\left(\frac{1}{\sqrt{-d^{2}x^{2}+1}}\right)\operatorname{csgn}(d)+bd\arctan\left(\frac{dx\operatorname{csgn}(d)}{\sqrt{-(dx+1)(dx-1)}}\right)-\sqrt{-d^{2}x^{2}+1}\;c\operatorname{csgn}(d)\right)\sqrt{-dx+1}\;\sqrt{dx+1}}{\sqrt{-d^{2}x^{2}+1}\;d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/x/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] (-csgn(d)*arctanh(1/(-d^2*x^2+1)^(1/2))*a*d^2-(-d^2*x^2+1)^(1/2)*c*csgn(d)+
arctan(csgn(d)*d*x/(-(d*x+1)*(d*x-1))^(1/2))*b*d)*(-d*x+1)^(1/2)*(d*x+1)^(1
/2)/d^2*csgn(d)/(-d^2*x^2+1)^(1/2)
```

maxima [A] time = 0.97, size = 57, normalized size = 1.19

$$-a\log\left(\frac{2\sqrt{-d^{2}x^{2}+1}}{|x|}+\frac{2}{|x|}\right)+\frac{b\arcsin\left(dx\right)}{d}-\frac{\sqrt{-d^{2}x^{2}+1}\,c}{d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima
")
```

```
[Out] -a*log(2*sqrt(-d^2*x^2 + 1)/abs(x) + 2/abs(x)) + b*arcsin(d*x)/d - sqrt(-d^
2*x^2 + 1)*c/d^2
```

mupad [B] time = 3.92, size = 122, normalized size = 2.54

$$a\left(\ln\left(\frac{\left(\sqrt{1-d\,x}-1\right)^{2}}{\left(\sqrt{d\,x+1}-1\right)^{2}}-1\right)-\ln\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)\right)-\frac{\sqrt{1-d\,x}\;\left(\frac{c}{d^{2}}+\frac{c\,x}{d}\right)}{\sqrt{d\,x+1}}-\frac{4\,b\,\mathrm{atan}\left(\frac{d\left(\sqrt{1-d\,x}-1\right)}{\left(\sqrt{d\,x+1}-1\right)\sqrt{d^{2}}}\right)}{\sqrt{d^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/(x*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] a*(log(((1 - d*x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 - 1) - log(((1 - d*x
)^(1/2) - 1)/((d*x + 1)^(1/2) - 1))) - ((1 - d*x)^(1/2)*(c/d^2 + (c*x)/d))/
(d*x + 1)^(1/2) - (4*b*atan((d*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2) - 1
)*(d^2)^(1/2))))/(d^2)^(1/2)
```

sympy [C] time = 55.20, size = 245, normalized size = 5.10

$$\frac{iaG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2} \\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0 \end{matrix} \middle| \frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{aG_{6,6}^{2,6}\left(\begin{matrix} 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0 \end{matrix} \middle| \frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{ibG_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1 \\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix} \middle| \frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}+\frac{bG \ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/x/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] I*a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)),
1/(d**2*x**2))/(4*pi**(3/2)) - a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()), ((
1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi**(3/2))
 - I*b*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), (
)), 1/(d**2*x**2))/(4*pi**(3/2)*d) + b*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1
), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*
pi**(3/2)*d) - I*c*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0,
1/4, 1/2, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**2) - c*meijerg(((-1, -3/4
, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(-2
*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**2)
```

3.152 $\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=48

$$-\frac{a\sqrt{1-d^2x^2}}{x}-b\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)+\frac{c\sin^{-1}(dx)}{d}$$

```
[Out] c*arcsin(d*x)/d-b*arctanh((-d^2*x^2+1)^(1/2))-a*(-d^2*x^2+1)^(1/2)/x
```

Rubi [A] time = 0.18, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {1609, 1807, 844, 216, 266, 63, 208}

$$-\frac{a\sqrt{1-d^2x^2}}{x}-b\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)+\frac{c\sin^{-1}(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(x^2*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((a*Sqrt[1 - d^2*x^2])/x) + (c*ArcSin[d*x])/d - b*ArcTanh[Sqrt[1 - d^2*x^2
]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1609

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
```

```
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\sqrt{1+dx}}\,dx &= \int \frac{a+bx+cx^2}{x^2\sqrt{1-d^2x^2}}\,dx\\
&= -\frac{a\sqrt{1-d^2x^2}}{x} - \int \frac{-b-cx}{x\sqrt{1-d^2x^2}}\,dx\\
&= -\frac{a\sqrt{1-d^2x^2}}{x} + b\int \frac{1}{x\sqrt{1-d^2x^2}}\,dx + c\int \frac{1}{\sqrt{1-d^2x^2}}\,dx\\
&= -\frac{a\sqrt{1-d^2x^2}}{x} + \frac{c\sin^{-1}(dx)}{d} + \frac{1}{2}b\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-d^2x}}\,dx, x, x^2\right)\\
&= -\frac{a\sqrt{1-d^2x^2}}{x} + \frac{c\sin^{-1}(dx)}{d} - \frac{b\,\text{Subst}\left(\int \frac{1}{\frac{1}{d^2}-\frac{x^2}{d^2}}\,dx, x, \sqrt{1-d^2x^2}\right)}{d^2}\\
&= -\frac{a\sqrt{1-d^2x^2}}{x} + \frac{c\sin^{-1}(dx)}{d} - b\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.06, size = 48, normalized size = 1.00

$$-\frac{a\sqrt{1-d^2x^2}}{x} - b\tanh^{-1}\left(\sqrt{1-d^2x^2}\right) + \frac{c\sin^{-1}(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(x^2*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((a*Sqrt[1 - d^2*x^2])/x) + (c*ArcSin[d*x])/d - b*ArcTanh[Sqrt[1 - d^2*x^2
]]
```

fricas [A] time = 0.84, size = 84, normalized size = 1.75

$$\frac{bdx\log\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{x}\right) - \sqrt{dx+1}\sqrt{-dx+1}\,ad - 2\,cx\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{dx}\right)}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] (b*d*x*log((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/x) - sqrt(d*x + 1)*sqrt(-d*x
+ 1)*a*d - 2*c*x*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/(d*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac
")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,-4,0,%%%{4,[2,2]%%%}] at parameters values [70,22
]Warning, choosing root of [1,0,-4,0,%%%{4,[2,2]%%%}] at parameters values
[42,56]1/d*(-2*c*(-1/2*pi-atan(sqrt(d*x+1)*((-1/2*(-2*sqrt(-d*x+1)+2*sqrt(2
))/sqrt(d*x+1))^2-1)/(-2*sqrt(-d*x+1)+2*sqrt(2))))-b*d*ln(abs(2*sqrt(d*x+1)
/(-2*sqrt(-d*x+1)+2*sqrt(2))+2-1/2*(-2*sqrt(-d*x+1)+2*sqrt(2))/sqrt(d*x+1))
)+b*d*ln(abs(2*sqrt(d*x+1)/(-2*sqrt(-d*x+1)+2*sqrt(2))-2-1/2*(-2*sqrt(-d*x+
1)+2*sqrt(2))/sqrt(d*x+1)))-4*a*d^2*(2*sqrt(d*x+1)/(-2*sqrt(-d*x+1)+2*sqrt(
2))-1/2*(-2*sqrt(-d*x+1)+2*sqrt(2))/sqrt(d*x+1))/(-(2*sqrt(d*x+1)/(-2*sqrt(
-d*x+1)+2*sqrt(2))-1/2*(-2*sqrt(-d*x+1)+2*sqrt(2))/sqrt(d*x+1))^2+4))
```

maple [C] time = 0.02, size = 97, normalized size = 2.02

$$\frac{\left(-bdx\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-d^2x^2+1}}\right)\mathrm{csgn}(d)-\sqrt{-d^2x^2+1}\;ad\,\mathrm{csgn}(d)+cx\arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^2x^2+1}}\right)\right)\sqrt{-dx+1}\;\sqrt{dx+1}\;\mathrm{csg}}{\sqrt{-d^2x^2+1}\;dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/x^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] (-csgn(d)*d*arctanh(1/(-d^2*x^2+1)^(1/2))*x*b-csgn(d)*d*(-d^2*x^2+1)^(1/2)*
a+arctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))*x*c)*(-d*x+1)^(1/2)*(d*x+1)^(1/2
)*csgn(d)/(-d^2*x^2+1)^(1/2)/x/d
```

maxima [A] time = 0.97, size = 57, normalized size = 1.19

$$-b\log\left(\frac{2\sqrt{-d^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{c\arcsin(dx)}{d}-\frac{\sqrt{-d^2x^2+1}\,a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -b*log(2*sqrt(-d^2*x^2 + 1)/abs(x) + 2/abs(x)) + c*arcsin(d*x)/d - sqrt(-d^
2*x^2 + 1)*a/x
```

mupad [B] time = 3.74, size = 114, normalized size = 2.38

$$b\left(\ln\left(\frac{\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-1\right)-\ln\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)\right)-\frac{4\,c\,\mathrm{atan}\left(\frac{d\left(\sqrt{1-d\,x}-1\right)}{\left(\sqrt{d\,x+1}-1\right)\sqrt{d^2}}\right)}{\sqrt{d^2}}-\frac{a\,\sqrt{1-d\,x}\,\sqrt{d\,x+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/(x^2*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] b*(log(((1 - d*x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 - 1) - log(((1 - d*x
)^(1/2) - 1)/((d*x + 1)^(1/2) - 1))) - (4*c*atan((d*((1 - d*x)^(1/2) - 1))/
(((d*x + 1)^(1/2) - 1)*(d^2)^(1/2))))/(d^2)^(1/2) - (a*(1 - d*x)^(1/2)*(d*x
 + 1)^(1/2))/x
```

sympy [C] time = 49.85, size = 221, normalized size = 4.60

$$\frac{iadG_{6,6}^{5,3}\left(\begin{matrix} \frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2 \\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0 \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{adG_{6,6}^{2,6}\left(\begin{matrix} \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0 \end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{ibG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2} \\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0 \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{bG_{6}^{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/x**2/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] I*a*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,))
, 1/(d**2*x**2))/(4*pi**(3/2)) + a*d*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1), (
)), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi**(3
/2)) + I*b*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2),
(0,)), 1/(d**2*x**2))/(4*pi**(3/2)) - b*meijerg(((0, 1/4, 1/2, 3/4, 1, 1),
()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi*
*(3/2)) - I*c*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1
, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d) + c*meijerg(((-1/2, -1/4, 0, 1/4,
 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(-2*I*pi)/(d**2*x**
2))/(4*pi**(3/2)*d)
```

3.153 $\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=71

$$-\frac{1}{2}\left(ad^2+2c\right)\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)-\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{b\sqrt{1-d^2x^2}}{x}$$

```
[Out] -1/2*(a*d^2+2*c)*arctanh((-d^2*x^2+1)^(1/2))-1/2*a*(-d^2*x^2+1)^(1/2)/x^2-b
*(-d^2*x^2+1)^(1/2)/x
```

Rubi [A] time = 0.19, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1609, 1807, 807, 266, 63, 208}

$$-\frac{1}{2}\left(ad^2+2c\right)\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)-\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{b\sqrt{1-d^2x^2}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(x^3*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -(a*Sqrt[1 - d^2*x^2])/(2*x^2) - (b*Sqrt[1 - d^2*x^2])/x - ((2*c + a*d^2)*A
rcTanh[Sqrt[1 - d^2*x^2]])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1609

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\sqrt{1+dx}}\,dx &= \int \frac{a+bx+cx^2}{x^3\sqrt{1-d^2x^2}}\,dx\\
&= -\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{1}{2}\int \frac{-2b-\left(2c+ad^2\right)x}{x^2\sqrt{1-d^2x^2}}\,dx\\
&= -\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{b\sqrt{1-d^2x^2}}{x}-\frac{1}{2}\left(-2c-ad^2\right)\int \frac{1}{x\sqrt{1-d^2x^2}}\,dx\\
&= -\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{b\sqrt{1-d^2x^2}}{x}-\frac{1}{4}\left(-2c-ad^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-d^2x}}\,dx,x,x^2\right)\\
&= -\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{b\sqrt{1-d^2x^2}}{x}-\frac{1}{2}\left(a+\frac{2c}{d^2}\right)\text{Subst}\left(\int \frac{1}{\frac{1}{d^2}-\frac{x^2}{d^2}}\,dx,x,\sqrt{1-d^2x^2}\right)\\
&= -\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{b\sqrt{1-d^2x^2}}{x}-\frac{1}{2}\left(2c+ad^2\right)\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 56, normalized size = 0.79

$$
-\frac{\sqrt{1-d^2x^2}\,(a+2bx)}{2x^2}-\frac{1}{2}\left(ad^2+2c\right)\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(x^3*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -1/2*((a + 2*b*x)*Sqrt[1 - d^2*x^2])/x^2 - ((2*c + a*d^2)*ArcTanh[Sqrt[1 -
d^2*x^2]])/2
```

fricas [A] time = 0.70, size = 65, normalized size = 0.92

$$
\frac{\left(ad^2+2\,c\right)x^2\log\left(\frac{\sqrt{dx+1}\,\sqrt{-dx+1}-1}{x}\right)-(2\,bx+a)\sqrt{dx+1}\,\sqrt{-dx+1}}{2\,x^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/2*((a*d^2 + 2*c)*x^2*log((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/x) - (2*b*x +
 a)*sqrt(d*x + 1)*sqrt(-d*x + 1))/x^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac
")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,-4,0,%%%{4,[2,2]%%%}] at parameters values [70,22
]Warning, choosing root of [1,0,-4,0,%%%{4,[2,2]%%%}] at parameters values
[42,56]1/d*(-1/2*(a*d^3+2*c*d)*ln(abs(2*sqrt(d*x+1)/(-2*sqrt(-d*x+1)+2*sqrt
(2))+2-1/2*(-2*sqrt(-d*x+1)+2*sqrt(2))/sqrt(d*x+1)))+1/2*(a*d^3+2*c*d)*ln(a
bs(2*sqrt(d*x+1)/(-2*sqrt(-d*x+1)+2*sqrt(2))-2-1/2*(-2*sqrt(-d*x+1)+2*sqrt(
2))/sqrt(d*x+1)))-(2*a*d^3*(2*sqrt(d*x+1)/(-2*sqrt(-d*x+1)+2*sqrt(2))-1/2*(
-2*sqrt(-d*x+1)+2*sqrt(2))/sqrt(d*x+1))^3-4*b*d^2*(2*sqrt(d*x+1)/(-2*sqrt(-
d*x+1)+2*sqrt(2))-1/2*(-2*sqrt(-d*x+1)+2*sqrt(2))/sqrt(d*x+1))^3+8*a*d^3*(2
*sqrt(d*x+1)/(-2*sqrt(-d*x+1)+2*sqrt(2))-1/2*(-2*sqrt(-d*x+1)+2*sqrt(2))/sq
rt(d*x+1))+16*b*d^2*(2*sqrt(d*x+1)/(-2*sqrt(-d*x+1)+2*sqrt(2))-1/2*(-2*sqrt
(-d*x+1)+2*sqrt(2))/sqrt(d*x+1)))/((2*sqrt(d*x+1)/(-2*sqrt(-d*x+1)+2*sqrt(2
))-1/2*(-2*sqrt(-d*x+1)+2*sqrt(2))/sqrt(d*x+1))^2-4)^2)
```

maple [C] time = 0.02, size = 108, normalized size = 1.52

$$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(a\,d^2x^2\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-d^2x^2+1}}\right)+2c\,x^2\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-d^2x^2+1}}\right)+2\sqrt{-d^2x^2+1}\,bx+\sqrt{-d^2x^2+\cdots}\right.}{2\sqrt{-d^2x^2+1}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/x^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] -1/2*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*csgn(d)^2*(arctanh(1/(-d^2*x^2+1)^(1/2))*
x^2*a*d^2+2*arctanh(1/(-d^2*x^2+1)^(1/2))*x^2*c+2*(-d^2*x^2+1)^(1/2)*x*b+(-
d^2*x^2+1)^(1/2)*a)/(-d^2*x^2+1)^(1/2)/x^2
```

maxima [A] time = 0.97, size = 98, normalized size = 1.38

$$-\frac{1}{2}\,ad^2\log\left(\frac{2\sqrt{-d^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-c\log\left(\frac{2\sqrt{-d^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\frac{\sqrt{-d^2x^2+1}\,b}{x}-\frac{\sqrt{-d^2x^2+1}\,a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -1/2*a*d^2*log(2*sqrt(-d^2*x^2 + 1)/abs(x) + 2/abs(x)) - c*log(2*sqrt(-d^2*
x^2 + 1)/abs(x) + 2/abs(x)) - sqrt(-d^2*x^2 + 1)*b/x - 1/2*sqrt(-d^2*x^2 +
1)*a/x^2
```

mupad [B] time = 5.85, size = 312, normalized size = 4.39

$$c\left(\ln\left(\frac{\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-1\right)-\ln\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)\right)-\frac{\frac{a\,d^2\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-\frac{a\,d^2}{2}+\frac{15\,a\,d^2\left(\sqrt{1-d\,x}-1\right)^4}{2\left(\sqrt{d\,x+1}-1\right)^4}}{\frac{16\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-\frac{32\left(\sqrt{1-d\,x}-1\right)^4}{\left(\sqrt{d\,x+1}-1\right)^4}+\frac{16\left(\sqrt{1-d\,x}-1\right)^6}{\left(\sqrt{d\,x+1}-1\right)^6}}+\frac{a\,d^2\,\ln\left(\frac{\left(\sqrt{1-\cdots}\right.}{\left(\sqrt{d\cdots}\right.}\right.}{2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/(x^3*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] c*(log(((1 - d*x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 - 1) - log(((1 - d*x
)^(1/2) - 1)/((d*x + 1)^(1/2) - 1))) - ((a*d^2*((1 - d*x)^(1/2) - 1)^2)/((d
*x + 1)^(1/2) - 1)^2 - (a*d^2)/2 + (15*a*d^2*((1 - d*x)^(1/2) - 1)^4)/(2*((
d*x + 1)^(1/2) - 1)^4))/((16*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)
^2 - (32*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (16*((1 - d*x)^
```

```
(1/2) - 1)^6)/((d*x + 1)^(1/2) - 1)^6) + (a*d^2*log(((1 - d*x)^(1/2) - 1)^2
/((d*x + 1)^(1/2) - 1)^2 - 1))/2 - (a*d^2*log(((1 - d*x)^(1/2) - 1)/((d*x +
 1)^(1/2) - 1)))/2 - (b*(1 - d*x)^(1/2)*(d*x + 1)^(1/2))/x + (a*d^2*((1 - d
*x)^(1/2) - 1)^2)/(32*((d*x + 1)^(1/2) - 1)^2)
```

sympy [C] time = 80.29, size = 218, normalized size = 3.07

$$\frac{iad^2G_{6,6}^{5,3}\left(\begin{matrix}\frac{7}{4},\frac{9}{4},1 & 2,2,\frac{5}{2}\\ \frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2} & 0\end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{ad^2G_{6,6}^{2,6}\left(\begin{matrix}1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2,1 & \\ \frac{5}{4},\frac{7}{4} & 1,\frac{3}{2},\frac{3}{2},0\end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{ibdG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/x**3/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] I*a*d**2*meijerg(((7/4, 9/4, 1), (2, 2, 5/2)), ((3/2, 7/4, 2, 9/4, 5/2), (0
,)), 1/(d**2*x**2))/(4*pi**(3/2)) - a*d**2*meijerg(((1, 5/4, 3/2, 7/4, 2, 1
), ()), ((5/4, 7/4), (1, 3/2, 3/2, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*
pi**(3/2)) + I*b*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7
/4, 2), (0,)), 1/(d**2*x**2))/(4*pi**(3/2)) + b*d*meijerg(((1/2, 3/4, 1, 5/
4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(-2*I*pi)/(d**2*x**
2))/(4*pi**(3/2)) + I*c*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1
, 5/4, 3/2), (0,)), 1/(d**2*x**2))/(4*pi**(3/2)) - c*meijerg(((0, 1/4, 1/2,
 3/4, 1, 1), ()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(-2*I*pi)/(d**2*
x**2))/(4*pi**(3/2))
```

3.154 $\int \frac{x(a+bx+cx^2)}{\sqrt{-1+dx}\,\sqrt{1+dx}}\,dx$

Optimal. Leaf size=87

$$\frac{\sqrt{dx-1}\,\sqrt{dx+1}\,\left(2\left(3ad^2+2c\right)+3bd^2x\right)}{6d^4}+\frac{b\cosh^{-1}(dx)}{2d^3}+\frac{cx^2\sqrt{dx-1}\,\sqrt{dx+1}}{3d^2}$$

```
[Out] 1/2*b*arccosh(d*x)/d^3+1/3*c*x^2*(d*x-1)^(1/2)*(d*x+1)^(1/2)/d^2+1/6*(3*b*d
^2*x+6*a*d^2+4*c)*(d*x-1)^(1/2)*(d*x+1)^(1/2)/d^4
```

Rubi [A] time = 0.15, antiderivative size = 151, normalized size of antiderivative = 1.74, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1610, 1809, 780, 217, 206}

$$-\frac{\left(1-d^2x^2\right)\left(2\left(3ad^2+2c\right)+3bd^2x\right)}{6d^4\sqrt{dx-1}\,\sqrt{dx+1}}+\frac{b\sqrt{d^2x^2-1}\,\tanh^{-1}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{2d^3\sqrt{dx-1}\,\sqrt{dx+1}}-\frac{cx^2\left(1-d^2x^2\right)}{3d^2\sqrt{dx-1}\,\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x + c*x^2))/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -(c*x^2*(1 - d^2*x^2))/(3*d^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) - ((2*(2*c + 3*
a*d^2) + 3*b*d^2*x)*(1 - d^2*x^2))/(6*d^4*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + (
b*Sqrt[-1 + d^2*x^2]*ArcTanh[(d*x)/Sqrt[-1 + d^2*x^2]])/(2*d^3*Sqrt[-1 + d*
x]*Sqrt[1 + d*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
```

```
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{x\left(a+bx+cx^2\right)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{-1+d^2x^2}\int \frac{x\left(a+bx+cx^2\right)}{\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{cx^2\left(1-d^2x^2\right)}{3d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{x\left(2c+3ad^2+3bd^2x\right)}{\sqrt{-1+d^2x^2}}\,dx}{3d^2\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{cx^2\left(1-d^2x^2\right)}{3d^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{\left(2\left(2c+3ad^2\right)+3bd^2x\right)\left(1-d^2x^2\right)}{6d^4\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(b\sqrt{-1+d^2x^2}\right)\int -}{2d^2\sqrt{-1+dx}\sqrt{}}$$

$$= -\frac{cx^2\left(1-d^2x^2\right)}{3d^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{\left(2\left(2c+3ad^2\right)+3bd^2x\right)\left(1-d^2x^2\right)}{6d^4\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(b\sqrt{-1+d^2x^2}\right)\text{Sub}}{2d^2\sqrt{-}}$$

$$= -\frac{cx^2\left(1-d^2x^2\right)}{3d^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{\left(2\left(2c+3ad^2\right)+3bd^2x\right)\left(1-d^2x^2\right)}{6d^4\sqrt{-1+dx}\sqrt{1+dx}} + \frac{b\sqrt{-1+d^2x^2}\,\tanh}{2d^3\sqrt{-1+dx}}$$

Mathematica [A] time = 0.35, size = 149, normalized size = 1.71

$$\frac{\sqrt{-(dx-1)^2}\sqrt{dx+1}\left(3d^2(2a+bx)+2c\left(d^2x^2+2\right)\right)+6\sqrt{dx-1}\sin^{-1}\left(\frac{\sqrt{1-dx}}{\sqrt{2}}\right)(d(2ad-b)+2c)-12\sqrt{1-dx}\ \text{ta}}{6d^4\sqrt{1-dx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*x + c*x^2))/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (Sqrt[-(-1 + d*x)^2]*Sqrt[1 + d*x]*(3*d^2*(2*a + b*x) + 2*c*(2 + d^2*x^2))
+ 6*(2*c + d*(-b + 2*a*d))*Sqrt[-1 + d*x]*ArcSin[Sqrt[1 - d*x]/Sqrt[2]] - 1
2*(c + d*(-b + a*d))*Sqrt[1 - d*x]*ArcTanh[Sqrt[(-1 + d*x)/(1 + d*x)]])/(6*
d^4*Sqrt[1 - d*x])
```

fricas [A] time = 0.58, size = 73, normalized size = 0.84

$$-\frac{3\,bd\log\left(-dx+\sqrt{dx+1}\,\sqrt{dx-1}\right)-\left(2\,cd^2x^2+3\,bd^2x+6\,ad^2+4\,c\right)\sqrt{dx+1}\,\sqrt{dx-1}}{6\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] -1/6*(3*b*d*log(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)) - (2*c*d^2*x^2 + 3*b*d^
2*x + 6*a*d^2 + 4*c)*sqrt(d*x + 1)*sqrt(d*x - 1))/d^4
```

giac [A] time = 1.30, size = 105, normalized size = 1.21

$$\frac{\sqrt{dx+1}\,\sqrt{dx-1}\left((dx+1)\left(\frac{2\,(dx+1)c}{d^3}+\frac{3\,bd^{10}-4\,cd^9}{d^{12}}\right)+\frac{3\left(2\,ad^{11}-bd^{10}+2\,cd^9\right)}{d^{12}}\right)-\frac{6\,b\log\left(\sqrt{dx+1}-\sqrt{dx-1}\right)}{d^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(sqrt(d*x + 1)*sqrt(d*x - 1)*((d*x + 1)*(2*(d*x + 1)*c/d^3 + (3*b*d^10
- 4*c*d^9)/d^12) + 3*(2*a*d^11 - b*d^10 + 2*c*d^9)/d^12) - 6*b*log(sqrt(d*x
 + 1) - sqrt(d*x - 1))/d^2)/d
```

maple [C] time = 0.03, size = 137, normalized size = 1.57

$$\frac{\sqrt{dx-1}\,\sqrt{dx+1}\,\left(2\sqrt{d^2x^2-1}\,c\,d^2x^2\mathrm{csgn}(d)+3\sqrt{d^2x^2-1}\,b\,d^2x\,\mathrm{csgn}(d)+6\sqrt{d^2x^2-1}\,a\,d^2\mathrm{csgn}(d)+3bd\ln\right.}{6\sqrt{d^2x^2-1}\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] 1/6*(d*x-1)^(1/2)*(d*x+1)^(1/2)*(2*csgn(d)*x^2*c*d^2*(d^2*x^2-1)^(1/2)+3*cs
gn(d)*(d^2*x^2-1)^(1/2)*x*b*d^2+6*csgn(d)*(d^2*x^2-1)^(1/2)*a*d^2+4*csgn(d)
*(d^2*x^2-1)^(1/2)*c+3*ln((csgn(d)*(d^2*x^2-1)^(1/2)+d*x)*csgn(d))*b*d)*csg
n(d)/d^4/(d^2*x^2-1)^(1/2)
```

maxima [A] time = 0.43, size = 100, normalized size = 1.15

$$\frac{\sqrt{d^2x^2-1}\,cx^2}{3\,d^2}+\frac{\sqrt{d^2x^2-1}\,bx}{2\,d^2}+\frac{\sqrt{d^2x^2-1}\,a}{d^2}+\frac{b\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}\,d\right)}{2\,d^3}+\frac{2\,\sqrt{d^2x^2-1}\,c}{3\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] 1/3*sqrt(d^2*x^2 - 1)*c*x^2/d^2 + 1/2*sqrt(d^2*x^2 - 1)*b*x/d^2 + sqrt(d^2*
x^2 - 1)*a/d^2 + 1/2*b*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*d)/d^3 + 2/3*sqrt(
d^2*x^2 - 1)*c/d^4
```

mupad [B] time = 12.35, size = 318, normalized size = 3.66

$$\frac{\sqrt{d\,x-1}\,\left(\frac{2\,c}{3\,d^4}+\frac{c\,x^3}{3\,d}+\frac{c\,x^2}{3\,d^2}+\frac{2\,c\,x}{3\,d^3}\right)}{\sqrt{d\,x+1}}+\frac{2\,b\,\mathrm{atanh}\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)}{d^3}-\frac{\frac{14\,b\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)^3}{\left(\sqrt{d\,x+1}-1\right)^3}+\frac{14\,b\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)^5}{\left(\sqrt{d\,x+1}-1\right)^5}+\frac{2\,b\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)^7}{\left(\sqrt{d\,x+1}-1\right)^7}+}{d^3-\frac{4\,d^3\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+\frac{6\,d^3\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)^4}{\left(\sqrt{d\,x+1}-1\right)^4}-\frac{4\,d^3\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)^6}{\left(\sqrt{d\,x+1}-1\right)^6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x + c*x^2))/((d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] (2*b*atanh(((d*x - 1)^(1/2) - 1i)/((d*x + 1)^(1/2) - 1)))/d^3 - ((14*b*((d*
x - 1)^(1/2) - 1i)^3)/((d*x + 1)^(1/2) - 1)^3 + (14*b*((d*x - 1)^(1/2) - 1i
)^5)/((d*x + 1)^(1/2) - 1)^5 + (2*b*((d*x - 1)^(1/2) - 1i)^7)/((d*x + 1)^(1
/2) - 1)^7 + (2*b*((d*x - 1)^(1/2) - 1i))/((d*x + 1)^(1/2) - 1))/(d^3 - (4*
d^3*((d*x - 1)^(1/2) - 1i)^2)/((d*x + 1)^(1/2) - 1)^2 + (6*d^3*((d*x - 1)^(
1/2) - 1i)^4)/((d*x + 1)^(1/2) - 1)^4 - (4*d^3*((d*x - 1)^(1/2) - 1i)^6)/((
d*x + 1)^(1/2) - 1)^6 + (d^3*((d*x - 1)^(1/2) - 1i)^8)/((d*x + 1)^(1/2) - 1
)^8) + ((d*x - 1)^(1/2)*((2*c)/(3*d^4) + (c*x^3)/(3*d) + (c*x^2)/(3*d^2) +
(2*c*x)/(3*d^3)))/(d*x + 1)^(1/2) + (a*(d*x - 1)^(1/2)*(d*x + 1)^(1/2))/d^2
```

sympy [C] time = 78.83, size = 308, normalized size = 3.54

$$\frac{aG_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1 \\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|{\frac{1}{d^2x^2}}\right)}{4\pi^{\frac{3}{2}}d^2}+\frac{iaG_{6,6}^{2,6}\left(\begin{matrix} -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0 \end{matrix}\middle|{\frac{e^{2i\pi}}{d^2x^2}}\right)}{4\pi^{\frac{3}{2}}d^2}+\frac{bG_{6,6}^{6,2}\left(\begin{matrix} -\frac{3}{4},- \\ -1,-\frac{3}{4},-\frac{1}{2},- \end{matrix}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2+b*x+a)/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] a*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0), ())
, 1/(d**2*x**2))/(4*pi**(3/2)*d**2) + I*a*meijerg(((-1, -3/4, -1/2, -1/4, 0
, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(2*I*pi)/(d**2*x**
2))/(4*pi**(3/2)*d**2) + b*meijerg(((-3/4, -1/4), (-1/2, -1/2, 0, 1)), ((-1
, -3/4, -1/2, -1/4, 0, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**3) - I*b*mei
jerg(((-3/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-3/2, -1, -1, 0
)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**3) + c*meijerg(((-5/4, -
3/4), (-1, -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/4, -1/2, 0), ()), 1/(d**2*x*
*2))/(4*pi**(3/2)*d**4) + I*c*meijerg(((-2, -7/4, -3/2, -5/4, -1, 1), ()),
((-7/4, -5/4), (-2, -3/2, -3/2, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**
(3/2)*d**4)
```

3.155 $\int \frac{a+bx+cx^2}{\sqrt{-1+dx}\,\sqrt{1+dx}}\,dx$

Optimal. Leaf size=52

$$\frac{\left(2ad^2+c\right)\cosh^{-1}(dx)}{2d^3}+\frac{\sqrt{dx-1}\,\sqrt{dx+1}\,(2b+cx)}{2d^2}$$

```
[Out] 1/2*(2*a*d^2+c)*arccosh(d*x)/d^3+1/2*(c*x+2*b)*(d*x-1)^(1/2)*(d*x+1)^(1/2)/
d^2
```

Rubi [B] time = 0.07, antiderivative size = 135, normalized size of antiderivative = 2.60, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {901, 1815, 641, 217, 206}

$$\frac{\sqrt{d^2x^2-1}\left(2ad^2+c\right)\tanh^{-1}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{2d^3\sqrt{dx-1}\,\sqrt{dx+1}}-\frac{b\left(1-d^2x^2\right)}{d^2\sqrt{dx-1}\,\sqrt{dx+1}}-\frac{cx\left(1-d^2x^2\right)}{2d^2\sqrt{dx-1}\,\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((b*(1 - d^2*x^2))/(d^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x])) - (c*x*(1 - d^2*x^2
))/(2*d^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + ((c + 2*a*d^2)*Sqrt[-1 + d^2*x^2]
*ArcTanh[(d*x)/Sqrt[-1 + d^2*x^2]])/(2*d^3*Sqrt[-1 + d*x]*Sqrt[1 + d*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 901

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[((d + e*x)^FracPart[m]*(f + g*x)^Fr
acPart[m])/(d*f + e*g*x^2)^FracPart[m], Int[(d*f + e*g*x^2)^m*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0]
 && EqQ[e*f + d*g, 0]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{-1+d^2x^2}\int \frac{a+bx+cx^2}{\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}}$$
$$= -\frac{cx\left(1-d^2x^2\right)}{2d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{c+2ad^2+2bd^2x}{\sqrt{-1+d^2x^2}}\,dx}{2d^2\sqrt{-1+dx}\sqrt{1+dx}}$$
$$= -\frac{b\left(1-d^2x^2\right)}{d^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{cx\left(1-d^2x^2\right)}{2d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(\left(c+2ad^2\right)\sqrt{-1+d^2x^2}\right)\int \frac{1}{\sqrt{-1+}}}{2d^2\sqrt{-1+dx}\sqrt{1+dx}}$$
$$= -\frac{b\left(1-d^2x^2\right)}{d^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{cx\left(1-d^2x^2\right)}{2d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(\left(c+2ad^2\right)\sqrt{-1+d^2x^2}\right)\text{Subst}\left(\right.}{2d^2\sqrt{-1+dx}\,\sqrt{}}$$
$$= -\frac{b\left(1-d^2x^2\right)}{d^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{cx\left(1-d^2x^2\right)}{2d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(c+2ad^2\right)\sqrt{-1+d^2x^2}\tanh^{-1}\left(\right.}{2d^3\sqrt{-1+dx}\sqrt{1+dx}}$$

Mathematica [B] time = 0.22, size = 126, normalized size = 2.42

$$\frac{4\sqrt{1-dx}\tanh^{-1}\left(\sqrt{\frac{dx-1}{dx+1}}\right)(d(ad-b)+c)+d\sqrt{-(dx-1)^2}\sqrt{dx+1}\,(2b+cx)+2\sqrt{dx-1}\,(2bd-c)\sin^{-1}\left(\frac{\sqrt{1-dx}}{\sqrt{2}}\right)}{2d^3\sqrt{1-dx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (d*(2*b + c*x)*Sqrt[-(-1 + d*x)^2]*Sqrt[1 + d*x] + 2*(-c + 2*b*d)*Sqrt[-1 +
 d*x]*ArcSin[Sqrt[1 - d*x]/Sqrt[2]] + 4*(c + d*(-b + a*d))*Sqrt[1 - d*x]*Ar
cTanh[Sqrt[(-1 + d*x)/(1 + d*x)]])/(2*d^3*Sqrt[1 - d*x])
```

fricas [A] time = 0.76, size = 61, normalized size = 1.17

$$\frac{(cdx+2\,bd)\sqrt{dx+1}\,\sqrt{dx-1}-\left(2\,ad^2+c\right)\log\left(-dx+\sqrt{dx+1}\,\sqrt{dx-1}\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*((c*d*x + 2*b*d)*sqrt(d*x + 1)*sqrt(d*x - 1) - (2*a*d^2 + c)*log(-d*x +
 sqrt(d*x + 1)*sqrt(d*x - 1)))/d^3
```

giac [A] time = 1.25, size = 80, normalized size = 1.54

$$\frac{\sqrt{dx+1}\,\sqrt{dx-1}\left(\frac{(dx+1)c}{d^2}+\frac{2\,bd^5-cd^4}{d^6}\right)-\frac{2\left(2\,ad^2+c\right)\log\left(\sqrt{dx+1}-\sqrt{dx-1}\right)}{d^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(d*x + 1)*sqrt(d*x - 1)*((d*x + 1)*c/d^2 + (2*b*d^5 - c*d^4)/d^6)
- 2*(2*a*d^2 + c)*log(sqrt(d*x + 1) - sqrt(d*x - 1))/d^2)/d
```

maple [C] time = 0.02, size = 120, normalized size = 2.31

$$\frac{\sqrt{dx-1}\,\sqrt{dx+1}\,\left(2a\,d^2\ln\left(\left(dx+\sqrt{d^2x^2-1}\,\operatorname{csgn}(d)\right)\operatorname{csgn}(d)\right)+\sqrt{d^2x^2-1}\,cdx\operatorname{csgn}(d)+2\sqrt{d^2x^2-1}\,bd\operatorname{cs}\right.}{2\sqrt{d^2x^2-1}\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] 1/2*(d*x-1)^(1/2)*(d*x+1)^(1/2)*(csgn(d)*d*(d^2*x^2-1)^(1/2)*x*c+2*ln((d*x+
(d^2*x^2-1)^(1/2)*csgn(d))*csgn(d))*a*d^2+2*csgn(d)*d*(d^2*x^2-1)^(1/2)*b+l
n((d*x+(d^2*x^2-1)^(1/2)*csgn(d))*csgn(d))*c)*csgn(d)/d^3/(d^2*x^2-1)^(1/2)
```

maxima [B] time = 0.43, size = 90, normalized size = 1.73

$$\frac{a\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}\,d\right)}{d}+\frac{\sqrt{d^2x^2-1}\,cx}{2\,d^2}+\frac{\sqrt{d^2x^2-1}\,b}{d^2}+\frac{c\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}\,d\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] a*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*d)/d + 1/2*sqrt(d^2*x^2 - 1)*c*x/d^2 +
sqrt(d^2*x^2 - 1)*b/d^2 + 1/2*c*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*d)/d^3
```

mupad [B] time = 12.40, size = 312, normalized size = 6.00

$$\frac{b\sqrt{d\,x-1}\,\sqrt{d\,x+1}}{d^2}+\frac{2\,c\,\operatorname{atanh}\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)}{d^3}-\frac{4\,a\,\operatorname{atan}\left(\frac{d\left(\sqrt{d\,x-1}-\mathrm{i}\right)}{\left(\sqrt{d\,x+1}-1\right)\sqrt{-d^2}}\right)}{\sqrt{-d^2}}-\frac{\frac{14\,c\left(\sqrt{d\,x-1}-\mathrm{i}\right)^3}{\left(\sqrt{d\,x+1}-1\right)^3}+\frac{14\,c\left(\sqrt{d\,x-1}-\mathrm{i}\right)^5}{\left(\sqrt{d\,x+1}-1\right)^5}+\frac{2}{(}}{d^3-\frac{4\,d^3\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+\frac{6\,d^3\left(\sqrt{d\,x-1}-\mathrm{i}\right)^4}{\left(\sqrt{d\,x+1}-1\right)^4}-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] (2*c*atanh(((d*x - 1)^(1/2) - 1i)/((d*x + 1)^(1/2) - 1)))/d^3 - ((14*c*((d*
x - 1)^(1/2) - 1i)^3)/((d*x + 1)^(1/2) - 1)^3 + (14*c*((d*x - 1)^(1/2) - 1i
)^5)/((d*x + 1)^(1/2) - 1)^5 + (2*c*((d*x - 1)^(1/2) - 1i)^7)/((d*x + 1)^(1
/2) - 1)^7 + (2*c*((d*x - 1)^(1/2) - 1i))/((d*x + 1)^(1/2) - 1))/(d^3 - (4*
d^3*((d*x - 1)^(1/2) - 1i)^2)/((d*x + 1)^(1/2) - 1)^2 + (6*d^3*((d*x - 1)^(
1/2) - 1i)^4)/((d*x + 1)^(1/2) - 1)^4 - (4*d^3*((d*x - 1)^(1/2) - 1i)^6)/((
d*x + 1)^(1/2) - 1)^6 + (d^3*((d*x - 1)^(1/2) - 1i)^8)/((d*x + 1)^(1/2) - 1
)^8) - (4*a*atan((d*((d*x - 1)^(1/2) - 1i))/(((d*x + 1)^(1/2) - 1)*(-d^2)^(
1/2))))/(-d^2)^(1/2) + (b*(d*x - 1)^(1/2)*(d*x + 1)^(1/2))/d^2
```

sympy [C] time = 48.29, size = 277, normalized size = 5.33

$$\frac{aG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}-\frac{iaG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}+\frac{bG_{6,6}^{6,2}\left(\begin{matrix}-\frac{1}{4},\frac{1}{4} & 0,0\\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\right.}{4\pi^{\frac{3}{2}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] a*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), 1
/(d**2*x**2))/(4*pi**(3/2)*d) - I*a*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1),
```

```
()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**
(3/2)*d) + b*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1
/2, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**2) + I*b*meijerg(((-1, -3/4, -1
/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(2*I*pi
)/(d**2*x**2))/(4*pi**(3/2)*d**2) + c*meijerg(((-3/4, -1/4), (-1/2, -1/2, 0
, 1)), ((-1, -3/4, -1/2, -1/4, 0, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**3
) - I*c*meijerg(((-3/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-3/2
, -1, -1, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**3)
```

3.156 $\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\,\sqrt{1+dx}}\,dx$

Optimal. Leaf size=55

$$a\tan^{-1}\left(\sqrt{dx-1}\,\sqrt{dx+1}\right)+\frac{b\cosh^{-1}(dx)}{d}+\frac{c\sqrt{dx-1}\,\sqrt{dx+1}}{d^2}$$

```
[Out] b*arccosh(d*x)/d+a*arctan((d*x-1)^(1/2)*(d*x+1)^(1/2))+c*(d*x-1)^(1/2)*(d*x
+1)^(1/2)/d^2
```

Rubi [B] time = 0.18, antiderivative size = 135, normalized size of antiderivative = 2.45, number of steps used = 8, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1610, 1809, 844, 217, 206, 266, 63, 205}

$$\frac{a\sqrt{d^2x^2-1}\,\tan^{-1}\left(\sqrt{d^2x^2-1}\right)}{\sqrt{dx-1}\,\sqrt{dx+1}}+\frac{b\sqrt{d^2x^2-1}\,\tanh^{-1}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{d\sqrt{dx-1}\,\sqrt{dx+1}}-\frac{c\left(1-d^2x^2\right)}{d^2\sqrt{dx-1}\,\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(x*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((c*(1 - d^2*x^2))/(d^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x])) + (a*Sqrt[-1 + d^2*
x^2]*ArcTan[Sqrt[-1 + d^2*x^2]])/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + (b*Sqrt[-
1 + d^2*x^2]*ArcTanh[(d*x)/Sqrt[-1 + d^2*x^2]])/(d*Sqrt[-1 + d*x]*Sqrt[1 +
d*x])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\,\sqrt{1+dx}}\,dx &= \frac{\sqrt{-1+d^2x^2}\int \frac{a+bx+cx^2}{x\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\,\sqrt{1+dx}}\\
&= -\frac{c\left(1-d^2x^2\right)}{d^2\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{ad^2+bd^2x}{x\sqrt{-1+d^2x^2}}\,dx}{d^2\sqrt{-1+dx}\,\sqrt{1+dx}}\\
&= -\frac{c\left(1-d^2x^2\right)}{d^2\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\left(a\sqrt{-1+d^2x^2}\right)\int \frac{1}{x\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\left(b\sqrt{-1+d^2x^2}\right)\int \frac{1}{\sqrt{-1+}}\cdots}{\sqrt{-1+dx}\,\sqrt{1+d}}\cdots\\
&= -\frac{c\left(1-d^2x^2\right)}{d^2\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\left(a\sqrt{-1+d^2x^2}\right)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{-1+d^2x}}\,dx,x,x^2\right)}{2\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\left(b\sqrt{-1+\right.}\cdots}{}\\
&= -\frac{c\left(1-d^2x^2\right)}{d^2\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{b\sqrt{-1+d^2x^2}\,\tanh^{-1}\left(\frac{dx}{\sqrt{-1+d^2x^2}}\right)}{d\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\left(a\sqrt{-1+d^2x^2}\right)\mathrm{Subs}\cdots}{d^2\sqrt{-1}\cdots}\\
&= -\frac{c\left(1-d^2x^2\right)}{d^2\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{a\sqrt{-1+d^2x^2}\,\tan^{-1}\left(\sqrt{-1+d^2x^2}\right)}{\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{b\sqrt{-1+d^2x^2}\,\tanh^{-}\cdots}{d\sqrt{-1+dx}\,\sqrt{}\cdots}
\end{aligned}
$$

Mathematica [B] time = 0.42, size = 128, normalized size = 2.33

$$
\frac{\frac{ad^2\sqrt{d^2x^2-1}\,\tan^{-1}\left(\sqrt{d^2x^2-1}\right)+cd^2x^2-2c\sqrt{1-d^2x^2}\,\sin^{-1}\left(\frac{\sqrt{1-dx}}{\sqrt{2}}\right)-c}{\sqrt{dx-1}\,\sqrt{dx+1}}-2(c-bd)\tanh^{-1}\left(\sqrt{\frac{dx-1}{dx+1}}\right)}{d^2}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)/(x*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] ((-c + c*d^2*x^2 - 2*c*Sqrt[1 - d^2*x^2]*ArcSin[Sqrt[1 - d*x]/Sqrt[2]] + a*
d^2*Sqrt[-1 + d^2*x^2]*ArcTan[Sqrt[-1 + d^2*x^2]])/(Sqrt[-1 + d*x]*Sqrt[1 +
 d*x]) - 2*(c - b*d)*ArcTanh[Sqrt[(-1 + d*x)/(1 + d*x)]])/d^2
```

fricas [A] time = 0.60, size = 73, normalized size = 1.33

$$\frac{2\,ad^2\arctan\left(-dx+\sqrt{dx+1}\,\sqrt{dx-1}\right)-bd\log\left(-dx+\sqrt{dx+1}\,\sqrt{dx-1}\right)+\sqrt{dx+1}\,\sqrt{dx-1}\,c}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] (2*a*d^2*arctan(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)) - b*d*log(-d*x + sqrt(d
*x + 1)*sqrt(d*x - 1)) + sqrt(d*x + 1)*sqrt(d*x - 1)*c)/d^2
```

giac [A] time = 1.29, size = 71, normalized size = 1.29

$$-2\,a\arctan\left(\frac{1}{2}\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right)-\frac{b\log\left(\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right)}{d}+\frac{\sqrt{dx+1}\,\sqrt{dx-1}\,c}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -2*a*arctan(1/2*(sqrt(d*x + 1) - sqrt(d*x - 1))^2) - b*log((sqrt(d*x + 1) -
 sqrt(d*x - 1))^2)/d + sqrt(d*x + 1)*sqrt(d*x - 1)*c/d^2
```

maple [C] time = 0.02, size = 95, normalized size = 1.73

$$\frac{\left(-a\,d^2\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)\mathrm{csgn}(d)+bd\ln\left(\left(dx+\sqrt{(dx+1)\,(dx-1)}\,\mathrm{csgn}(d)\right)\mathrm{csgn}(d)\right)+\sqrt{d^2x^2-1}\,c\,\mathrm{csgn}(d)\right)}{\sqrt{d^2x^2-1}\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/x/(d*x-1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] (-csgn(d)*arctan(1/(d^2*x^2-1)^(1/2))*a*d^2+(d^2*x^2-1)^(1/2)*c*csgn(d)+ln(
(csgn(d)*((d*x+1)*(d*x-1))^(1/2)+d*x)*csgn(d))*b*d)*(d*x-1)^(1/2)*(d*x+1)^(
1/2)/d^2*csgn(d)/(d^2*x^2-1)^(1/2)
```

maxima [A] time = 0.96, size = 56, normalized size = 1.02

$$-a\arcsin\left(\frac{1}{d|x|}\right)+\frac{b\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}\,d\right)}{d}+\frac{\sqrt{d^2x^2-1}\,c}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -a*arcsin(1/(d*abs(x))) + b*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*d)/d + sqrt(d
^2*x^2 - 1)*c/d^2
```

mupad [B] time = 3.97, size = 118, normalized size = 2.15

$$\frac{c\,\sqrt{d\,x-1}\,\sqrt{d\,x+1}}{d^2}-\frac{4\,b\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)}{\left(\sqrt{d\,x+1}-1\right)\,\sqrt{-d^2}}\right)}{\sqrt{-d^2}}-a\,\left(\ln\left(\frac{\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+1\right)-\ln\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)\right)\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/(x*(d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] (c*(d*x - 1)^(1/2)*(d*x + 1)^(1/2))/d^2 - (4*b*atan((d*((d*x - 1)^(1/2) - 1
i))/(((d*x + 1)^(1/2) - 1)*(-d^2)^(1/2))))/(-d^2)^(1/2) - a*(log(((d*x - 1)
^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + 1) - log(((d*x - 1)^(1/2) - 1i)/((
d*x + 1)^(1/2) - 1)))*1i
```

sympy [C] time = 47.40, size = 240, normalized size = 4.36

$$-\frac{aG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2} \\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0 \end{matrix} \middle| \frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{iaG_{6,6}^{2,6}\left(\begin{matrix} 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0 \end{matrix} \middle| \frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{bG_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1 \\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix} \middle| \frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d} - \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/x/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] -a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)), 1
/(d**2*x**2))/(4*pi**(3/2)) + I*a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()), (
(1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2))
 + b*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ())
, 1/(d**2*x**2))/(4*pi**(3/2)*d) - I*b*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1
), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*p
i**(3/2)*d) + c*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4
, 1/2, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**2) + I*c*meijerg(((-1, -3/4,
 -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(2*I
*pi)/(d**2*x**2))/(4*pi**(3/2)*d**2)
```

3.157 $\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\,\sqrt{1+dx}}\,dx$

Optimal. Leaf size=55

$$\frac{a\sqrt{dx-1}\,\sqrt{dx+1}}{x}+b\tan^{-1}\left(\sqrt{dx-1}\,\sqrt{dx+1}\right)+\frac{c\cosh^{-1}(dx)}{d}$$

```
[Out] c*arccosh(d*x)/d+b*arctan((d*x-1)^(1/2)*(d*x+1)^(1/2))+a*(d*x-1)^(1/2)*(d*x
+1)^(1/2)/x
```

Rubi [B] time = 0.18, antiderivative size = 135, normalized size of antiderivative = 2.45, number of steps used = 8, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1610, 1807, 844, 217, 206, 266, 63, 205}

$$-\frac{a\left(1-d^2x^2\right)}{x\sqrt{dx-1}\,\sqrt{dx+1}}+\frac{b\sqrt{d^2x^2-1}\,\tan^{-1}\left(\sqrt{d^2x^2-1}\right)}{\sqrt{dx-1}\,\sqrt{dx+1}}+\frac{c\sqrt{d^2x^2-1}\,\tanh^{-1}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{d\sqrt{dx-1}\,\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(x^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((a*(1 - d^2*x^2))/(x*Sqrt[-1 + d*x]*Sqrt[1 + d*x])) + (b*Sqrt[-1 + d^2*x^
2]*ArcTan[Sqrt[-1 + d^2*x^2]])/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + (c*Sqrt[-1
+ d^2*x^2]*ArcTanh[(d*x)/Sqrt[-1 + d^2*x^2]])/(d*Sqrt[-1 + d*x]*Sqrt[1 + d*
x])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{-1+d^2x^2}\int \frac{a+bx+cx^2}{x^2\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{b+cx}{x\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(b\sqrt{-1+d^2x^2}\right)\int \frac{1}{x\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(c\sqrt{-1+d^2x^2}\right)\int \frac{1}{\sqrt{-1+\cdots}}}{\sqrt{-1+dx}\sqrt{1+d\cdots}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(b\sqrt{-1+d^2x^2}\right)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{-1+d^2x}}\,dx, x, x^2\right)}{2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(c\sqrt{-1+\cdots}\right.}{\cdots}$$

$$= -\frac{a\left(1-d^2x^2\right)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{c\sqrt{-1+d^2x^2}\tanh^{-1}\left(\frac{dx}{\sqrt{-1+d^2x^2}}\right)}{d\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(b\sqrt{-1+d^2x^2}\right)\mathrm{Subst}\cdots}{d^2\sqrt{-1\cdots}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{b\sqrt{-1+d^2x^2}\tan^{-1}\left(\sqrt{-1+d^2x^2}\right)}{\sqrt{-1+dx}\sqrt{1+dx}} + \frac{c\sqrt{-1+d^2x^2}\tanh^{-1}\cdots}{d\sqrt{-1+dx}\sqrt{\cdots}}$$

Mathematica [A] time = 0.19, size = 89, normalized size = 1.62

$$\frac{a\left(d^2x^2-1\right)+bx\sqrt{d^2x^2-1}\tan^{-1}\left(\sqrt{d^2x^2-1}\right)}{x\sqrt{dx-1}\sqrt{dx+1}} + \frac{2c\tanh^{-1}\left(\sqrt{\frac{dx-1}{dx+1}}\right)}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)/(x^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (a*(-1 + d^2*x^2) + b*x*Sqrt[-1 + d^2*x^2]*ArcTan[Sqrt[-1 + d^2*x^2]])/(x*S
qrt[-1 + d*x]*Sqrt[1 + d*x]) + (2*c*ArcTanh[Sqrt[(-1 + d*x)/(1 + d*x)]])/d
```

fricas [A] time = 0.64, size = 82, normalized size = 1.49

$$\frac{ad^2x + 2\,bdx \arctan\left(-dx + \sqrt{dx+1}\,\sqrt{dx-1}\right) + \sqrt{dx+1}\,\sqrt{dx-1}\,ad - cx\log\left(-dx + \sqrt{dx+1}\,\sqrt{dx-1}\right)}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^2/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="frica
s")
```

```
[Out] (a*d^2*x + 2*b*d*x*arctan(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)) + sqrt(d*x +
1)*sqrt(d*x - 1)*a*d - c*x*log(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)))/(d*x)
```

giac [A] time = 1.35, size = 83, normalized size = 1.51

$$-\frac{2\,bd\arctan\left(\frac{1}{2}\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right) - \frac{8\,ad^2}{\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^4+4} + c\log\left(\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^2/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac"
)
```

```
[Out] -(2*b*d*arctan(1/2*(sqrt(d*x + 1) - sqrt(d*x - 1))^2) - 8*a*d^2/((sqrt(d*x
+ 1) - sqrt(d*x - 1))^4 + 4) + c*log((sqrt(d*x + 1) - sqrt(d*x - 1))^2))/d
```

maple [C] time = 0.02, size = 96, normalized size = 1.75

$$\frac{\left(-bdx\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)\mathrm{csgn}(d) + \sqrt{d^2x^2-1}\;ad\,\mathrm{csgn}(d) + cx\ln\left(\left(dx+\sqrt{d^2x^2-1}\;\mathrm{csgn}(d)\right)\mathrm{csgn}(d)\right)\right)\sqrt{dx-}}{\sqrt{d^2x^2-1}\;dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/x^2/(d*x-1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] (-csgn(d)*d*arctan(1/(d^2*x^2-1)^(1/2))*x*b+csgn(d)*d*(d^2*x^2-1)^(1/2)*a+l
n((d*x+(d^2*x^2-1)^(1/2)*csgn(d))*csgn(d))*x*c)*(d*x-1)^(1/2)*(d*x+1)^(1/2)
*csgn(d)/(d^2*x^2-1)^(1/2)/x/d
```

maxima [A] time = 0.97, size = 56, normalized size = 1.02

$$-b\arcsin\left(\frac{1}{d|x|}\right) + \frac{c\log\left(2\,d^2x + 2\,\sqrt{d^2x^2-1}\,d\right)}{d} + \frac{\sqrt{d^2x^2-1}\,a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^2/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -b*arcsin(1/(d*abs(x))) + c*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*d)/d + sqrt(d
^2*x^2 - 1)*a/x
```

mupad [B] time = 3.86, size = 118, normalized size = 2.15

$$\frac{a\,\sqrt{d\,x-1}\,\sqrt{d\,x+1}}{x} - \frac{4\,c\,\mathrm{atan}\left(\frac{d\left(\sqrt{d\,x-1}-\mathrm{i}\right)}{\left(\sqrt{d\,x+1}-1\right)\sqrt{-d^2}}\right)}{\sqrt{-d^2}} - b\left(\ln\left(\frac{\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+1\right) - \ln\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)\right)1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/(x^2*(d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] (a*(d*x - 1)^(1/2)*(d*x + 1)^(1/2))/x - (4*c*atan((d*((d*x - 1)^(1/2) - 1i)
)/(((d*x + 1)^(1/2) - 1)*(-d^2)^(1/2))))/(-d^2)^(1/2) - b*(log(((d*x - 1)^(
1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + 1) - log(((d*x - 1)^(1/2) - 1i)/((d*
x + 1)^(1/2) - 1)))*1i
```

sympy [C] time = 45.93, size = 216, normalized size = 3.93

$$-\frac{adG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{iadG_{6,6}^{2,6}\left(\begin{matrix}\frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0\end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{bG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2}\\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0\end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{ibG}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/x**2/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] -a*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,)),
 1/(d**2*x**2))/(4*pi**(3/2)) - I*a*d*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1),
()), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3
/2)) - b*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0
,)), 1/(d**2*x**2))/(4*pi**(3/2)) + I*b*meijerg(((0, 1/4, 1/2, 3/4, 1, 1),
()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**
(3/2)) + c*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0
), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d) - I*c*meijerg(((-1/2, -1/4, 0, 1/4,
1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(2*I*pi)/(d**2*x**2)
)/(4*pi**(3/2)*d)
```

3.158 $\int \frac{a+bx+cx^2}{x^3\sqrt{-1+dx}\,\sqrt{1+dx}}\,dx$

Optimal. Leaf size=83

$$\frac{1}{2}\left(ad^2+2c\right)\tan^{-1}\left(\sqrt{dx-1}\,\sqrt{dx+1}\right)+\frac{a\sqrt{dx-1}\,\sqrt{dx+1}}{2x^2}+\frac{b\sqrt{dx-1}\,\sqrt{dx+1}}{x}$$

```
[Out] 1/2*(a*d^2+2*c)*arctan((d*x-1)^(1/2)*(d*x+1)^(1/2))+1/2*a*(d*x-1)^(1/2)*(d*
x+1)^(1/2)/x^2+b*(d*x-1)^(1/2)*(d*x+1)^(1/2)/x
```

Rubi [A] time = 0.19, antiderivative size = 129, normalized size of antiderivative = 1.55, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1610, 1807, 807, 266, 63, 205}

$$\frac{\sqrt{d^2x^2-1}\,\left(ad^2+2c\right)\tan^{-1}\left(\sqrt{d^2x^2-1}\right)}{2\sqrt{dx-1}\,\sqrt{dx+1}}-\frac{a\left(1-d^2x^2\right)}{2x^2\sqrt{dx-1}\,\sqrt{dx+1}}-\frac{b\left(1-d^2x^2\right)}{x\sqrt{dx-1}\,\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(x^3*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -(a*(1 - d^2*x^2))/(2*x^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) - (b*(1 - d^2*x^2))
/(x*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + ((2*c + a*d^2)*Sqrt[-1 + d^2*x^2]*ArcTa
n[Sqrt[-1 + d^2*x^2]])/(2*Sqrt[-1 + d*x]*Sqrt[1 + d*x])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
```

```
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{a+bx+cx^2}{x^3\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{-1+d^2x^2}\int \frac{a+bx+cx^2}{x^3\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{2b+\left(2c+ad^2\right)x}{x^2\sqrt{-1+d^2x^2}}\,dx}{2\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b\left(1-d^2x^2\right)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(\left(2c+ad^2\right)\sqrt{-1+d^2x^2}\right)\int \frac{}{x\sqrt{-}}}{2\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b\left(1-d^2x^2\right)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(\left(2c+ad^2\right)\sqrt{-1+d^2x^2}\right)\text{Subst}}{4\sqrt{-1+dx}\sqrt{1}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b\left(1-d^2x^2\right)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(\left(2c+ad^2\right)\sqrt{-1+d^2x^2}\right)\text{Subst}}{2d^2\sqrt{-1+d}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b\left(1-d^2x^2\right)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(2c+ad^2\right)\sqrt{-1+d^2x^2}\tan^{-1}\left(\right.}{2\sqrt{-1+dx}\sqrt{1+dx}}$$

Mathematica [A] time = 0.14, size = 82, normalized size = 0.99

$$\frac{\left(d^2x^2-1\right)(a+2bx)+x^2\sqrt{d^2x^2-1}\left(ad^2+2c\right)\tan^{-1}\left(\sqrt{d^2x^2-1}\right)}{2x^2\sqrt{dx-1}\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(x^3*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] ((a + 2*b*x)*(-1 + d^2*x^2) + (2*c + a*d^2)*x^2*Sqrt[-1 + d^2*x^2]*ArcTan[S
qrt[-1 + d^2*x^2]])/(2*x^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x])
```

fricas [A] time = 0.59, size = 69, normalized size = 0.83

$$\frac{2\,bdx^2+2\left(ad^2+2\,c\right)x^2\arctan\left(-dx+\sqrt{dx+1}\sqrt{dx-1}\right)+(2\,bx+a)\sqrt{dx+1}\sqrt{dx-1}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^3/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/2*(2*b*d*x^2 + 2*(a*d^2 + 2*c)*x^2*arctan(-d*x + sqrt(d*x + 1)*sqrt(d*x -
 1)) + (2*b*x + a)*sqrt(d*x + 1)*sqrt(d*x - 1))/x^2
```

giac [B] time = 1.35, size = 145, normalized size = 1.75

$$-\frac{\left(ad^3+2\,cd\right)\arctan\left(\frac{1}{2}\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right)+\frac{2\left(ad^3\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^6-4\,bd^2\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^4-4\,ad^3\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2}{\left(\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^4+4\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^3/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac"
)
```

```
[Out] -((a*d^3 + 2*c*d)*arctan(1/2*(sqrt(d*x + 1) - sqrt(d*x - 1))^2) + 2*(a*d^3*
(sqrt(d*x + 1) - sqrt(d*x - 1))^6 - 4*b*d^2*(sqrt(d*x + 1) - sqrt(d*x - 1))
^4 - 4*a*d^3*(sqrt(d*x + 1) - sqrt(d*x - 1))^2 - 16*b*d^2)/((sqrt(d*x + 1)
- sqrt(d*x - 1))^4 + 4)^2)/d
```

maple [C] time = 0.02, size = 103, normalized size = 1.24

$$-\frac{\sqrt{dx-1}\,\sqrt{dx+1}\,\left(a\,d^2x^2\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)+2c\,x^2\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)-2\sqrt{d^2x^2-1}\,bx-\sqrt{d^2x^2-1}\,a\right)\mathrm{csgn}(}{2\sqrt{d^2x^2-1}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/x^3/(d*x-1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] -1/2*(d*x-1)^(1/2)*(d*x+1)^(1/2)*csgn(d)^2*(arctan(1/(d^2*x^2-1)^(1/2))*x^2
*a*d^2+2*arctan(1/(d^2*x^2-1)^(1/2))*x^2*c-2*(d^2*x^2-1)^(1/2)*x*b-(d^2*x^2
-1)^(1/2)*a)/(d^2*x^2-1)^(1/2)/x^2
```

maxima [A] time = 0.97, size = 61, normalized size = 0.73

$$-\frac{1}{2}\,ad^2\arcsin\left(\frac{1}{d|x|}\right)-c\arcsin\left(\frac{1}{d|x|}\right)+\frac{\sqrt{d^2x^2-1}\,b}{x}+\frac{\sqrt{d^2x^2-1}\,a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^3/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -1/2*a*d^2*arcsin(1/(d*abs(x))) - c*arcsin(1/(d*abs(x))) + sqrt(d^2*x^2 - 1
)*b/x + 1/2*sqrt(d^2*x^2 - 1)*a/x^2
```

mupad [B] time = 9.89, size = 316, normalized size = 3.81

$$\frac{\frac{a\,d^2\,1\mathrm{i}}{32}+\frac{a\,d^2\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2 1\mathrm{i}}{16\left(\sqrt{d\,x+1}-1\right)^2}-\frac{a\,d^2\left(\sqrt{d\,x-1}-\mathrm{i}\right)^4 15\mathrm{i}}{32\left(\sqrt{d\,x+1}-1\right)^4}}{\frac{\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+\frac{2\left(\sqrt{d\,x-1}-\mathrm{i}\right)^4}{\left(\sqrt{d\,x+1}-1\right)^4}+\frac{\left(\sqrt{d\,x-1}-\mathrm{i}\right)^6}{\left(\sqrt{d\,x+1}-1\right)^6}}-c\left(\ln\left(\frac{\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+1\right)-\ln\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)\right)1\mathrm{i}-\frac{a\,d^2\ln\Bigg(\frac{(\sqrt{d}}{(\sqrt{d}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/(x^3*(d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] ((a*d^2*1i)/32 + (a*d^2*((d*x - 1)^(1/2) - 1i)^2*1i)/(16*((d*x + 1)^(1/2) -
 1)^2) - (a*d^2*((d*x - 1)^(1/2) - 1i)^4*15i)/(32*((d*x + 1)^(1/2) - 1)^4))
/(((d*x - 1)^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + (2*((d*x - 1)^(1/2) -
```

```
1i)^4)/((d*x + 1)^(1/2) - 1)^4 + ((d*x - 1)^(1/2) - 1i)^6/((d*x + 1)^(1/2)
- 1)^6) - c*(log(((d*x - 1)^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + 1) - lo
g(((d*x - 1)^(1/2) - 1i)/((d*x + 1)^(1/2) - 1)))*1i - (a*d^2*log(((d*x - 1)
^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + 1)*1i)/2 + (a*d^2*log(((d*x - 1)^(
1/2) - 1i)/((d*x + 1)^(1/2) - 1))*1i)/2 + (b*(d*x - 1)^(1/2)*(d*x + 1)^(1/2
))/x + (a*d^2*((d*x - 1)^(1/2) - 1i)^2*1i)/(32*((d*x + 1)^(1/2) - 1)^2)
```

sympy [C] time = 74.80, size = 212, normalized size = 2.55

$$-\frac{ad^2G_{6,6}^{5,3}\left(\begin{matrix}\frac{7}{4},\frac{9}{4},1 & 2,2,\frac{5}{2}\\ \frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2} & 0\end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{iad^2G_{6,6}^{2,6}\left(\begin{matrix}1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2,1 & \\ \frac{5}{4},\frac{7}{4} & 1,\frac{3}{2},\frac{3}{2},0\end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{bdG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/x**3/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] -a*d**2*meijerg(((7/4, 9/4, 1), (2, 2, 5/2)), ((3/2, 7/4, 2, 9/4, 5/2), (0,
)), 1/(d**2*x**2))/(4*pi**(3/2)) + I*a*d**2*meijerg(((1, 5/4, 3/2, 7/4, 2,
1), ()), ((5/4, 7/4), (1, 3/2, 3/2, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*
pi**(3/2)) - b*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4
, 2), (0,)), 1/(d**2*x**2))/(4*pi**(3/2)) - I*b*d*meijerg(((1/2, 3/4, 1, 5/
4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(2*I*pi)/(d**2*x**2
))/(4*pi**(3/2)) - c*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5
/4, 3/2), (0,)), 1/(d**2*x**2))/(4*pi**(3/2)) + I*c*meijerg(((0, 1/4, 1/2,
3/4, 1, 1), ()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/(d**2*x*
*2))/(4*pi**(3/2))
```

3.159 $\int \frac{a+bx+cx^2}{x^4\sqrt{-1+dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=116

$$\frac{\sqrt{dx-1}\sqrt{dx+1}\left(2ad^2+3c\right)}{3x}+\frac{a\sqrt{dx-1}\sqrt{dx+1}}{3x^3}+\frac{1}{2}bd^2\tan^{-1}\left(\sqrt{dx-1}\sqrt{dx+1}\right)+\frac{b\sqrt{dx-1}\sqrt{dx+1}}{2x^2}$$

```
[Out] 1/2*b*d^2*arctan((d*x-1)^(1/2)*(d*x+1)^(1/2))+1/3*a*(d*x-1)^(1/2)*(d*x+1)^(
1/2)/x^3+1/2*b*(d*x-1)^(1/2)*(d*x+1)^(1/2)/x^2+1/3*(2*a*d^2+3*c)*(d*x-1)^(1
/2)*(d*x+1)^(1/2)/x
```

Rubi [A] time = 0.22, antiderivative size = 171, normalized size of antiderivative = 1.47, number of steps used = 7, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {1610, 1807, 835, 807, 266, 63, 205}

$$-\frac{\left(1-d^2x^2\right)\left(2ad^2+3c\right)}{3x\sqrt{dx-1}\sqrt{dx+1}}-\frac{a\left(1-d^2x^2\right)}{3x^3\sqrt{dx-1}\sqrt{dx+1}}-\frac{b\left(1-d^2x^2\right)}{2x^2\sqrt{dx-1}\sqrt{dx+1}}+\frac{bd^2\sqrt{d^2x^2-1}\tan^{-1}\left(\sqrt{d^2x^2-1}\right)}{2\sqrt{dx-1}\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(x^4*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -(a*(1 - d^2*x^2))/(3*x^3*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) - (b*(1 - d^2*x^2))
/(2*x^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) - ((3*c + 2*a*d^2)*(1 - d^2*x^2))/(3*
x*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + (b*d^2*Sqrt[-1 + d^2*x^2]*ArcTan[Sqrt[-1
+ d^2*x^2]])/(2*Sqrt[-1 + d*x]*Sqrt[1 + d*x])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
```

```
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{a+bx+cx^2}{x^4\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{-1+d^2x^2}\int \frac{a+bx+cx^2}{x^4\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{3x^3\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{3b+\left(3c+2ad^2\right)x}{x^3\sqrt{-1+d^2x^2}}\,dx}{3\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{3x^3\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{2\left(3c+2ad^2\right)+3bd^2x}{x^2\sqrt{-1+d^2x^2}}}{6\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{3x^3\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{\left(3c+2ad^2\right)\left(1-d^2x^2\right)}{3x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(bd^2\right.}{}$$

$$= -\frac{a\left(1-d^2x^2\right)}{3x^3\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{\left(3c+2ad^2\right)\left(1-d^2x^2\right)}{3x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(bd^2\right.}{}$$

$$= -\frac{a\left(1-d^2x^2\right)}{3x^3\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{\left(3c+2ad^2\right)\left(1-d^2x^2\right)}{3x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(b\sqrt{\right.}}{}$$

$$= -\frac{a\left(1-d^2x^2\right)}{3x^3\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{\left(3c+2ad^2\right)\left(1-d^2x^2\right)}{3x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{bd^2}{}$$

Mathematica [A] time = 0.13, size = 94, normalized size = 0.81

$$\frac{\left(d^2x^2-1\right)\left(a\left(4d^2x^2+2\right)+3x(b+2cx)\right)+3bd^2x^3\sqrt{d^2x^2-1}\tan^{-1}\left(\sqrt{d^2x^2-1}\right)}{6x^3\sqrt{dx-1}\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(x^4*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] ((-1 + d^2*x^2)*(3*x*(b + 2*c*x) + a*(2 + 4*d^2*x^2)) + 3*b*d^2*x^3*Sqrt[-1
 + d^2*x^2]*ArcTan[Sqrt[-1 + d^2*x^2]])/(6*x^3*Sqrt[-1 + d*x]*Sqrt[1 + d*x]
)
```

fricas [A] time = 0.63, size = 90, normalized size = 0.78

$$\frac{6\,bd^2x^3\arctan\left(-dx+\sqrt{dx+1}\,\sqrt{dx-1}\right)+2\left(2\,ad^3+3\,cd\right)x^3+\left(2\left(2\,ad^2+3\,c\right)x^2+3\,bx+2\,a\right)\sqrt{dx+1}\,\sqrt{dx-\cdots}}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^4/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/6*(6*b*d^2*x^3*arctan(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)) + 2*(2*a*d^3 +
3*c*d)*x^3 + (2*(2*a*d^2 + 3*c)*x^2 + 3*b*x + 2*a)*sqrt(d*x + 1)*sqrt(d*x -
 1))/x^3
```

giac [B] time = 1.35, size = 197, normalized size = 1.70

$$-\frac{3\,bd^3\arctan\left(\frac{1}{2}\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right)+\frac{2\left(3\,bd^3\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^{10}-12\,cd^2\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^{8}-96\,ad^4\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^{4}-96\cdots\right.}{\left(\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^4+\cdots\right.}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^4/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac"
)
```

```
[Out] -1/3*(3*b*d^3*arctan(1/2*(sqrt(d*x + 1) - sqrt(d*x - 1))^2) + 2*(3*b*d^3*(s
qrt(d*x + 1) - sqrt(d*x - 1))^10 - 12*c*d^2*(sqrt(d*x + 1) - sqrt(d*x - 1))
^8 - 96*a*d^4*(sqrt(d*x + 1) - sqrt(d*x - 1))^4 - 96*c*d^2*(sqrt(d*x + 1) -
 sqrt(d*x - 1))^4 - 48*b*d^3*(sqrt(d*x + 1) - sqrt(d*x - 1))^2 - 128*a*d^4
- 192*c*d^2)/((sqrt(d*x + 1) - sqrt(d*x - 1))^4 + 4)^3)/d
```

maple [C] time = 0.02, size = 123, normalized size = 1.06

$$-\frac{\sqrt{dx-1}\,\sqrt{dx+1}\,\left(3b\,d^2x^3\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)-4\sqrt{d^2x^2-1}\,a\,d^2x^2-6\sqrt{d^2x^2-1}\,c\,x^2-3\sqrt{d^2x^2-1}\,bx-2\sqrt{d\cdots}\right.}{6\sqrt{d^2x^2-1}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/x^4/(d*x-1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] -1/6*(d*x-1)^(1/2)*(d*x+1)^(1/2)*csgn(d)^2*(3*arctan(1/(d^2*x^2-1)^(1/2))*x
^3*b*d^2-4*(d^2*x^2-1)^(1/2)*x^2*a*d^2-6*(d^2*x^2-1)^(1/2)*x^2*c-3*(d^2*x^2
-1)^(1/2)*b*x-2*(d^2*x^2-1)^(1/2)*a)/(d^2*x^2-1)^(1/2)/x^3
```

maxima [A] time = 0.98, size = 86, normalized size = 0.74

$$-\frac{1}{2}\,bd^2\arcsin\left(\frac{1}{d|x|}\right)+\frac{2\,\sqrt{d^2x^2-1}\,ad^2}{3\,x}+\frac{\sqrt{d^2x^2-1}\,c}{x}+\frac{\sqrt{d^2x^2-1}\,b}{2\,x^2}+\frac{\sqrt{d^2x^2-1}\,a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^4/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -1/2*b*d^2*arcsin(1/(d*abs(x))) + 2/3*sqrt(d^2*x^2 - 1)*a*d^2/x + sqrt(d^2*
x^2 - 1)*c/x + 1/2*sqrt(d^2*x^2 - 1)*b/x^2 + 1/3*sqrt(d^2*x^2 - 1)*a/x^3
```

mupad [B] time = 9.44, size = 304, normalized size = 2.62

$$\frac{\frac{b\,d^2\,1\mathrm{i}}{32}+\frac{b\,d^2\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2\,1\mathrm{i}}{16\,{\left(\sqrt{d\,x+1}-1\right)}^2}-\frac{b\,d^2\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^4\,15\mathrm{i}}{32\,{\left(\sqrt{d\,x+1}-1\right)}^4}}{\frac{{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+\frac{2\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^4}{{\left(\sqrt{d\,x+1}-1\right)}^4}+\frac{{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^6}{{\left(\sqrt{d\,x+1}-1\right)}^6}}-\frac{b\,d^2\,\ln\left(\frac{{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+1\right)\,1\mathrm{i}}{2}+\frac{b\,d^2\,\ln\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)\,1\mathrm{i}}{2}+\frac{c\,\sqrt{d\,x-1}\,\sqrt{d\,x-}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/(x^4*(d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] ((b*d^2*1i)/32 + (b*d^2*((d*x - 1)^(1/2) - 1i)^2*1i)/(16*((d*x + 1)^(1/2) -
 1)^2) - (b*d^2*((d*x - 1)^(1/2) - 1i)^4*15i)/(32*((d*x + 1)^(1/2) - 1)^4))
/(((d*x - 1)^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + (2*((d*x - 1)^(1/2) -
1i)^4)/((d*x + 1)^(1/2) - 1)^4 + ((d*x - 1)^(1/2) - 1i)^6/((d*x + 1)^(1/2)
- 1)^6) - (b*d^2*log(((d*x - 1)^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + 1)*
1i)/2 + (b*d^2*log(((d*x - 1)^(1/2) - 1i)/((d*x + 1)^(1/2) - 1))*1i)/2 + (c
*(d*x - 1)^(1/2)*(d*x + 1)^(1/2))/x + ((d*x - 1)^(1/2)*(a/3 + (2*a*d^2*x^2)
/3 + (2*a*d^3*x^3)/3 + (a*d*x)/3))/(x^3*(d*x + 1)^(1/2)) + (b*d^2*((d*x - 1
)^(1/2) - 1i)^2*1i)/(32*((d*x + 1)^(1/2) - 1)^2)
```

sympy [C] time = 129.78, size = 219, normalized size = 1.89

$$-\frac{ad^3G_{6,6}^{5,3}\left(\begin{matrix}\frac{9}{4},\frac{11}{4},1 & \frac{5}{2},\frac{5}{2},3\\ 2,\frac{9}{4},\frac{5}{2},\frac{11}{4},3 & 0\end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{iad^3G_{6,6}^{2,6}\left(\begin{matrix}\frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2},1 & \\ \frac{7}{4},\frac{9}{4} & \frac{3}{2},2,2,0\end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{bd^2G_{6,6}^{5,3}\left(\begin{matrix}\frac{7}{4},\frac{9}{4},1 & 2,2,\frac{5}{2}\\ \frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2} & 0\end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/x**4/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] -a*d**3*meijerg(((9/4, 11/4, 1), (5/2, 5/2, 3)), ((2, 9/4, 5/2, 11/4, 3), (
0,)), 1/(d**2*x**2))/(4*pi**(3/2)) - I*a*d**3*meijerg(((3/2, 7/4, 2, 9/4, 5
/2, 1), ()), ((7/4, 9/4), (3/2, 2, 2, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(
4*pi**(3/2)) - b*d**2*meijerg(((7/4, 9/4, 1), (2, 2, 5/2)), ((3/2, 7/4, 2,
9/4, 5/2), (0,)), 1/(d**2*x**2))/(4*pi**(3/2)) + I*b*d**2*meijerg(((1, 5/4,
 3/2, 7/4, 2, 1), ()), ((5/4, 7/4), (1, 3/2, 3/2, 0)), exp_polar(2*I*pi)/(d
**2*x**2))/(4*pi**(3/2)) - c*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1,
 5/4, 3/2, 7/4, 2), (0,)), 1/(d**2*x**2))/(4*pi**(3/2)) - I*c*d*meijerg(((1
/2, 3/4, 1, 5/4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(2*I*
pi)/(d**2*x**2))/(4*pi**(3/2))
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
   hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```